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THE ANALYSIS OF THE TURBULENCE DRIVEN PLATE RESPONSE IN PRESENCE OF ADDED MASSES

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Abstract
In the present work analytical and experimental responses of a simple flexural plate are discussed. The plate is excited by a wall pressure distribution due to a turbulent boundary layer. It is deeply investigated the influence of the transducer mass on the response. Specifically, it was analysed if the random and convective nature of the load was such to alter the behaviour of the sensors. Exact analytical expansions were assembled in order to model the singularity of the response at the locations of the transducer mass and further to highlight the role played by the correlation areas. The results of the numerical model cross checked with the available experimental results obtained at the ælab/DPA wind-tunnel.

INTRODUCTION

Previous theoretical results compared with wind-tunnel measurements have been devoted to the analysis of the plate response under a turbulent boundary layer excitation. Specifically these past activities have demonstrated the possible role played by the correlation area when analysing the effect of transducer mass(es), [1,2].

The present work represents a further step toward the same research goal. It has been performed by using a theoretical model build to get the stochastic response of a reference plate with a single concentrated mass and the configurations with two added masses. The solution responses have been built by using modal expansions with exact integrations, since a simply supported test plate was considered under the turbulent boundary layer excitation model proposed by Corcos (separable variables), [3]. The models of the plate and the turbulent boundary layer are the simplest possible, but the formulation of the problem contains all relevant parameters and it could be quickly exported in numerical predictive codes. The mass singularities were introduced by using Dirac functions directly in the generalised mass matrix that is the mass operator projected on the modal base. The enclosed results seem to demonstrate the role played by the correlation area associated with such random and convective load. In fact, for the asymptotical flow speeds available in the ælab/DPA wind-tunnel facility, two main effects were measured and predicted: (i) the validity of the standard literature formula for analysing the effect of the transducer mass [4], and (ii) the transducer mass is not able to sense the presence of another mass outside of its correlation area, [2]. Particular attention was devoted to the application of the exact and validated theoretical model for increasing flow speeds. A comparison between configurations loaded with stochastic convective and non-convective excitations is discussed, too. The fundamental investigation on a plate response is reported in [4], where the effect of an added mass in the presence of a stochastic pressure distribution is discussed. Asymptotical relationships, obtained under the Asymptotical Modal Analysis (A.M.A.) hypotheses, are used to take into account the variations introduced by this added mass on the overall response of a test plate. The same relationships determine the measurable frequency range with reference to a given value of the accelerometer mass. It was studied if those relations can be applied also on the plate response when driven by a TBL.

ANALYTICAL PLATE RESPONSE

The fluctuations of the wall pressure field due to the TBL can be characterized by the cross spectral density function:

$$X_{pp}(\xi;\omega) = \langle p(\mathbf{r};\omega)p(\mathbf{r}+\xi;\omega)\rangle \tag{1}$$

where the symbol < > denotes the statistical average, p is the pressure and ω denotes the excitation radian frequency. The vector **r** denotes the distance of a given point from the origin of the reference system, while the vector ξ denotes the distance between two given points, Fig.1. The X_{pp} function is complex and can be expressed as follows:

$$X_{pp}(\xi;\omega) = S_p(\omega)\Gamma(\xi;\omega) \tag{2}$$

that is by using the product of the power spectral density (the auto power) S_p, and a function Γ depending on the geometry; both are frequency dependent. Γ will represent the Fourier transform of the correlation between two points whose distance is $\xi(\xi_x,\xi_y)$: the square modulus of Γ is the coherence function. Corcos proposed a simplified formulation of the cross spectrum, [3]:

$$\begin{gathered} X_{pp}\left(\xi_x,\xi_y,\omega\right) = S_p(\omega)\exp\left(-\alpha_x\left|\frac{\omega\xi_x}{U_c}\right|\right)\exp\left(-\alpha_y\left|\frac{\omega\xi_y}{U_c}\right|\right)\exp\left(-\frac{i\omega\xi_x}{U_c}\right) \\ L_x(\omega) = \frac{U_c}{\alpha_x\omega}; \quad L_y(\omega) = \frac{U_c}{\alpha_y\omega}; \quad \frac{U_c}{U_\infty} = k \end{gathered} \tag{3}$$

The coefficients α_x and α_y indicate the loss of coherence in longitudinal and transverse directions; U_c is the convection speed; i is the imaginary unit. The symbol k denotes a constant value; U_∞ is the undisturbed flow speed; L_x and L_y are the correlation lengths. For smooth walls, commonly accepted values are: α_x=0.116, α_y=0.7, and k=0.8, [5]. Enhanced models are due to Efimtsov, [6].

