ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

USE OF VIBROACOUSTIC DIAGNOSIS IN REDUCING THE UNCERTAINTY OF TECHNICAL RISK ESTIMATION

Robert Gumiński*, Stanisław Radkowski

Institute of Machine Design Fundamentals, Warsaw University of Technology
ul. Narbutta 84, 02-524 Warsaw, Poland
ras@simr.pw.edu.pl, (e-mail address of lead author)

Abstract

The certainty of risk estimation, being part of technical risk analysis, depends on numerous factors, including the uncertainty of external conditions, the uncertainty of the model, which is related to the relevant level of knowledge of degradation and fatigue-related processes, parametrical uncertainty and the quantitative evaluation of the effect of propagation of uncertainty in the system. Evaluation of the consequences and gravity of loss while relying on uncertain results makes no sense and thus the model should account for the uncertainty of analysis, and also additional information should be introduced to reduce the uncertainty. Use of vibroacoustic diagnosis methods, which enables detection of both qualitative and quantitative changes of mechanical properties and kinematic-and-dynamic parameters of critical elements and kinematic nodes, can substantially reduce the uncertainty of technical risk analysis. It should be stressed that the studies dealing with modeling while accounting for uncertainty stress that there is a major deficit of available means for dealing with this sphere when compared to actual requirements.

While tackling the problem of the impact of diagnostic inspection results on reduction of uncertainty of risk estimation, one should above all review the possibility of applying the Proportional Hazards Model when describing the degradation and fatigue processes while also accounting for the impact of the covariates.

INTRODUCTION

The growing social aversion to incorrect decisions leads to emergence of understandable interest in research on and evaluation of uncertainty that accompanies subsequent stages of the construction process and that is associated with certain operational decisions.

The sources of uncertainty should be sought in such areas as: the randomization of loads and load capacity as the degradation process develops, the incomplete statistical information when estimating the parameters of a model, the adoption of simplified models of phenomena and processes, the procedures related to determining the variables which describe the processes of wear as well as degradation, the description of environmental impact, the modeling of operator's errors and especially the process of determining the relations and the feedback in the man-environment-technology system. In general one can say that uncertainty results from lack of sufficient knowledge about load capacity and actual loads to which technical systems are subjected. The basic problem is the relevant selection of variables so as to enable dimensioning of machines elements while accounting for the properties of materials and for the load as well as application of relevant methods of modeling and analysis of the process of emergence and development of defects. We propose using the information obtained from vibroacoustic diagnosis to determine the influence that respective stages of defect development have on the function of intensity of defects in examined elements or kinematic pairs. Use of Proportional Hazards Model will allow accounting for the results of the diagnostic experiment by extending the observation space to include the dimension of system variables.

PROPORTIONAL HAZARDS MODEL

The reliability function can be determined in two ways: as a function of probability density that accounts for the influence of degradation processes and operating conditions, or as a function of defect intensity that depends on the base function of defect intensity and on the systemic variables accounting for operating conditions. As regards the models that account for diagnostic research, the systemic variables will account for the influence that the additional information obtained during diagnostic inspection has on uncertainty of reliability evaluation.

Thus, as presented in [5] the application of proportional models offers the possibility of examining the influence of external loads and environmental conditions, as well as manufacturing and assembly quality (relying on systemic variables) on the system's reliability while the evaluation of the influence of changes of these factors and the impact of internal loads that depend on degradation and wear and tear processes call for application of more complex models.

The features of Proportional Hazards Model result from the following assumptions:

1. The ratio of intensity of damage for two different values of a systemic variable is independent of time;
2. Intensity of defects for various values of the systemic variable is described by the following distribution.

