

AR POLE TRAJECTORY IN CONDITION MONITORING STUDIES

Suguna Thanagasundram¹, Kalu Ram Gurung¹, Y.Feng^{*1} and Fernando Soares Schlindwein¹

¹University of Leicester, Department of Engineering, University Road, Leicester LE1 7RH, UK <u>st89@le.ac.uk</u>

Abstract

In this paper a novel approach is proposed for vibration based fault detection studies by the tracking of pole movements in the complex z domain. Vibration signals obtained from the ball bearings from a High Vacuum (HV) and Low Vacuum (LV) ends of a dry vacuum pump run in normal and fault conditions are modeled as time variant AR (Autoregressive) series. The positions of the poles which are the roots of the AR coefficient polynomial vary for every frame of vibration data. It is a known fact that as defects such as spalls and cracks start to appear on the ball bearings, the amplitude of the vibrations of characteristic defect frequencies increase. Faults can be predicted by movement of poles in the complex plane as the pole positions are expected to move closer to the unit circle as the severity of the defect increases. The area of the region swept by the migratory poles loci and their distances from the unit circle can be useful fault indicators. From the position of the poles inside the unit circle, classification and quantification of the main spectral peak of defect frequencies can be easily performed, leading to the possibility of having frame to frame monitoring of spectral parameters of interest. The AR pole positions also allow an easier quantitative estimation of the spectral parameters. The pole representation facilitates the easier understanding of the spectral characteristics of the process because of the one-to-one correspondence between the poles and the AR spectral peaks. This method has interesting potential applications in condition monitoring and diagnostic applications. The description of the movement of the poles is shown to be particularly important for the study of harmonic components of the signal. This analysis has been validated with actual data obtained from the pump and initial results obtained are very promising.

INTRODUCTION

Vibration analysts often rely on PSDs (Power Spectral Densities) of vibration data to monitor the health of moving parts of machinery. Common failures such as bearing faults and gear problems can be detected by trending major frequency components and their amplitudes. Most of the frequency domain methods used in industry today are based on the FFT (Fast Fourier Transform) technique. The major shortcoming of the spectral estimation technique is that a large number of frequency components have to be monitored due to the complexity of the system. A standard approach in evaluating an instantaneous frequency implies the computation of the whole spectrum first and then estimation of the amplitude of a particular frequency of interest. For instance, if ball bearing defect frequencies [1] such as BPFO (Ball Pass Frequency of Outer Race), BPFI (Ball Pass Frequency of Inner Race), BSF (Ball Spin Frequency) and FTF (Fundamental Train Frequency, also known as Cage Frequency) are to be detected, FFT spectra are computed for frame sizes of concern and then the spectra are filtered to monitor the presence of the fault frequencies. Such a process can be firstly be time consuming as whole frames of data have to be estimated. Secondly, it can be power intensive as it involves processing time proportional to $N \log_2 N$ computations where N is the sample size. Another concern of the FFT technique is that a large enough sample size has to be used for the spectral estimation for reasonable resolution capabilities as the resolution of the FFT tool is inversely proportional to the frame size utilized. This might not be appropriate in real time applications.

An interesting alternative is to evaluate directly the frequencies of interest. In this case only those frequencies have to estimated instead of the whole spectrum. This provides a reduction in computing time and effort facilitating real time estimation. In this study, characteristic bearing defect frequencies are extracted from the pole frequencies of a parametric AR model [2]. Using the AR estimation method, it becomes unnecessary for the calculation of the whole spectrum. Instead just evaluation of the pole frequencies of interest from the derived AR parameters would suffice as AR modelling allows spectral decomposition. This often just involves the calculation of the AR coefficients and the variance of the input vibration signal. Small order AR models can efficiently estimate the pole frequencies which correspond to the poles of the bearing defect frequencies. The AR technique also only requires a fraction of samples as that required by the FFT method for the same resolution. When compared to the traditional FFT method, the resolution of the AR technique is higher due to its implicit extrapolated autocorrelation sequence. This means that smaller sample sizes can be used for PSD estimation. Power and frequency of each bearing defect spectral component can be extracted from the position and residual of each pole. The time varying behaviour of the spectral components can also be studied by tracking the movement of the AR poles. This investigation reports a study of the detection of an inner race bearing fault of a dry vacuum pump through the mapping of AR poles from its vibration signal. This computational method seems to be very attractive for condition monitoring applications as the method provides a more

immediate comprehension of the spectral process characteristics when expressed in terms of poles and AR spectral components.