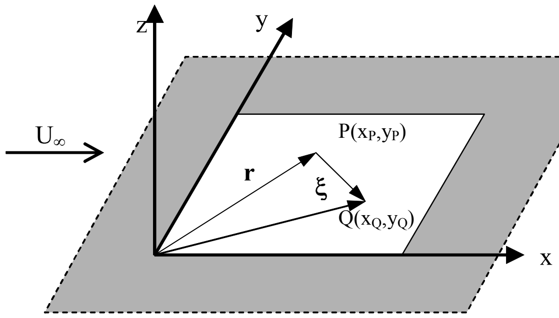

Fig.1 – Sketch of the Elastic Plate (white area) in the Rigid Aerodynamic Baffle.

The plate is thin, flat, rectangular and isotropic; it is simply supported on all four edges and mounted in an infinite rigid plane baffle flush with the TBL, Fig.1. It is assumed that any fluid-loading effect on the structural dynamic response can be neglected, too. The plate belongs to the xy plane, and the flexural out-of-plane displacements, named w(x,y;t), are along the z axis. The general basic equation can be written as follows for the flexural plate transverse displacements under a generic pressure load p(x,y;t):

$$D\nabla^4 w(x,y;t) + \rho h \frac{\partial^2 w(x,y;t)}{\partial t^2} = p(x,y;t) \quad \text{with} \quad D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{4}$$

The side lengths of the plate are a and b, stream and cross wise, respectively, and h is the thickness; D is the flexural stiffness and E, ν and ρ denote the material constants. Along the sides all the displacements are zeroed. The displacement cross spectral density between any arbitrary pair of points, $A(x_A,y_A)$ and $B(x_B,y_B)$, in terms of modal expansion and due to an assigned stochastic distributed excitation, is given as follows, [7]:

$$X_w\left(x_A,y_A,x_B,y_B,\omega\right) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left[\frac{\psi_j\left(x_A,y_A\right)\psi_k\left(x_B,y_B\right)}{L_j^*(\omega)L_k(\omega)}\right]\left[\frac{S_p(\omega)(ab)^2}{\gamma_j\gamma_k}\right]A_{Q_jQ_k}(\omega) \tag{5}$$

with

$$A_{Q_jQ_k}(\omega) = \int_0^a\int_0^a\int_0^b\int_0^b\left[\frac{X_{pp}\left(x,y,x',y',\omega\right)}{S_p(\omega)(ab)^2}\psi_j(x,y)\psi_k\left(x',y'\right)\right]dydy'dxdx' \tag{6}$$

The integrals defined by the symbol $A_{Q_jQ_k}$ are well known as the modal acceptances; ψ_j and ω_j are the mode shapes and the natural radian frequencies; m_j and n_j are the integers denoting the number of half wavelengths for the j-th mode; η is the structural damping.

$$\psi_j(x,y) = \sin\left(\frac{m_j \pi x}{a}\right)\sin\left(\frac{n_j \pi y}{b}\right); \gamma_j = \frac{ab}{4}; m_{Gj} = \frac{\rho hab}{4}; \quad L_j(\omega) = m_{Gj}\left[\omega_j^2 - \omega^2 + i\eta\omega_j^2\right];$$
$$L_j(\omega) = H_j^{-1}(\omega); \qquad \omega_j = \left(\frac{Eh^2}{12\rho(1-\nu^2)}\right)^{\frac{1}{2}}\left[\left(\frac{m_j \pi x}{a}\right)^2 + \left(\frac{n_j \pi y}{b}\right)^2\right] \tag{7}$$

