ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

DYNAMIC ANALYSIS OF LARGE IN-SPACE DEPLOYABLE MEMBRANE ANTENNAS

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Abstract

This paper presents a vibration analysis of an eight-meter diameter membrane reflectarray antenna, which is composed of a thin membrane and a deployable frame. The analysis process has two main steps. In the first step, a two-variable-parameter (2-VP) membrane model is developed to determine the in-plane stress distribution due to pre-tensioning, which eventually yields the differential stiffness of the membrane. In the second step, the membrane differential stiffness is incorporated in a dynamic equation governing the transverse vibration of the membrane-frame assembly, in a synthesis that is facilitated by the Distributed Transfer Function Method (DTFM). This eventually produces the natural frequencies and mode shapes of the antenna. The combination of the 2-VP model and the DTFM synthesis provides an accurate prediction of the in-plane stress distribution and modes of vibration for the antenna.

1. INTRODUCTION

Deployable telecommunication reflectarray antennas are being developed at the Jet Propulsion Laboratory for future space missions. The major components of a membrane antenna are one or several layers of thin-membrane pre-tensioned by a space deployable frame. Advantages of using thin-film membranes in such an antenna design include ultra lightweight and small packaging volume for large deployed apertures in operation. Inspired by recent successes in developing membrane antennas, which include several X-, Ku-, and Ka-band reflectarray antennas [1, 2], space science missions that will employ multiple-band reflectarray antennas with aperture sizes over 8 meters are being considered.

This paper discusses a vibration analysis process developed for analyzing an eight-meter diameter membrane antenna as shown in Figure 1. The membrane aperture of the antenna has very little out-of-plane bending stiffness. The out-of-plane stiffness of this membrane aperture comes from pre-tensioning. This stiffness, called the differential stiffness, is a function of the membrane stress distribution. Therefore, the dynamic analysis of a piece of membrane has two steps. The first step is static analysis to obtain the stress distribution due to pre-tensioning and the second step is modal analysis of the tensioned membrane with differential stiffness incorporated.

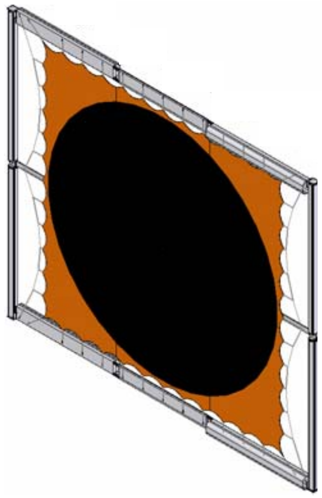

Figure 1 – Design drawing of the eight-meter diameter membrane antenna

A thin membrane, because of its low resistance to bending, is easy to wrinkle. This renders the stress distribution and dynamic behaviors of the membrane sensitive to loading and boundary conditions, which in turn imposes challenging problems in modeling and analysis. Using the finite element method to analyze the stress distribution on a piece of thin-membrane is very problematic because wrinkles are usually introduced by uneven stress distribution. Wrinkles cause numerical instability and prevent successful numerical convergence. In order to resolve this problem, an innovative analytical methodology, namely two-variable-parameter (2-VP) method, has been developed for calculating the differential stiffness on a pre-tensioned thin-membrane. Another analytical method called Distributed Transfer Function Method (DTFM) is then used to assemble the membrane with other antenna components and perform the modal analysis.

2. MODELING AND VIBRATION ANALYSIS

The antenna in consideration is a deployable frame with a mounted thin-film membrane, as shown in Fig. 1. In modeling and analysis of such a structure, the frame is described by the DTFM [3, 4]; the membrane is described by 2-VP model [5-7] that systematically characterizes taut, wrinkled and slack states of the membrane. Assembly of the frame, as well as the membrane, leads to a dynamic equation and the solution of which gives the natural frequencies and mode shapes of the antenna structure.

2.1 Frame Members

The deployable frame is modeled as an assemblage of members rigidly connected at nodes. By the DTFM [4], the dynamic response of a frame member in three-dimensional free vibration can be described by the dynamic equilibrium equation

$$\begin{pmatrix} p_i \\ p_j \end{pmatrix} = K_m(s) \begin{pmatrix} \alpha_i \\ \alpha_j \end{pmatrix} \tag{1}$$

where α_i and p_i are the vectors of nodal displacements and forces of the membrane at node *i*, $K_m(s)$ is the dynamic stiffness matrix, and *s* is the complex parameter that comes from Laplace transform with respect to time. For *s* = 0, $K_m(0)$ gives a stiffness matrix of the boom member. Unlike finite element modeling, $K_m(s)$ obtained herein is always of exact and closed form [3].