Based on the above assumptions we can obtain the following equation:

$$\lambda(t, z, \beta) = \lambda_0(t)\, r(z, \beta) \tag{1}$$

where:

t – time

z – systemic variable

β - unknown parameter accounting for the influence of the systemic variable

$\lambda_0(t)$ – intensity of defects for the value of the systemic variable adopted as the reference level

If we assume, according to Cox model (1997) [1], that:

$$r(z,\beta) = e^{z\beta} \tag{2}$$

then we obtain:

$$\lambda(t,z,\beta) = \lambda_0(t)e^{z\beta} \tag{3}$$

The model that could be expressed by means of equation (1) is called the Proportional Hazards Model and we can generalize it to cover any number of systemic variables:

$$\lambda(t, z_1,...z_n, \beta_1,..., \beta_n) = \lambda_0(t)\, r(z_1,...z_n, \beta_1,..., \beta_n) \tag{4}$$

after incorporating the Cox model we will get:

$$\lambda(t, z_1,...z_n, \beta_1,..., \beta_n) = \lambda_0(t)e^{z_1\beta_1+...+z_n\beta_n} \tag{5}$$

The exponential form of function $r(z,\beta)$ guarantees that the intensity function assumes non-negative values irrespective of the value of coefficients.

The distribution used most frequently when analyzing reliability is the exponential distribution for which the intensity of defects is constant and thus also the relationship of intensity for two groups of data related to defined defects will also be constant, which meets the requirements of the Proportional Hazards Model. By assuming exponential distribution we rule out the possibility of accounting for the influence of time. For this reason we should consider the possibility of using Weibull distribution, however the conditions of its application must be assumed in such a way so that the assumptions of the Proportional Hazards Model are met. Accordingly the intensity of defects for Weibull distribution will have the following form:

$$\lambda = \frac{\alpha}{\eta^{\alpha}} t^{\alpha-1} \tag{6}$$

while the intensity ratio will be:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{\alpha_1}{\eta_1^{\alpha_1}} t^{\alpha_1 - 1}}{\frac{\alpha_2}{\eta_2^{\alpha_2}} t^{\alpha_2 - 1}} = \frac{\alpha_1}{\alpha_2} \cdot \frac{\eta_2^{\alpha_2}}{\eta_1^{\alpha_1}} \cdot \frac{t^{\alpha_1 - 1}}{t^{\alpha_2 - 1}} \tag{7}$$

The above relationship shows that the assumptions of the proportional model will be met for Weibull distribution only if the shape parameter remains constant for both data groups. Application Proportional Hazards Model in control diagnostic of bearings have been presented by authors in [4]

Bayesian Updating is another approach enabling one to account for the information obtained not only during control diagnosis but also during observation of degradation and fatigue processes.

BAYESIAN UPDATING

The issue attracting the biggest interest from the point of view of value of diagnostic information is the use of the results of a diagnostic experiment for explaining the mechanism of influence of random factors. Let us note that generally this function can be analyzed as a conditional random variable. The conditions can be imposed by both, the relevantly defined external and internal factors, maintenance and repair activities, as well as the phase of defect development. Adoption of the last model results in use of Bayesian formulas, used to determine the influence of a posteriori information in the examination of probability distribution parameter.

Let us consider the problem of estimation of the probability density function while using the conditional probability model $f(x/W)$ for a given set of data $D=\{x_1,...,x_N\}$. Since the proper value of parameter W is uncertain, thus in the case of Bayesian model we assume that the analyzed figure is a random variable with a defined distribution as per Jaynes' rule [6]: if there are no reasons for adopting the a priori distribution, then monotonous distribution should be assumed. When observing the D data set, we can see the updating of the data for conditional a posteriori distribution $f(W/D)$ as per Bayesian formula:

$$f(W/D) = \frac{f(D/W) \cdot f(W)}{f(D)} \tag{8}$$

A good illustration of the discussed method of using Bayesian formula to evaluate the parameters of distribution based on diagnostic information is demonstrated by Cruse in [2] where Bayesian theorem is used to determine the value of the parameters describing the growth of fatigue-related defect while accounting for the observation of crack development.

The essence of this approach involves updating the estimated parameters of a probabilistic model in order to achieve higher compliance of results of modeling and observation.

In accordance with the assumptions presented above, we assume that the parameters that are either unknown or uncertain are random variables. The uncertainty of estimation of parameters can be associated with the changes of random variables with the use of Bayesian theorem.

