ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

ACOUSTIC AND STRONGER PRESSURE WAVE INTERACTIONS WITH FLAMES

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Abstract

Pressure wave interactions with combustion has been a subject of a rich pedigree, beginning with Lord Rayleigh's work in the nineteenth century and the known relevance of these effects in gas turbines and combustors where catastrophic failures can occur due to resonance. In this paper some of the fundamental length and time scales in pressure wave interactions with premixed flames are discussed. These basic relationships are fundamental in understanding the essentials of pressure wave interactions with combustion. There are four main types of interactions - long wavelength disturbances where the flame is essentially passive to the pressure wave oscillations. Such frequencies would typically be in the 1 – 10 Hz range. Very long wavelength type disturbances (typically 0.1 – 1 Hz) would be a special case where the flame is a contact discontinuity. The very common type of interactions are for higher frequency (hundreds of Hz) pressure oscillations and in this case, though the flame movement is still essentially passive to the pressure changes, there can be changes in the energy fluctuations at the flame which then cause Rayleigh's criterion to be obeyed and energy to be added to the acoustic disturbance leading to resonance. For even higher frequencies still (the last region of interactions – KHz to MHz), the pressure wave disturbances can at this level severely disturb the premixed flame since the pressure disturbance length scale is on a par with the flame thickness, and the interactions can then such as to single out a chemically derived resonant high frequency. Some systems are considered where these basic principles are applied, and theoretical transfer functions for predicting the resonance characteristics for combustion devices are discussed.

INTRODUCTION

For a pressure wave interacting with a premixed flame (figure 1), analysis has identified key length and time ratios in flame–pressure interactions [1-4], defined as follows:

$$\tau \equiv \frac{\text{diffusion time}}{\text{acoustic time}}; \quad N \equiv \frac{\text{characteristic length of pressure disturbance}}{\text{diffusion length}} , \tag{1a,b}$$

and which characterize each type of interaction. By defining the Mach number of flame propagation, i.e.

$$M \equiv \frac{u_0'}{a_0'} , \tag{2}$$

(where u_0' is the initial burning velocity and a_0' the sound speed), it follows that

$$\tau \equiv \frac{1}{NM} , \tag{3}$$

If the flame is characterized by one overall Arrhenius reaction with a non-dimensional activation energy defined as

$$\theta \equiv \frac{E_{A'}'}{R'T_b'} , \tag{4}$$

(where $E_{A'}'$ is the dimensional energy, R' is the universal gas constant and T_b' is the steady burnt temperature), then the characteristic length-scale of the pressure disturbances determines four distinct cases of pressure–premixed flame interaction.

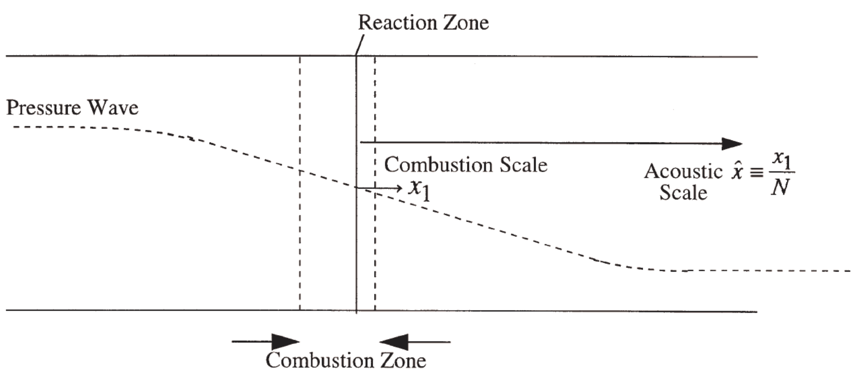

Figure 1 – Typical length- and time-scales for pressure interactions with premixed combustion fronts.