The auto spectrum of the displacement at a selected point is given as follows:

$$S_w(x_A, y_A, \omega) = \sum_{j=1}^{\infty}\left[\frac{\psi_j^{\,2}(x_A, y_A)}{\left|L_j\right|^2} S_p(\omega)(ab)^2 A_{Q_jQ_j}(\omega)\right] + \\ 2\,\mathrm{Re}\left\{\sum_{j=1}^{\infty}\sum_{k=j+1}^{\infty}\left[\frac{\psi_j(x_A, y_A)\psi_k(x_A, y_A)}{L_j^*(\omega)L_k(\omega)}\right](ab)^2\left[S_p(\omega)A_{Q_jQ_k}(\omega)\right]\right\} \tag{8}$$

It has also to be noted that it is possible to evaluate the mean response in terms of acceleration without any added mass by using the following relationship:

$$\overline{S}_a(\omega) = \frac{\omega^4}{ab}\int_{ab} S_W(\omega, x, y)\,dxdy \tag{9}$$

Due to the orthogonality of the modes, the modal cross terms do not give any contribution to these mean spectra. Similar quantities could be found by accepting an average over a selected number of points, NG:

$$\overline{S}_a(\omega) = \frac{1}{NG}\sum_{i=1}^{NG} S_a(\omega, x_i, y_i) \tag{10}$$

The computational effort in evaluating Eq.(10) is obviously much greater than for Eq.(9). At increasing excitation frequency the differences between the two mean spectra become indistinguishable, and then the detailed local response is no longer required, [1,2].

The mass of the system in Eq.(4) can be modified for taking into account the presence of a concentrated mass by using the Dirac δ function:

$$D\nabla^4 w(x,y;t) + \left[\rho h + \frac{m_{acc}}{ab}\delta(x-\overline{x}, y-\overline{y})\right]\frac{\partial^2 w(x,y;t)}{\partial t^2} = p(x,y;t) \tag{11}$$

When evaluating the generalised mass matrix member, there will be several possibilities, according to the solution of the following integral for the mode pair i and j:

$$m_{Gij} = \int_0^b\int_0^a [\rho h + \frac{m_{acc}}{ab}\delta(x-\overline{x}, y-\overline{y})]\psi_i(x,y)\psi_i(x,y)\,dxdy \tag{12}$$

For the cases herein presented (single and double added masses), it was assumed that the modal base (mode shapes and natural frequencies) is unaffected by the (single and double) small added mass. The exact mode shapes and natural frequencies of the original uniform plate have been used as base expansion for calculating the mode shapes and natural frequencies of the plate with the added mass, [2,8,9 and 10]. At a random point $T(x_T,y_T)$, a Dirac δ–function can be imposed in order to simulate the presence of a concentrated mass (m_{acc}), [10].

The generalised mass matrix, can be written as follows:

$$m_{G_{i,j}} = \begin{cases} \rho h \dfrac{ab}{4} + \psi_i^2(x_T, y_T) m_{acc} & \text{if } i = j \\ \psi_i(x_T, y_T)\psi_j(x_T, y_T) m_{acc} & \text{if } i \neq j \end{cases} ; LF_{i,j}(\omega) = \begin{cases} m_{G_j}\left[\omega_j^2 - \omega^2 + i\eta\omega_j^2\right] & \text{if } i = j \\ m_{Gi,j}\left[-\omega^2\right] & \text{if } i \neq j \end{cases} ; \quad (13)$$

In matrix form:

$$S_{and}(\omega, x_A, y_A, x_A, y_A; x_T, y_T) = \omega^4 [\psi(x_A, y_A)][LF^{-1}(\omega)][B(\omega)][LF^H(\omega)]^{-1} [\psi(x_A, y_A)]^T \quad (14)$$

The *hermitian* of a given matrix (transpose and complex conjugate) is denoted by H. S_{and} denotes the response in terms of acceleration at the point $A(x_A,y_A)$ with an added mass at $T(x_T,y_T)$.

An inversion of two complex matrices, LF and LF^H, is required for each excitation frequency. These matrices are both square of order NM, where NM are the number of modes retained for getting the response. The choice of the given added masses was based on the same characteristic ratios presented in [4], as the mass of the plate over the added mass, for sake of completeness.

