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INFLUENCE OF GYROSCOPIC EFFECT ON THE CRITICAL SPEED OF THE ROTATION OF THE HETEROGENEOUS ROD

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Abstract
The problem of the loss of stability of the rotation with a constant angular velocity ω elastic rod from the variable rigid $EJ = EJ(x)$ and density $\rho = \rho(x)$ by axis *x* is examined.

INTRODUCTION

The influence of gyroscopic effect (GE) on the stability of the rotation of the outrigger shaft, which simulates the unguided rocket stabilized by rotation, is investigated. The distribution of the rigid and density characteristics of rod starts in the form of continuous smooth functions in this case. The distributed centrifugal force and the distributed bending moment, caused by GE, act to the rod after loss of stability. We bring the task to the integration of ordinary differential equation of the 4th order with the variable coefficients relative to the dimensionless function of sagging, using known differential relationships between the internal power factors and the intensity of external loads with the bend.

The number of examples for different laws of variation in the density and hardness along the axis of rod is examined. The analysis of the obtained results shows that the calculation GE with a study of those being fast-turning is gross it influences critical frequencies to the side of their increase. The degree of influence in essence is connected with the number of the frequency and parameter of the function of heterogeneity, which characterize extension shaft. Influence GE is reduced with an increase in the extension shaft.

Disregard GE leads to the essential errors with a study of the highest frequencies. The degree of influence GE is determined by the function of the heterogeneity for the rods of variable along the axis of hardness. The monotonic growth of functions leads to an increase in the influence GE, monotonic decrease - to the decrease. The position is different for the rods of permanent hardness and variable density: the degree of influence GE is practically identical with different laws of variation density.

PROBLEM FORMULATION

The distributed centrifugal force of intensity $\omega^2 \rho(x) A(x) w(x)$ and the distributed bending moment of intensity $\omega^2 \rho(x) J(x) w'(x)$ (caused by the gyroscopic effect (GE)) acts on swollen as a result of the loss of stability rod. It is designated here: x – axial coordinate, $w = w(x)$ – rod deflection, $A = A(x)$, $J = J(x)$ – the area and the moment of the inertia of the cross section of rod respectively, prime indicates differentiation with respect to x.

We will obtain the equation of equilibrium in the form

$$M''(x) + \omega^2 \left\{ \left[\rho(x) J(x) w'(x)\right]' + \rho(x) A(x) w(x) \right\} = 0, \tag{1}$$

where $M = M(x)$ – the bending moment

$$M(x) = -E(x) J(x) w''(x). \tag{2}$$

Using known differential relationships between the internal power factors and the intensities of external loads with the bend of rods. We bring the task to the integration of the 4-th order with the variable coefficients relative to the function of the sagging $w = w(x)$

$$\left[E(x) J(x) w''(x)\right]'' - \omega^2 \left[\rho(x) J(x) w'(x)\right]' - \omega^2 \rho(x) A(x) w(x) = 0, \tag{3}$$

excluding from equation (1) moment with the aid of idea (2).
By introduction of the dimensionless parameters and functions

$$\xi = \frac{x}{l}, \quad W = \frac{w}{l}, \quad G = \frac{EJ}{E_* J_*}, \quad S = \frac{\rho A}{\rho_* A_*}, \quad R = k \frac{\rho J}{\rho_* J_*}, \quad k = \left(\frac{i_*}{l}\right)^2.$$

where l – length of rod; $E_*, \rho_*, A_*, J_*, i_*$ – some characteristic values of the module of elasticity, density of material, area, moment and radius of inertia of the cross section of rod, equation (3) is reduced to form

$$\left[G(\xi) W''(\xi)\right]'' - p^2 \left\{ \left[R(\xi) W'(\xi)\right]' - S(\xi) W(\xi) \right\} = 0, \tag{4}$$

where $p = \omega l^2 \sqrt{\frac{\rho_* A_*}{E_* J_*}}$ – the dimensionless parameter of the frequency of rotation. Functions $G = G(\xi)$, $S = S(\xi)$, $R = R(\xi)$, characterize the heterogeneity of the rod from the different sides: $G(\xi)$ – variable along the axis ξ rigidity, $S(\xi)$ – density of material, $R(\xi)$ – the influence of gyroscopic effect.

PROBLEM SOLUTION

The analytical method of integrating the equations of the type (2) in the absence of the term, which contains function $R = R(\xi)$, i.e., the method of constructing the solution of this problem without the calculation GE, is represented, for example, in articles [1, 2, 3].