Then, while assuming that we will estimate the parameters of the a priori distribution of parameter *a-f(a)* and that *D* is an observation set, the parameters of the a posteriori distribution will have the following notation:

$$f(a/D) = \frac{f(D/a)f(a)}{f(D)} \tag{9}$$

where: $f(D) = \int_{-\infty}^{\infty} f(D/a)f(a)da$

Additionally it is assumed that the denominator which contains the integral of the function of the a posteriori probability density is constant and equals *1* and that *f(D/a)* is the probability of observation that can be expressed by a likelihood function. Equation (9) can be expressed in the following form:

$$f(a/D) = K_B \cdot L[D/a] \cdot f(a) \tag{10}$$

where:

K_B - the normalizing constant

$L[D/a]$ - likelihood function

As has been indicated in the quoted paper, such an approach allows reduction of uncertainty of evaluation conducted on the basis of a small number of results obtained in comparable conditions.

Let us note that Bayesian formula (9) can be written as a ratio of a posteriori and a priori distribution:

$$f(a/D) = \frac{f(D/a)f(a)}{f(D)} \propto L(D/a)f(a) \tag{11}$$

In addition let us note that according to Jeffreys's law [7] the a priori density of probability is proportionate to the square root of the determinant of the Fisher information matrix:

$$f(a) \propto (\det \mathrm{I}(a))^{1/2} \tag{12}$$

where:

$\mathrm{I}(a) = -E\left[\frac{\partial^2 \ln L(D/a)}{\partial a^2}\right]$ - is calculated as the matrix of average second derivatives of the logarithm of likelihood function determined on the basis of the experiment's results.

Thus ultimately formula (11) has the following form:

$$f(a/D) \propto L(D/a)(\det \mathrm{I}(a))^{1/2} \tag{13}$$

While analyzing the experience from applying Bayesian formula in medical diagnosis [3], we can extend the presented method of using the a posteriori information to the task of similar evaluation of results obtained in a diagnostic experiment.

Example

A redesign is being reliability tested. Test data from the original design is available (on the based Weibull++ ReliaSoft). The original time-to-failure data set is as follows:

1268	5405	6217	7949	9636	12016
3640	5425	6736	8540	9847	12145
4127	5814	7456	9158	10006	12477
4173	5822	7523	9351	11747	12620
5264	6206	7865	9624	11817	13650

The test of the redesign yielded the following data set after 11911 hours of testing.

Time	F/S
7908	F
8843	F
11911	F
11911	S
11911	S
11911	S

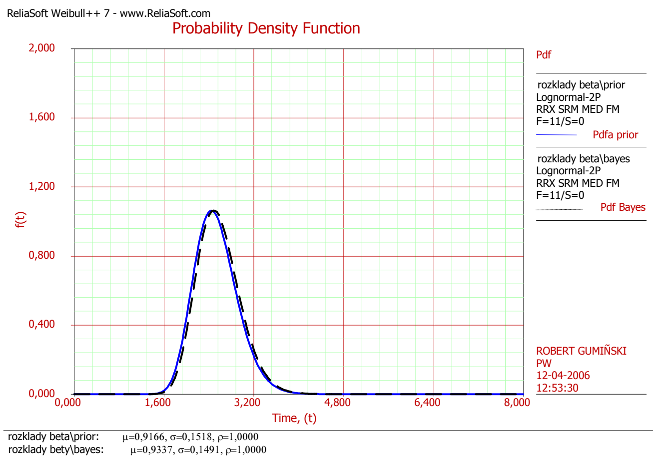

Figure 1 – The shape parameter a priori and a posteriori probability density function.

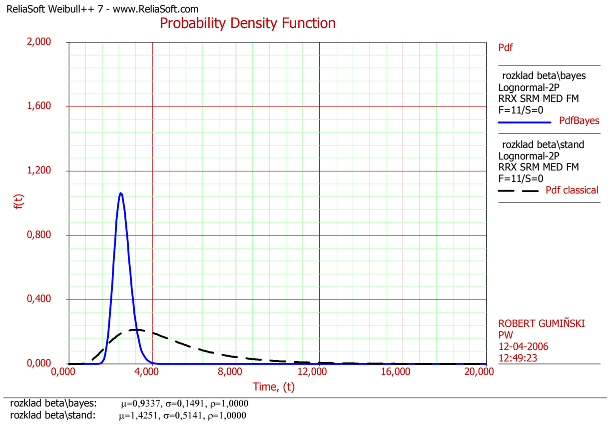

Figure 2 – The shape parameter probability density function obtained by classical statistic methods and Bayesian Updating