The presence of two added masses can obtained simply modifying the expression of the generalised mass matrix; if one is at $Q1(x_{Q1},y_{Q1})$, and the second at $Q2(x_{Q2},y_{Q2})$, one gets:

$$m_{G_{i,j}} = \begin{cases} \rho h \dfrac{ab}{4} + \psi_i^2(x_{Q1}, y_{Q1}) m_{acc} + \psi_i^2(x_{Q2}, y_{Q2}) m_{acc} & \text{if } i = j \\ \psi_i(x_{Q1}, y_{Q1})\psi_j(x_{Q1}, y_{Q1}) m_{acc} + \psi_i(x_{Q2}, y_{Q2})\psi_j(x_{Q2}, y_{Q2}) m_{acc} & \text{otherwise} \end{cases} ; \quad (15)$$

EXPERIMENTAL MEASUREMENTS

This work was preceded by a set of experimental measurements, here briefly recalled [1,2]. The measurements were carried out in the wind tunnel at the Department of Aeronautical Engineering, DPA.

The characteristics of the aluminium plate are: a=0.768 m; b=0.328 m; h=0.0016 m; E=7.0 10^{10} Pa; ν=0.33; ρ=2700 kg m^{-3}. The plate was excited by the TBL produced in the wind tunnel at three different flow speeds: 25, 33 and 40 m/s. The last speed is the maximum possible in the DPA facility. The sensor array was composed by PCB accelerometers (mod.352B10) weighing 0.7 g each. No specific aerodynamic measurements for characterising the TBL loading the plate were performed.

In order to investigate the effect of an added mass over the response, the selected strategy was the same used in [4] and detailed in [1 and 2]. The aim of this experimental acquisition was to get a generic kinematic response, R. At the i-th point, this can be so written:

$$R = R(U, m_{add}, \omega, P_i) \quad (16)$$

There is the possibility to average over two types of points, for each asymptotical flow speed (U), added mass (m_{acc}) and excitation frequency (ω). The first type of points, R_O, is represented by those without added mass but this mass is present at another one. The second type, R_E, takes into account the presence of an added mass in addition to the accelerometer. Two different means over the responses can be defined, accordingly. For analyzing the effect of the sensor mass, a response ratio (named γ) was accordingly defined:

$$\gamma(U, m_{acc}, \omega) = R_E(U, m_{acc}, \omega)\left[R_O(U, m_{acc}, \omega)\right]^{-1}; \tag{17}$$

This last ratio is the same as theoretically defined and measured in [4] for estimating the effect of the sensor mass; for sake of completeness, it is here reported the proposed approximation obtained with the Asymptotical Modal Analysis (AMA) for a given plate:

$$g(\omega, m_{acc}) \approx \left(\frac{2}{\pi^2}\frac{\rho h}{m_{acc}}\lambda(\omega)^2\right)^2\left[1+\left(\frac{2}{\pi^2}\frac{\rho h}{m_{acc}}\lambda(\omega)^2\right)^2\right]^{-1}; \tag{18}$$

where m_{acc} denotes again the added mass, λ is the plate flexural wavelength, ρ is the mass density of the plate, h is the plate thickness and ω is the radian excitation frequency.

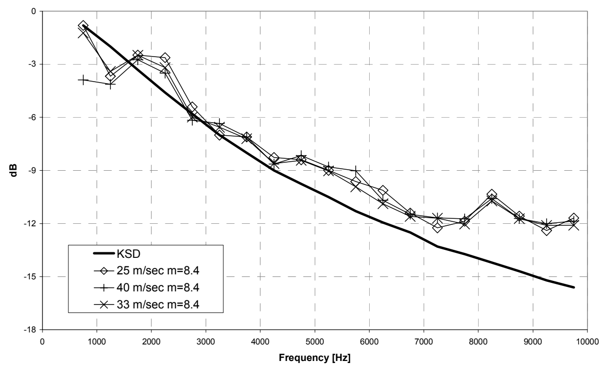

Figure 2: Response ratios [dB] vs. Freq.[Hz]– thick black line, g as in Eq.(19); thin lines obtained by experimental measurements, γ as in Eq.(18) - from [2].

In the related figures this equation will be always labelled as KSD. Eq.(19) allows the use of the cited miniature accelerometers in the whole frequency range of analysis, 0-10kHz (tolerance $\leq$ 1dB). For both the ratio, γ or g, the 0 dB value means no influence in the plate response due to the sensor mass. The results in Fig.2 demonstrated that for the measured configurations, the relation in Eq.(19) can be used also for random and convective load, and further there is no dependence on the flow speed. The lack of agreement above 5Khz is due to the approximation associated with AMA, [4].