Using a somewhat modified approach, we will search for the general solution of equation (4) in the form

$$W(\xi) = \sum C_j W_j(\xi), \qquad (5)$$

where $C_j \quad (j = \overline{1, 4})$ – arbitrary constants;

$$W_j(\xi) = f_j(\xi) + \int H(\xi, z)\tilde{g}_j(z) f_j(z)\,dz + \sum^{\infty} \int H(\xi, z)\left[\int P^{(n)}(z, \eta)\tilde{g}_j(\eta) f_j(\eta)\,d\eta\right]dz$$

– fundamental system of solution of equation (4), $P^{(n)}$ – n-th iteration of nucleus

$$P(\xi, z) = -\frac{\tilde{g}_1(\xi) f_1(\xi)}{f_1(z)} + \frac{\tilde{g}_2(\xi) f_2(\xi)}{f_2(z)} + i\left[\frac{\tilde{g}_3(\xi) f_3(\xi)}{f_3(z)} - \frac{\tilde{g}_4(\xi) f_4(\xi)}{f_4(z)}\right],$$

$$H(\xi, z) = -\frac{f_1(\xi)}{f_1(z)} + \frac{f_2(\xi)}{f_2(z)} - i\left[\frac{f_3(\xi)}{f_3(z)} - \frac{f_4(\xi)}{f_4(z)}\right],$$

$$f_j(\xi) = g(\xi)\delta_j(\xi)\exp(\xi_j m_j(\xi, 0)),$$

$$\bar{f}_1(\xi) = g(\xi)\delta_3(\xi)\exp(m_1(\xi, 0)), \qquad \bar{f}_2(\xi) = g(\xi)\delta_3(\xi)\exp(-m_1(\xi, 0)),$$

$$\bar{f}_3(\xi) = g(\xi)\delta_1(\xi)\exp(im_3(\xi, 0)), \qquad \bar{f}_4(\xi) = g(\xi)\delta_1(\xi)\exp(-im_3(\xi, 0)),$$

$$g(\xi) = S^{-3/8}(\xi)G^{-1/8}(\xi)\alpha^{-1/2}(\xi), \qquad \delta_j(\xi) = \left[\alpha(\xi) + \varepsilon_j^2 \cdot \gamma(\xi)\right]^{-1/4},$$

$$m_j(\xi) = \int_0^{\xi} b_j(\xi)\,d\xi, \qquad b_j(\xi) - \beta(\xi)\delta_j^{-2}(\xi), \qquad \alpha(\xi) - \left[1 + \gamma^2(\xi)\right]^{1/2},$$

$$\beta(\xi) = p^2\left[\frac{S(\xi)}{G(\xi)}\right]^{1/4}, \qquad \gamma(\xi) = \frac{R(\xi)}{ZpS^{1/2}(\xi)G^{1/2}(\xi)}, \qquad \tilde{g}_j(\xi) = \frac{g_j(\xi)}{4G(\xi)\beta^3(\xi)},$$

$$g_j(\xi) = K(\xi) + \varepsilon_j L(\xi) + \varepsilon_j T(\xi), \qquad K(\xi) = \frac{(G(\xi)q''(\xi))''}{q(\xi)},$$

$$L = G\beta\left[\frac{G''}{G}\left(\frac{\beta'}{\beta} + 2\frac{q'}{q}\right) + \frac{G'}{G}\left(6\frac{q''}{q} + 2\frac{\beta''}{\beta} + 6q\frac{q'\beta'}{g\beta}\right) + 4\frac{q'''}{q} + \frac{\beta'''}{\beta} + 6q\frac{q''\beta'}{g\beta} + 4\frac{q'\beta''}{q\beta}\right],$$

$$T = G\beta^2\left[\frac{G''}{G} + 6\frac{G'}{G}\left(\frac{\beta'}{\beta} + \frac{q'}{q}\right) + 6\frac{q''}{q} + 12\frac{q'\beta'}{q\beta} + 3\left(\frac{\beta'}{\beta}\right)^2 + 4\frac{\beta''}{\beta}\right],$$

$\varepsilon_1 = 1$, $\varepsilon_2 = -1$, $\varepsilon_3 = i$, $\varepsilon_4 = -i$ (roots of equation $\varepsilon^4 = 1$).

Having available general solution for saggings (5), it is possible to obtain the remaining kinematic and power factors of the task by successive differentiation: the angles of rotation of cross sections $W'(\xi)$, bending moments $M(\xi)$ and the transverse forces $Q(\xi)$, necessary subsequently for the formulation of boundary conditions.

RESULTS

Results for four versions of the support are represented below: rod with the pinched ends, console, the hinged support of ends and free ends. In particular, the last case simulates the flight of rocket being fast-turned relative to axis.