2.2 Membrane Model—Two Variable Parameter Method

In this study, the membrane of the deployable antenna is modeled as a thin plate with in-plane displacements *u*, *v* and transverse deflection *w*. It is assumed that the membrane does not resist negative in-plane principal stresses and that in vibration the in-plane membrane stresses remain unchanged [7]. With these assumptions, the plane deformation of the membrane satisfies the equilibrium equations

$$\frac{\partial}{\partial x}\sigma_x + \frac{\partial}{\partial y}\tau_{xy} = 0, \quad \frac{\partial}{\partial y}\sigma_y + \frac{\partial}{\partial x}\tau_{xy} = 0 \tag{2}$$

and the constraint conditions

$$\sigma_1 \geq 0, \quad \sigma_2 \geq 0 \tag{3}$$

where σ_1, σ_2 are the principal stresses of the membrane; the transverse vibration of the membrane is governed by

$$D\nabla^4 w + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + \rho \frac{\partial^2 w}{\partial t^2} = 0 \tag{4}$$

where ρ and *D* are the mass density (mass per unit area) and flexural rigidity of the plate, respectively; and N_x, N_y, N_{xy} are the internal forces of the membrane that are related to the membrane in-plane stresses by $N_x = \sigma_x h, \quad N_y = \sigma_y h, \quad N_{xy} = \sigma_{xy} h$.

Solution of the free vibration problem for a wrinkled membrane takes two steps:

Step 1. Solve the wrinkling problem of the membrane defined by Eq. (2) with conditions (4), for the in-plan stresses. This is done through the use of 2-VP membrane model [7], which describes the stress-strain relation by a constitutive matrix $\left[\tilde{D}(\lambda_1, \lambda_2)\right]$ that is a function of nonnegative control parameters λ_1 and λ_2. As shown in [6], the control parameters are such that they can be adjusted to characterize three states of the membrane:

(S1) Taut state $\lambda_1 = 0, \quad \lambda_2 = 0$ (5a)

(S2) Wrinkled state $\lambda_1 = 0, \quad \lambda_2 > 0$ (5b)

(S3) Slack state $\lambda_1 > 0, \quad \lambda_2 > 0$ (5c)

With $\left[\tilde{D}(\lambda_1,\lambda_2)\right]$ and a parametric variational principle [6, 7], the wrinkling problem is converted to an equivalent nonlinear complementary problem (NCP) in optimization theory. Solution of the NCP yields the in-plane stresses of the membrane. One advantage of the 2-VP model with NCP in wrinkling analysis is that it guarantees convergent solutions, without need for stress iteration.

Step 2. Determine the transverse vibration of the membrane. With the membrane in-plane stresses obtained in Step 1, the internal forces (N_x, N_y, N_{xy}) of the membrane are known. Thus, a finite element discretization of Eq. (4) yields

$$M_m \frac{d^2}{dt^2}W + \left(K_b + K_g\right)W = 0 \tag{6}$$

Where, W is a vector of nodal displacements by which the transverse displacement $w(x,t)$ of the membrane is interpolated, M_m is the effective mass matrix of the membrane, K_b is a bending stiffness matrix, and K_g is a differential (or geometric) stiffness matrix characterizing the internal forces N_x, N_y, N_{xy}. Solution of Eq. (6) gives the natural frequencies and mode shapes of the wrinkled membrane.

2.3 Assembly of Antenna Structure

The membrane is mounted onto the frame with catenaries and springs, as illustrated in Fig. 2, where d_1, d_2 and d_3 are spaces that set up tension forces in catenary and springs (for the membrane antenna considered in Fig. 1, more space parameters can be introduced). The space parameters are adjusted such that the membrane is evenly loaded along its boundary, and has the desired normal stresses in its central area. The catenary cables are under tension and viewed as elastic bars.

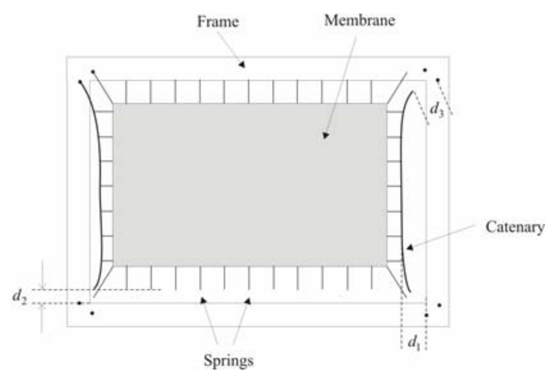

Figure 2 – Schematic of frame-membrane assembly

Assume that the membrane antenna in *static equilibrium* only experience in-plane deformations. As such, the equilibrium equations for the antenna are:

Member
$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} u_{c1} \\ v_1 \end{Bmatrix} = \begin{Bmatrix} f_c \\ 0 \end{Bmatrix} \tag{7a}$$

Frame with catenary
$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} u_{c2} \\ v_2 \end{Bmatrix} = \begin{Bmatrix} -f_c \\ 0 \end{Bmatrix} \tag{7b}$$

where A_{ij} are the membrane stiffness matrices that are obtained through use of the 2-VP mentioned in Section 2.2; B_{ij} are the stiffness matrices of the frame-catenary substructure that are obtained by the DTFM with s = 0; u_{c1} and u_{c2} are vectors of displacements that are involved in the coupling between the membrane and frame; and v_1 and v_2 are vectors of displacements that are not directly involved in the coupling. The f_c is a vector of elastic coupling forces, and is of the form

$$f_c = K_c \left(u_{c2} + d_c - u_{c1} \right) \tag{8}$$

where K_c is a matrix consisting of the coefficients of the springs that connect the membrane to the frame or catenary, and d_c is a vector of space parameters as illustrated in Fig. 2. Substituting Eq. (8) into Eqs. (7) gives

$$\begin{bmatrix} A_{11}+K_c & -K_c & A_{12} & 0 \\ -K_c & B_{11}+K_c & 0 & B_{12} \\ A_{21} & 0 & A_{22} & 0 \\ 0 & B_{21} & 0 & B_{22} \end{bmatrix} \begin{Bmatrix} u_{c1} \\ u_{c2} \\ v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} K_c d_c \\ -K_c d_c \\ 0 \\ 0 \end{Bmatrix} \tag{9}$$

from which, the membrane in-plane deformation and the differential stiffness matrix K_g can be computed. As can be seen from Eq. (9), the spaces serve as external loads.

Now consider small vibration of the antenna from its equilibrium configuration. For the membrane, its transverse vibration is governed by Eq. (6); its in-plane vibration is described by

$$M_A \frac{d^2}{dt^2} \begin{Bmatrix} \Delta u_{c1} \\ \Delta v_1 \end{Bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \Delta u_{c1} \\ \Delta v_1 \end{Bmatrix} = \begin{Bmatrix} q_c \\ 0 \end{Bmatrix} \tag{10}$$

and for the frame, its three-dimensional motion is governed by

$$M_B \frac{d^2}{dt^2} \begin{Bmatrix} u_{c2} \\ v_2 \\ w_2 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} \Delta u_{c2} \\ \Delta v_2 \\ w_2 \end{Bmatrix} = \begin{Bmatrix} -q_c \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

Here Δu_{c1}, Δu_{c2}, Δv_1 and Δv_2 are perturbations of u_{c1}, u_{c2}, v_1 and v_2, respectively;

w_2 is a vector of out-plane displacements of the frame; M_A and M_B are the inertia matrices; and q_c is a vector of coupling forces given by

$$q_c = K_c\left(\Delta u_{c2} - \Delta u_{c1}\right). \tag{12}$$

According to the above discussion, the free vibration of the membrane antenna takes the following two steps. First, solve Eq. (9) to determine the differential stiffness matrix K_g. Second, solve Eqs. (6), (10), (11) and (12) for the natural frequencies and mode shapes of the structure. Note that the transverse vibration and in-plane deformation of the membrane are not coupled. This is because the geometric nonlinearity of the membrane is neglected in the current linear vibration analysis. The three-dimensional motion of the frame, on the other hand, is coupled with the in-plane motion of the membrane.

3. NUMERICAL EXAMPLES

In order to verify the aforementioned membrane modeling method, a scaled engineering model of the reflectarray antenna membrane has been assembled to experimentally determine its natural frequencies. Figure 3 shows the test set up. This model is a 1.5-meter by 1.5-meter square Liquid Crystal Polymer (LCP) membrane with copper coating on one side. The catenaries are assembled to the four membrane edges to evenly tension the membrane [8]. The catenaries are tensioned by constant force springs, as shown in Fig. 3, to assure that the membrane stress is kept at the design level without being affected by the environment, such as temperature changes.