(i) $N >> 1/M : \tau << 1$. Large length-scale disturbances. Pressure gradients *not important throughout* the combustion region (including inner reaction zone and outer combustion zones—preheat and equilibrium). The effect of the pressure disturbances are felt only in the outer Eulerian (hydrodynamic) zones, where conservation of momentum and energy implies the acoustic equations for small-amplitude disturbances. For high-speed subsonic fronts, jump conditions emerge from Rankine–Hugoniot jump conditions across the whole combustion region.

(ii) $N = 1/M : \tau = 1$. Pressure gradients *not important* in the combustion region; inner reaction zone not affected by pressure field. The effect of the pressure disturbances is predominantly in the outer combustion zones (preheat and equilibrium), where the equations and jump conditions govern the connection between the mass flux and pressure transients.

(iii) $N = 1/\theta^2 M : \tau = \theta^2$. Pressure gradients *still not felt* in the combustion region; however, fast time-scale now causes the pressure changes to affect the inner reaction zone. A different equation to that in (i) above now determines the connection between the mass flux and the pressure changes.

(iv) $N = 1 : \tau = 1/M$. Pressure gradients *now important* in the combustion region, which experiences the full effect of any pressure wave passing through. Non-constant wave speed with nonlinearities for large-amplitude disturbances; the pressure changes are of an ultra-short length-scale.

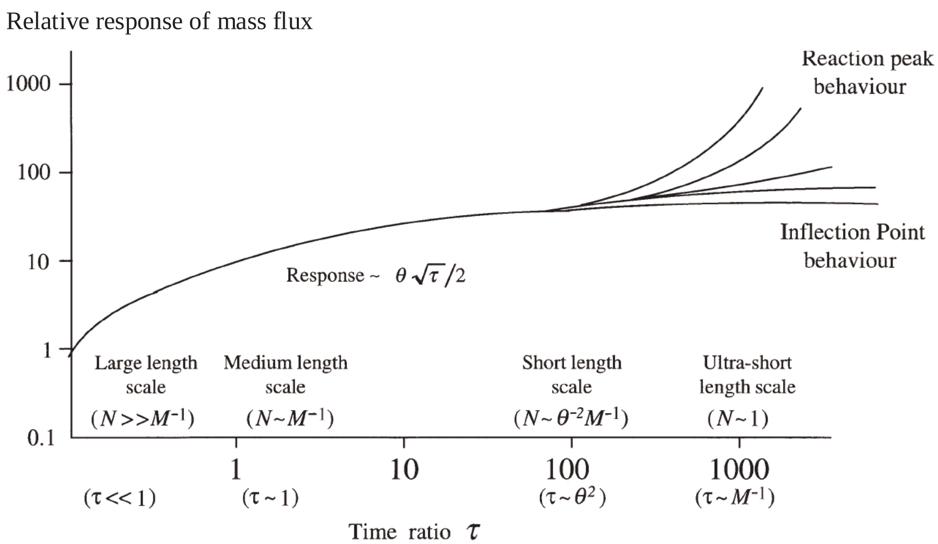

Figure 2 – Schematic of mass flux response $((m_0 - 1)/[(1 - \gamma^{-1})(p_0 - 1)])$ *to small-amplitude disturbances for different time- and length-scales.*

The schematic (figure 2) shows the different order of magnitude of mass flux response, according to the ratio τ (which is equal to $1/NM$). This diagram pertains to low-speed flames and small-amplitude disturbances. On the left of the diagram, the flame can be regarded as a contact discontinuity where the whole combustion region is swept along with the fluid disturbance. On the right of the diagram, the pressure disturbance is of an ultra-short length-scale such that the pressure gradient is ‘felt’ within the combustion region. For sharp pressure increases, the (forward) inflection point responds at a faster rate than the reaction peak point, thus ‘thinning’ the flame, which will have an overall increase in burning velocity before eventually settling down to a new steady-state structure.