THEORY BEHIND AR MODELLING

An AR process of model order p can be described by Eq.(1) where a_k are the AR parameters, e[t] is white noise with zero mean and variance σ^2 and t is the discrete-time index [2]. The same equation expressed in the z-transform domain is stated as Eq.(2).

$$x[t] = -\sum_{k=1}^{p} a_{k} x[t-k] + e[t]$$
(1)

$$X[z] = -X(z)\sum_{k=1}^{p} a_{k}z^{-k} + E[z]$$
⁽²⁾

If H[z] is defined as AR polynomial of the model transfer function relating the input to output, Eq.(2) can be rewritten as Eq.(3) where X[z] and E[z] are the z-transforms of x[t] and e[t] respectively and H[z] is defined by Eq.(4).

$$X[z] = H(z)E[z]$$
(3)

$$H(z) = \frac{1}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$
(4)

The poles, z_k , are obtained by finding the roots of the AR coefficient polynomial in the denominator of H[z]. An AR model's transfer function contains poles in the denominator plus only trivial zeroes in the numerator at z=0, so it is referred to as an "all-pole" model. Since the coefficients of H(z) are real, the roots must be real or complex conjugate pairs. The number of poles in z plane equals to the AR model order.

$$H(z) = \frac{1}{(z+z_0)(z+z_1)\dots(z+z_p)} = \frac{1}{z \cdot \prod_{k=0}^p (1+z_k z^{-1})}$$
(5)

There are four main methods for the estimation of the a_k coefficients: Yule-Walker, Burg, Least Squares Forward method and Least Squares Forward Backward method [2]. The PSD of the output signal, $P_x(f)$, is related to the PSD of the input signal which is noise. $P_x(f)$ can be obtained with the variance σ^2 once the a_k coefficients are known.

$$P_{x}(f) = \frac{\sigma^{2} \Delta t}{\left|1 + \sum_{k=1}^{p} a_{k} z^{-k}\right|_{z=e^{j2\pi f \Delta t}}^{2}}$$
(6)

Each pair of complex conjugate poles in Eq.(5) has a one to one relation with the AR spectrum, $P_x(f)$, in the z domain. A p^{th} order AR model with p poles will have m peak frequencies where m = p/2 when p is even and m = (p+1)/2 when p is odd. Not all poles give rise to peaks in the AR spectrum. Only the poles which are close to the unit circle give rise to sharp peaks in the AR frequency spectrum (see Figure 1). The other poles are equally distributed around the unit circle to create an equiripple 'flat' PSD estimation. The closer the poles are to the unit circle, the bigger are the amplitude of the peaks. For stability, all poles must lie within the z-plane unit circle, thus the magnitude of each must be less than unity. The advantage of the pole tracking method is that the symmetry of the z plane representation can be conveniently exploited and we can disregard the poles in the negative imaginary plane, halving the number of poles to be tracked.



Figure 1: Monitoring the peaks obtained from the AR model. In this case, AR model order was 8 and sampling frequency was 2000 Hz. Notice that it is only the pole nearest to the unit circle that gives rise to a sharp peak in the AR spectrum.

Each pole z_k has a phase ϕ_k and a magnitude r_k (distance from the origin). By knowing the pole position inside the unit circle, the central frequency f_k of each peak can be obtained from the phase ϕ_k if the sampling frequency f_s is known (Eq.(7)). The position of the more significant poles vary with the every frame of data depending on whether they were obtained from no-fault or faulty conditions. The area mapped out by these critical migratory poles is given by Eq.(8).

$$f_k = 2 \cdot \pi \cdot \phi_k = \tan^{-1} \left(\operatorname{Im}(z_k) / \operatorname{Re}(z_k) \right) \times \frac{f_s}{2\pi}$$
(7)

$$Area = \frac{1}{2} \cdot (\Delta r_k)^2 \cdot \Delta \phi_k \tag{8}$$

RELATING BEARING CHARACTERISTIC DEFECT FREQUENCIES WITH POLE POSITIONS

The angles of the pole locations of the characteristic defect frequencies for the ball bearings [1] are fixed as they are determined by the application and the geometrical shape of the ball bearings used but the distances of the pole locations from the unit circle are determined by the levels of vibration they cause. It is known that as defects such as spalls and cracks start to appear on the ball bearings the amplitude of the vibrations increase (see Figure 2). We can determine what is the appropriate alarm level of vibration from standards such as the ISO 10816 [3] and ISO 7919 [4] and translate these to relative amplitudes and alarm levels and hence corresponding pole positions for the characteristic ball bearing defect frequencies.



Figure 2: Typical movement of pole as defect becomes more severe (from 1 to 3) and

amplitude of vibration of characteristic defect frequency begins to increase.

The Barden bearing specifications for the BOC Edwards IGX dry vacuum pump that was used as the testbed in this experimentation are: number of balls = 9, pitch diameter = 46.2 mm, ball diameter = 9.5mm and contact angle = 24.97 degrees. For a case of the pump rotating at a speed of 100 Hz, the theoretical ball bearing defect frequencies and their relative pole phase angles ϕ_k for a sampling rate f_s of 2000 Hz are shown in Figure 3. In reality, a bearing with an inner race fault has BPFI occurring at slightly less than 534.10 Hz as it was observed that the rotating speed of the pump's rotor shaft, on which the bearing case was directly connected to, was often less than the set speed of the pump due to rotor slip. It varied with external running conditions like the ultimate pressure of the inlet of the pump. The running speed of the pump had to be determined very accurately for the diagnostics scheme, as this was the frequency which was used in the calculations of the bearing defect frequencies. This effect was taken to consideration when translating the bearing defect frequencies into pole positions.