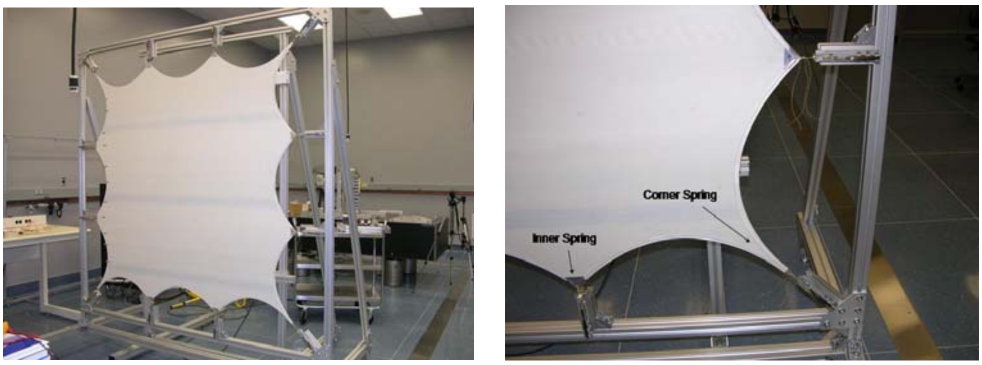

Figure 3 – Test setup(left),close up at tensioning springs (right)

The frame was assembled using high-stiffness/light-weight BOSCH aluminium extrusions to separate the frame modes from the membrane modes. A piece of membrane is very flimsy, non-contact excitation and measuring devices should be used to assure that pure membrane modes are measured. This test used a speaker to excite the membrane acoustically and a non-contact laser vibrometer to acquire the response modes and modal frequencies. Fig. 4 shows the first three mode shapes that were experimentally obtained. Table 1 gives modal frequencies obtained by test, Finite Element Analysis (FEA) and 2-VP method.

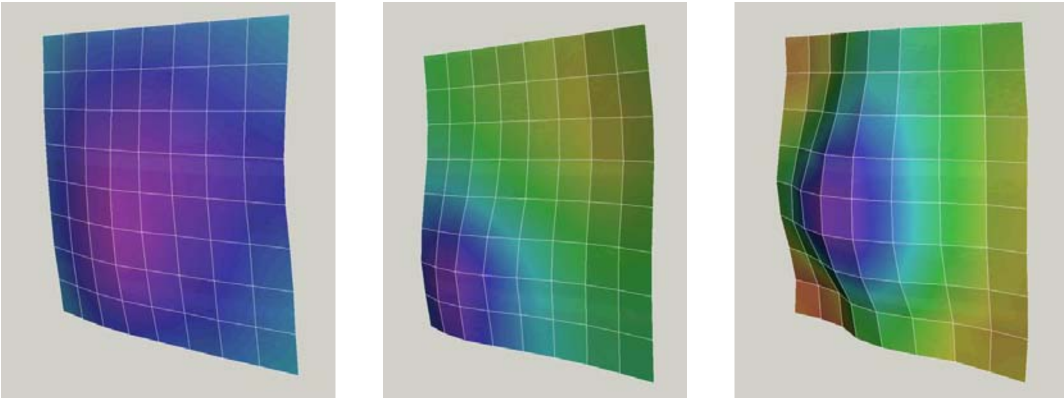

Figure 4 – First mode shape (left), second mode shape (middle), third mode shape (right)

Table 1 – model frequencies

Frequency	Test (Hz)	FEA (Hz)	2-VP (Hz)
f_1	2.44	2.43	2.24
f_2	4.75	4.49	4.21
f_3	7.44	--	7.04

Mode shapes acquired from 2-VP and FEA matched very well with tested mode shapes. Therefore, the 2-VP membrane analysis method has been verified to be suitable for analyzing membrane space structures.

4. CONCLUSIONS

Membrane reflectarrays and other membrane space structures are being developed for future space missions. Since a piece of membrane wrinkles when it is unevenly tensioned, modelling and analysis of membrane space structures remains a challenging problem. This paper presented the 2-VP model for membrane analysis. The 2-VP model can systematically characterize taut, wrinkled and slack states of the membrane, and in numerical simulation guarantees convergent solutions without the need for stress iteration. This paper also presents a DTFM synthesis for analysis of a space structure as an assembly of deployable frame components and thin-film membranes. The synthesis, which makes use of exact and closed-form transfer functions of frame components, offers high computational efficiency and precision. A sub-scale membrane system of 1.5-m by 1.5-m has been assembled for experimental study and the predictions by the 2-VP model have been correlated to the test data. The results obtained from this study indicate that the 2-VP model and the DTFM synthesis are promising analysis tools that are worthy of further development for design of future membrane space structures.

5. ACKNOLODGEMENTS

The authors wish to thank Drs. James Lesh, Faramaz Davarian, and Richard Lovick of JPL for their encouragement, guidance, and support. The authors also want to acknowledge Jose Perez, Kevin Knarr, Sam Szeto, and Julieta Morales for their contributions on both experimental and analytical aspects of this study.

The work described was performed at the Jet Propulsion Laboratory, California Institute of Technology and the University of Southern California, under a contract with the National Aeronautics and Space Administration.

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