FREE FLAME RESONANCE

For small-amplitude disturbances (where the amplitude ε is $O(\theta^{-1})$), and the initial combustion wave is a low-speed flame (with only negligible pressure changes across the deflagration), then the most instructive case is case (ii) in the above list. When a premixed flame is near an oscillatory pressure field, it can be shown [2] that there is an important coupling between the strength of the pressure disturbance $\overline{p}_{u0}$ and the fluctuating mass burning rate $\overline{m}_{u0}$ [1,5,6], given by

$$\overline{m}_{u0} = \frac{\left(1-\gamma^{-1}\right)\left(2r-Q\right)\left(\tfrac{1}{2}+r\right)\theta\overline{p}_{u0}}{4r} \quad , \tag{5}$$

where $r=\sqrt{\omega+\tfrac{1}{4}}$ and ω is a non-dimensional complex frequency (thus $\omega \equiv \omega_r + i\omega_i$ where ω_r is growth rate and ω_i is the modal frequency), Q is the non-dimensional heat release (typically $Q \approx 0.8$) and θ is the dimensionless activation energy (typically $\theta \approx 10$).

It should be noted that the result above for laminar flames can equally well be applied to the *thick turbulent flame regime*, where the structure is such that the reaction zone can be thought of as a thick turbulent brush. Using different lengthscales determined by viscous diffusion and heat transfer, the basic physics of the interactions described earlier for laminar flames—certainly as regards extinction—can be regarded as similar and thus used to predict the behaviour of thick turbulent flames in a changing pressure environment. For such an application to thick turbulent flames, a first approximation must be that the change in mass burning rate due to the small-scale increase in baroclinically generated vorticity within the flame will not be large. This is an acceptable assumption for case (ii) type interactions, since long-wavelength acoustics are considered and the pressure gradient is not ‘felt’ in the reaction zone.

Equation (5) is for case (ii) $\left(N=1/M : \tau=1\right)$. Clearly, as ω becomes large, then equation (5) implies

$$\overline{m}_{u0} \approx \tfrac{1}{2}\left(1-\gamma^{-1}\right)\theta\overline{p}_{u0}\sqrt{\omega} \ , \tag{6}$$

which, in figure 2, corresponds to the $\tfrac{1}{2}\theta\sqrt{\tau}$ part of that schematic. In [3] further investigations were undertaken for the case of very high frequency (that is, $\omega \sim O(\theta)$), and, as a result of the asymptotic analysis with this assumption (i.e. case (iii) $\left(N=1/\theta^2 M : \tau=\theta^2\right)$), the $\sqrt{\omega}$ ‘tail’ of equation (6) can be shown not to carry on to become larger and larger without limit for increasing frequency. Eventually, the response reaches a peak level, as shown in figure 3.

For $\theta \approx 10$ and $Q = 0.8$ (typical for hydrocarbon combustion), the response peak

$$\left[\frac{m_{u0}}{\left(1-\gamma^{-1}\right)p_{u0}}\right]\Bigg|_{\text{peak}} \approx 40 \ , \qquad (7)$$

at $\omega_{\text{i max}} \approx 0.75\theta^2 Q^2$, so that there is, in fact, a high-frequency natural resonance (that is, apart from any organ pipe resonance from equipment surrounding the flame).
In dimensional terms, the resonant frequency is given by

$$f'_{\text{resonance}} \equiv \frac{\omega'_{\text{i resonance}}}{2\pi} \approx \frac{0.75}{2\pi}\left(\frac{E'_A}{R'T'_b}\right)^2 \frac{u_0'^2}{\kappa'} \text{ Hz.} \qquad (8)$$