The standard procedure of satisfaction of boundary conditions reduces to the transcendental frequency equations of form

1) the pinched ends

$$\left[\frac{a(0)}{a(1)} + \frac{a(1)}{a(0)}\right]\mathrm{ch}m_1(1,0)\cos m_3(1,0) + \left[a(0)a(1) - \frac{1}{a(0)a(1)}\right]\mathrm{sh}m_1(1,0)\sin m_3(1,0) = 2;$$

2) console

$$\left[a(0)a^3(1) + \frac{1}{a(0)a^3(1)}\right]\mathrm{ch}m_1(1,0)\cos m_3(1,0) +$$
$$+\left[\frac{a(0)}{a^3(1)}\ \ \frac{a^3(1)}{a(0)}\right]\mathrm{sh}m_1(1,0)\sin m_3(1,0) = 2;$$

3) the free ends

$$\left[\frac{a^3(1)}{a^3(0)}+\frac{a^3(0)}{a^3(1)}\right]\mathrm{ch}\, m_1(1,0)\cos m_3(1,0)+$$

$$+\left[\frac{1}{a^3(0)a^3(1)}-a^3(0)a^3(1)\right]\mathrm{sh}\, m_1(1,0)\sin m_3(1,0)=2;$$

4) the hinged support of the ends

$$\sin m_3(1,0)=0,$$

where it is marked $a(\xi)=[\alpha(\xi)-\gamma(\xi)]^{1/2}$.

A number of the examples is examined:

1) the rod with the free ends of the round cross section, whose diameter changes along the axis according to the linear law

$$S(\xi)=[1-(1-r)\xi]^2,\quad G(\xi)=[1-(1-r)\xi]^4,\quad R(\xi)=k[1-(1-r)\xi]^4;$$

2) the rod of permanent rigidity with the free ends, whose density changes according to the linear law

$$S(\xi)=1-(1-r)\xi,\quad G(\xi)=1,\quad R(\xi)=k[1-(1-r)\xi];$$

3) the rod of permanent rigidity with the free ends, whose density changes according to the cosinusoidal law

$$S(\xi)=1+\mu\cos 2\pi\xi,\quad G(\xi)=1,\quad R(\xi)=k[1+\mu\cos 2\pi\xi],$$

where r – parameter of conicity (relation of the diameters of end sections),

$$\mu=\frac{\rho_a}{\rho_m},$$

ρ_a – the amplitude values of density, ρ_m – the average values of density.

The analysis of the obtained results shows that the calculation of gyroscopic effect with a study of those being fast-turning rods is gross it influences the critical frequencies of rotation to the side of their increase. The degree of influence in essence is connected with the number natural of the frequency in question and parameter «*k*» of the function of heterogeneity, which characterize extension rod, i.e., the ratio of the characteristic transverse dimension of rod to its length.

Thus, influence GE is reduced with an increase in lengthening rod. For example, for by 3-th the frequency of uniform rod with $k = 17 \cdot 10^{-4}$ a relative change in the frequency as a result of the calculation of gyroscopic forces composes 20,8%, with $k = 9{,}8 \cdot 10^{-4}$ – 10,6% and with $k = 6{,}25 \cdot 10^{-4}$ – 6,4%. Influence GE on the fundamental frequency insignificantly and does not exceed 1%. Thus, with a study of high frequencies the disregard GE leads to the essential errors. For the rods of variable rigidity along the axis the degree of influence GE is determined by the nature of the function $R(\xi)$: its monotonic growth leads to an increase in the influence GE, monotonic decrease – to the decrease. In particular, for the 1-st case with $r = 1$ and $k = 17 \cdot 10^{-4}$ 3-th critical frequency of rotation grows by 20,8%, and with $r = 0{,}25$ in all by 5,8%. The position is different for the rods of permanent hardness and variable density: the degree of influence GE is practically identical with different laws of variation in the density and with the significant heterogeneity. Thus, the 2-nd case with $r = 1$ (uniform rod) relative change by 3-th frequency with $k = 17 \cdot 10^{-4}$ composes 20,8%, and with $r = 4$ – 20,9%.

The parameter was used for evaluating the accuracy of the obtained natural frequencies of the rotation

$$\Delta = \int_0^1 \frac{\left| W_j(\xi) - f_j(\xi) \right|}{\left| f_1(\xi) \right|} d\xi,$$

that being the average along the length of rod relative error between the precise and approximate particular solutions of equation (4) /3/.

It is shown that error in the calculated frequencies does not exceed 10% with different laws of variation in the rigidity and density and with the significant degree of heterogeneity strong heterogeneity worsens the accuracy of results.

The obtained analytical solutions are especially effective with a study of the highest frequencies, when the influence of gyroscopic effect is substantial.

REFERENCES

[1] Gordon V. «The analytical decision of a problem of vibrations of one dimensionally nonhomogeneous elastic bodies» Proc. VI Intern. congress on sound and vibration, Lingby, Denmark, 3195-3198 (1999).

[2] Gordon V. «Free radial vibration of barrels» Proc. I Intern. conf. on military technology, Stockholm, Sweden, 257-265 (2003).

[3] Gordon V. «A study of the free bending vibrations of the rods of trapeziform longitudinal section» Allunion mezhvuz. coll. the applied problems of strength and plasticity. Gorky university, 105-113 (1988) (in Russian).