EXPERIMENTS FOR A SINGLE MASS

The past numerical and experimental data have to be completed in order to be sure of the full validity of the results, [1,2]. A final comparison was prepared among exact structural responses replicating the experimental measurements. The added mass was 8.4 g, and the number of modes used in the expansion was set to 980, for sake of convergence: the last mode resonates at 30697 Hz and the range of analysis can reach

8 kHz with full convergence. The mode shapes and natural frequencies of the configuration of the plate with a single mass were tested by using the finite element method. It is useful to recall the definition of the modal overlap factor: $\mu(\omega)=\eta(\omega)\,n\,\omega$; where n is the flexural modal density. For the present plate, the modal overlap factor, μ, becomes of unit order of magnitude at f_M=935 Hz; for $f>f_M$, one can assume that the modal behaviour disappears.

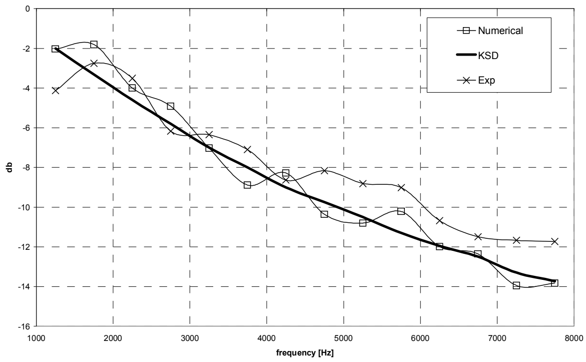

Figure 3: Response ratios [dB] vs. Freq.[Hz] for an added mass of 8.4 g at U=40 m/s. thick black line, g as in Eq.(19).

The mean numerical responses presented in Fig.3 and 4 have been obtained by averaging over 4 random positions. The comparison in Fig.3 again demonstrates that the approximation proposed in Eq.(19) is still useful when a random and convective load is used; further, the exact model in Eq.(13,14) is able to reproduce the effect of the added mass. Then, having a validated numerical tool for analysing the effect, it was prepared a new comparison at wind tunnel speed well higher than the maximum available at the DPA, Fig.4, finding again the same kind of results.

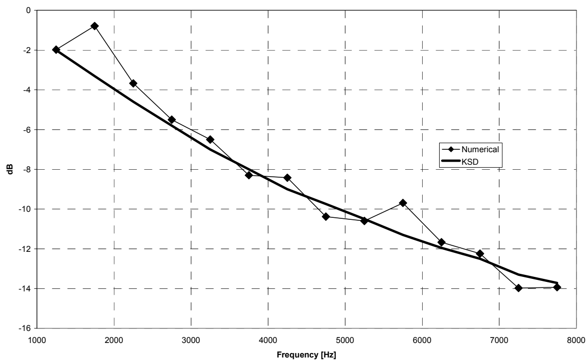

Figure 4: Response ratios [dB] vs. Freq.[Hz] for an added mass of 8.4 g at U=115 m/s. thick black line, g as in Eq.(19).

The present tests strengthen also the two masses analyses reported in [2]. The analytical responses under generic random convective and non-convective excitations were studied, too. The results are very preliminary and the analytical model seems to be able to take into account the role played by the correlation areas. The numerical runs are quite expensive in terms of computational time: 360 seconds per excitation frequency (FORTRAN code running on a Pentium IV-M / 256MB RAM /800 MHz).

SUMMARY

The exact results shown in the present work demonstrated that the effect of an added mass over the structural response of a simple plate is not influenced by the random and convective nature of the wall pressure distribution due to a turbulent boundary layer. Numerical calculations were performed on exact modal expansions in order to investigate the dynamic responses of a very simple plate. The tests have been developed by adding a single mass and calculating the response at the location of the added mass. This was simulated by using a Dirac δ function in the definition of the mass distribution and by modifying the generalised mass matrix to be used in the modal expansions. The validity of the analytical results here reported and compared with the experimental one demonstrated the applicability of the pursued theoretical approach.

ACKNOWLEDGMENTS

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