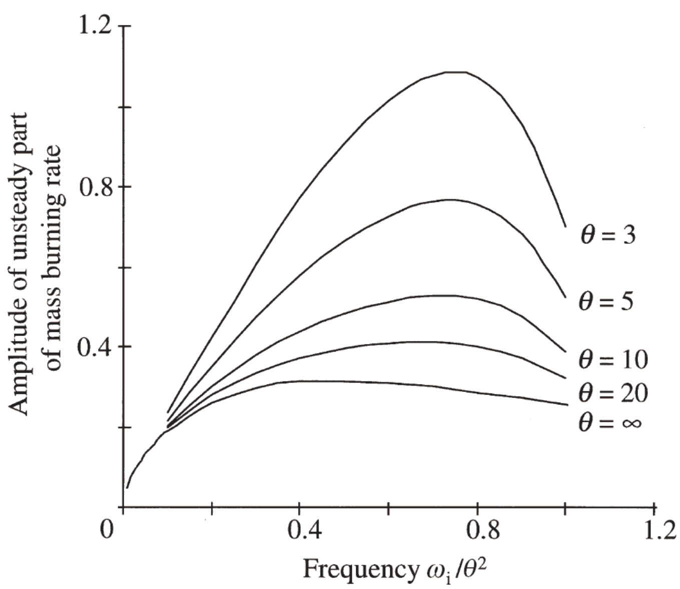

Figure 3 – Variation of mass burning ratio $M = m_{u0}/[\theta^2 Q(1 - \gamma{-}1)p_{u0}]$ with frequency and activation energy for high-frequency oscillations, $\tau \sim \theta^2$.

This is for an overall one-step reaction, where u'_{01} is the steady burning velocity and κ' is the thermal diffusivity. Thus a natural resonant high frequency (of the order of kHz) exists for most practical hydrocarbon flames and should be measurable experimentally, although this has not yet been confirmed. A pure tone generator directed at a laboratory premixed flame should then give resonance in the flame itself at a particular value of high frequency. Though there will be difficulties in measuring a pure harmonic at high frequency, and the theory here is based on single-step reaction kinetics, nevertheless it is surprising how well the single-step chemistry has worked for flame stability analysis, so, at least qualitatively, equation (8) should yield a reasonable approximation to the acoustic resonant frequency for premixed flames. Using a thermal diffusivity for air κ' at 1000 K of 1.69×10^{-4} m^2s^{-1} (see p. 839 of [7] for an estimate of thermal diffusion coefficient at high temperature), a typical burning velocity u_0' of 0.2 ms^{-1}, with $\theta \equiv E_A'/R'T_b'$ = 10, yields a typical resonant frequency prediction for premixed hydrocarbon combustion to be *ca*. 2.8 kHz.

If non-harmonic disturbances are allowed, then if the length-scale is small enough ($\tau \sim O(M^{-1})$), the pressure gradient becomes significant in the reaction zone, and there is severe distortion of this region, such that for a pressure drop, the flame broadens and slows down, and for a pressure rise, the reverse happens with the flame thinning and accelerating. Earlier investigations [4,8,9] show that sharp pressure changes can cause major changes in the flame structure—transient stretching or compression of the flame—and, consequently, the mass burning rate can alter substantially.

SMALL AMPLITUDE HEAT TRANSFER RESONANCE

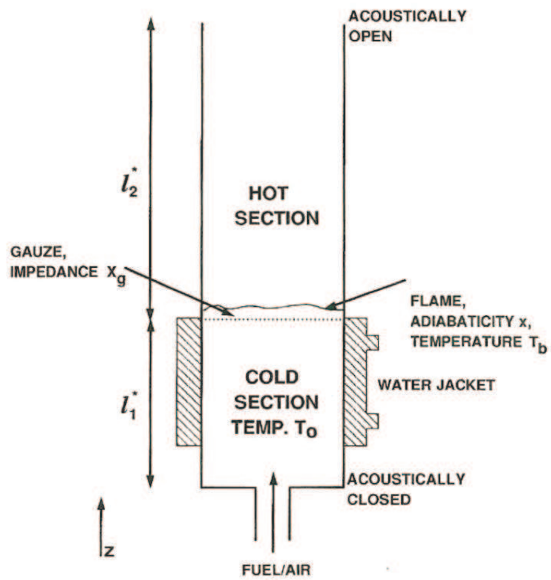

Figure 4 – Schematic of a typical Rijke burner.