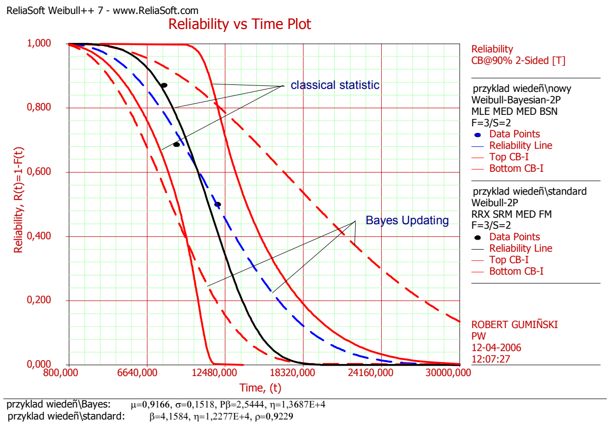

Figure 3 – 90% confidence bounds of reliability function obtained by classical statistic methods and Bayesian Updating

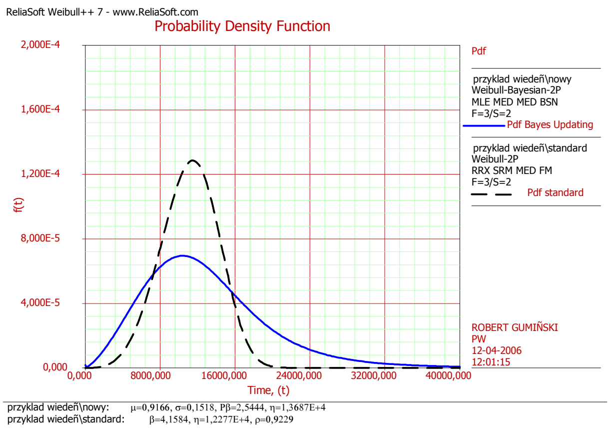

Figure 4 – The probability density function of the time to failure obtained by classical statistic methods and Bayesian Updating

SUMMARY

The quality of manufacturing and assembly as well as the occurring degradation processes can on the one hand lead to changes of intensity of defects and on the other be the reason of change of probability distribution parameters in the function of time and change of systemic variables. Use of vibroacoustic diagnosis methods offers a possibility for accounting for the influence of defect development phases, on the intensity of defects in the examined elements of kinematic nodes. Use of Proportional Hazards Model is possible in diagnosis conducted during inspections while operational diagnosis requires use of models in which the intensity of defects depends on two values: the systemic variable and the time of operation. Use of a posteriori information obtained as a result of observation or active diagnostic experiment and application of Bayesian model leads to reduction of uncertainty while estimating the impact of the degradation phase on an element's reliability and while estimating the level of technical risk. Next task is the investigation of the limitation of Bayesian Updating application in vibroacoustic diagnostic.

The research is financed from the funds of the Ministry of Science and Development of Information Technologies for years 2005-2008 as a requested research project

REFERENCES

[1] Cox D.R., Regression models and life tables, Journal of Royal Statistical Society, series B, Vol. 34 pp 187-220, (1997)

[2] Cruse T. A. Reliability – Based Mechanical Design, Marcel Dekker, Inc. New York (1997)

[3] Dębicka J. Risk Assessment method in life insurances, Economic Academy Publisher, Wrocław (2000) (in Polish)

[4] Gumiński R., Radkowski S. Use of Proportional Hazards Model in risk analysis in the light of global approach directives, Tehoss Gdańsk 2005r

[5] Gumiński R., Radkowski S., Sobczykiewicz W., Using of proportional hazards models in reliability analysis at example of human – technological system, XVIII Conference of Scientific Trends in the Development of Heavy Duty Machines, Zakopane (2005) (in Polish)

[6] Jaynes E.T.: Information Theory and Statistical Mechanics, Physical Reviews, Vol., 106, , str. 620÷630 (1957)

[7] Jeffreys H., Theory of probability, Data, Oxford University Press, Oxford 1961