Figure 3: Theoretical location of Characteristic Defect frequencies in the z plane can be worked out if the bearing dimensions are known with standard reference formulas. Pump was set to 100 Hz and sampling frequency used was 2000 Hz in this case.

SETUP AND DATA ACQUISTION

The pump has a single row of deep groove ceramic bearings at both the High Vacuum (HV) and Low Vacuum (LV) ends. Dry vacuum pumps are commonly used in semiconductor and cleanroom environments where corrosive gases are abundant, temperatures are high and effluents resulting from the processes can cause seizure of the pumps. With constant use, there are chances of the ceramic bearings failing over time because of their operation in such harsh running conditions. To simulate a faulty condition, a bearing which had a single point defect on the inner race was planted on the HV end. The acceleration was captured on the pump housing directly over the bearing casing on the HV for normal (no-faults) and faulty condition using a surface micromachined accelerometer ADXL105 which was mounted radially on the pump running at 100 Hz. The signals from the ADXL105 were filtered with a 8th order low pass elliptic anti-aliasing filter with a cut-off frequency of 10 kHz and an attenuation of 70 dB in the stop band. The analogue to digital conversion was performed with a 16-bit NI 6034E Analogue to Digital Converter (ADC) card. The sampling rate was set to 40 kHz and frame sizes of 5000 samples were acquired as per our requirements. The data was then downsampled to 2 kHz as it was known that the frequencies of interest lie in the range from 0-1000 Hz (refer to Figure 3). Prior to downsampling, the vibration signals were also preprocessed by amplitude demodulation [5]. If there is noise and other higher level vibrations generated by the other machine components, there is a chance that the harmonics of the defect frequencies may be buried in the spectrum of the other components. To overcome this limitation, the technique of amplitude demodulation is recommended. This involves two basic steps 1) bandpass filtering around one of the resonant peaks where there is structural resonance 2) applying the Hilbert transform to the bandpassed signal to obtain the squared envelope.

A 10th order AR model was found sufficient to study the behaviour of ADXL105 vibration signals mounted on the HV end for a pump operating in normal conditions and also for a pump fitted with a bearing with an inner race fault. Frame sizes of 5000 samples were used and 80 frames of data were tracked to monitor the movement of the BPFI pole corresponding to the inner race defect frequency. The pole positions of the BPFI pole per frame, distance of the BPFI from the origin, the angle mapped by BPFI pole and the cumulative area traversed by the BPFI pole were plotted for the 80 frames of vibration data both for normal and faulty conditions.



RESULTS

Figure 4: Pump running at 100 Hz. 80 frames of ADXL105 vibration data used. A 10th order AR model was used (a) Distribution of poles from a pump running in normal conditions (black) and for a case with a bearing which has an inner race fault (red). Note concentrated clustering of poles near unit circle for BPFI frequency at approximately 530 Hz for faulty data. Spread of poles is much larger for normal conditions. (b) Distance of BPFI pole from origin. The distance of the BPFI pole for faulty conditions markedly much higher than in normal conditions. (c) Angle BPFI pole traverses. For normal condition, there was much variation in the BPFI angles but for faulty condition the variance was much less. (d)
Cumulative area traversed by the BPFI poles. The area mapped out for normal condition was much larger than that for faulty condition.

The locus of the BPFI poles on the z plane were tracked for normal and faulty running conditions. The BPFI pole was chosen as the dominant pole as the fault condition studied was the case of a pump fitted with a bearing with an inner race fault. If other bearing defects are to be monitored, the corresponding critical pole positions relating to the characteristic defect frequencies of significance can be tracked. For the faulty condition, the spread of the poles was much concentrated and nearer to the unit circle. The distance of BPFI poles from the origin was much larger for the faulty condition than in the normal case. The mean distance of the BPFI pole was 0.7652 and 0.8981 for normal and faulty condition respectively. This will be seen as bigger amplitude of spectral peaks associated with critical poles (amplitude of spectral peaks is proportional to the inverse of the distance of pole to the unit circle) when the AR spectra is plotted for the faulty condition. The mean angle of the BPFI pole was 1.6856 radians and 1.6666 radians respectively for normal and faulty condition respectively. The average mean angles of the BPFI poles in both cases were nearly the same. However the variance of the angles of the BPFI poles ($\Delta \phi_k =$ 0.6908) for the normal condition was much larger than for the faulty case ($\Delta \phi_k =$ 0.1291). The variance of BPFI angles can be used as a useful indicator for fault classification to distinguish between the no-fault and defective cases as the difference between them is significant. The cumulative area traversed by the migratory normal BPFI poles (0.1845) was also markedly much bigger than the case for the faulty condition (0.0011).

CONCLUSION

The distance, angle and area mapped by the critical characteristic defect pole positions can be used to establish boundary conditions to distinguish between normal and faulty conditions. The proposed method has useful applications in fault classification as faults can be detected by studying the movement of poles in the complex plane without having to compute the whole PSD spectra which can be computationally intensive. The bearing defect frequencies can be decomposed to spectral pole positions and only the critical pole positions are required to be tracked.

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