In the earlier section ε was $O(\theta^{-1})$. If the amplitude of the acoustic disturbances is now much smaller and ε becomes of $O(M)$ with τ still $O(1)$, then a different set of acoustic balances emerges [10], such that a very small acoustic field involves a velocity fluctuation which now is at the same magnitude as the combustion generated flow velocities near the flame. These velocities are, through the gas law and continuity relations, inseparably connected to parallel temperature disturbances through the flame and what then emerges is a transfer function V between such velocity perturbations $\hat{u}_1$ (cold) and $\hat{u}_2$ (hot) given by:

$$\hat{u}_2 = -\frac{V}{T_0}\hat{u}_1 \tag{9}$$

where

$$V \approx -T_0 - \frac{(1-T_0)\left(\frac{1}{2}+r\right)\exp\left(-\left(\left(\frac{1}{2}+r\right)x\right)\right)}{2\left[re^{-2rx} + \frac{\omega}{\theta(1-T_0)}\right]} \;\; ; \;\; r \equiv \sqrt{\omega + \frac{1}{4}} \tag{10}$$

As in the earlier case, ω represents the complex frequency, θ is the dimensionless activation energy, and here T_0 is the ratio of unburnt to burnt temperature in the steady (burner-anchored flame) and x is the adiabaticity. It is given by

$$x \equiv \ln\left[\frac{T_{ad} - T_0}{T_{ad} - T_b}\right] \tag{11}$$

and is linked to the flame stand off distance. As the flame becomes less attached to the burner and the flame temperature T_b approaches the adiabatic flame temperature T_{ad}, so the adiabaticity x increases.

The impedance of the gauze holder of Rijke type burner systems (see figure 4) to the passage of pressure disturbances is small – the only appreciable effect is a phase shift, so that impedance term used in simulations of these burners is of the form iX_g. Considerable success [11, 12] has been achieved through the use of the velocity transfer function (equation (10)) to predict the resonance behaviour of flames in such devices Rijke type burner systems.

CONCLUDING REMARKS AND SIMPLE RULES FOR RESONANCE

(1) A necessary (but not sufficient) condition for resonance of practical combustion systems with acoustic waves (usually governed by small amplitude heat transfer resonance) is that the time scale t'_{ac} of acoustic waves travelling in the outer chamber (typical length ℓ'_{ac}) is of the same order as the time scale (t'_{comb}) for the much slower diffusion disturbance to travel through typical distances (ℓ'_{comb}) associated with combustion. With speed of sound a'_0, the two times can be written as

$$t'_{ac} = \frac{\ell'_{ac}}{a'_0} \;; \quad t'_{comb} = \frac{\kappa'}{u'^2_0} \quad , \tag{12a,b}$$

where κ' is thermal diffusion. Consequently for the two time scales to be the same

$$\ell'_{ac} = \frac{a'_0 \kappa'}{u_0'^2} \quad , \tag{13}$$

is the necessary condition for resonance, where u'_0 can be regarded as the burning velocity of the steady flame.

(2) The sufficient condition for resonance can only be found by solving the full system of the acoustic field coupled with the flame region. All important is a velocity transfer function (equation (10)) to describe the progress of the velocity fluctuations through the flame.

(3) Free flames also show resonance behaviour but at higher typical frequencies where $\tau = \theta^2$ (and $N = 1/\theta^2 M$) which corresponds to the condition (equivalent to equation (13))

$$\ell'_{ac} = \frac{a'_0 \kappa'}{\theta^2 u_0'^2} \quad . \tag{14}$$

In this case, the high frequency pressure disturbances cause the inner reaction zone to oscillate, and resonance to occur. An estimate (equation (8)) of the resonant frequency can be made.

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