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INVESTIGATION OF NONLINEAR ACOUSTIC PROPERTIES FOR PERFORATES

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Abstract
This paper presents the results of a study of non-linear acoustic properties of perforates and micro-perforates. The results are potentially of interest for perforates and other facing sheets used in aircraft engine liners as well as perforate pipes used in automotive mufflers. In the linear limit the perforate acoustic impedance is independent of the sound field but when the sound pressure level is high it will be dependent on the acoustic particle velocity in the holes. For pure tone excitation the impedance will be controlled by the acoustic particle velocity at that frequency. If the acoustic excitation is random or periodic with multiple harmonics the impedance at a certain frequency will depend on the particle velocity at other frequencies. In this paper a study has been made of harmonic interaction effects by using multiple pure tone excitation and random noise excitation.

1. INTRODUCTION

In Ref. 1 experimental techniques for determining acoustic impedance and flow resistance for perforates under non-linear conditions were discussed. Experiments were made using both pure tone and random excitation and the relevant parameters controlling the non-linearity were discussed. In Ref. 2 a study of harmonic interaction effects using two-tone excitations was made. In the linear case the impedance is independent of the sound field but when the sound pressure level is high the perforate impedance will be dependent on the acoustic particle velocity in the holes. For pure tone excitation it is obvious that the impedance will be controlled by the acoustic particle velocity at that frequency. If the acoustic excitation is random or periodic with multiple harmonics the impedance at a certain frequency may depend on the particle velocity at other frequencies. The total rms-value or the Reynolds number based on the particle velocity in the holes are for instance candidates for being the relevant parameters. In this paper the experimental techniques discussed in Ref. 1 and 2 are further developed for studying this problem. A study is made of the harmonic interaction effects by using multiple pure tone excitation. By changing the amplitude level of one to four additional pure tone components the effect on the impedance at the studied frequency is investigated.

Many investigations of nonlinear effects occurring when high amplitude sound waves are incident on perforated plates or orifice plates have been published, see e.g., Ref. 3-8. In many of the early works a standing wave tube with single frequency excitation was used. Ingård and Ising [5] in their classical study on nonlinearity of orifices used a differential pressure measurements and a hotwire probe in the hole, still with pure tone excitation. Maa [9-11], has studied linear and nonlinear behaviour of so-called micro-perforates. There is no generally accepted definition of what a micro-perforate is except that the holes are small with diameters smaller than around 0.5 mm in the case of circular holes. It is generally agreed that the non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings [12-15]. Tam et al [13-15], have studied slit resonators using direct numerical simulation in [14-15] also validated using impedance tube measurements.

2. MODELS FOR PERFORATE IMPEDANCE

In Ref. 7-8 a semi-empirical model for perforate resistance and reactance was presented. It summarizes the previously published models and adds a few improvements to the resistance and reactance end corrections and the resistive and reactive terms associated with nonlinearities and grazing flow. Removing the terms associated with grazing and through flow which are not relevant for the present study gives

$$\theta = \mathrm{Re}\left(\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')}+\frac{\delta_{re}}{F(\mu)}f_{\mathrm{int}}\right]\right)+\frac{1}{\sigma}\left[1-\frac{2J_1(kd)}{kd}\right]+\left(\frac{1-\sigma^2}{\sigma^2 C_D^2}\right)\cdot\frac{1}{2c}|v_n|, \qquad (1)$$

$$\mathrm{X} = \mathrm{Im}\left(\frac{jk}{\sigma C_D}\left[\frac{t}{F(\mu')}+\frac{0.5d}{F(\mu)}f_{\mathrm{int}}\right]\right)-\left(\frac{1-\sigma^2}{\sigma^2 C_D^2}\right)\cdot\frac{1}{2c}\cdot\frac{|v_n|}{3}, \qquad (2)$$

where θ is the normalized resistance and X is the normalized reactance, k is the wave number, σ is the porosity (percentage open area), C_D is the discharge coefficient, t is the plate thickness, μ is the adiabatic dynamic viscosity, $\mu = 2{,}179\mu$ is the dynamic viscosity close to a conducting wall, $\nu = \mu/\rho$ is the kinematic viscosity, J is the Bessel function, d is the hole diameter, c is the speed of sound, v_n is the peak value of the acoustic particle velocity incident on the sample. The rest of the parameters are defined as

$$K = \sqrt{-\frac{j\omega}{\nu}}, \tag{3}$$

$$F(\mu) = 1 - \frac{4J_1(Kd/2)}{Kd \cdot J_0(Kd/2)}, \tag{4}$$

$$\delta_{re} = 0.2d + 200d^2 + 16000d^3, \tag{5}$$

$$f_{\text{int}} = 1 - 1.47\sqrt{\sigma} + 0.47\sqrt{\sigma^3}. \tag{6}$$

The third term in (1) and the second term in (2) represent the nonlinear contributions proportional to the acoustic particle velocity. The second term in (1) is usually small compared to the first linear term and can be neglected. These equations will be used for comparison with the experimental results.

Comparison will also be made with the model suggested by Maa [9-11] for micro-perforates

$$\theta = \frac{32\nu}{\sigma\ c}\frac{t}{d^2}\left[\sqrt{1 + k_p^2/32} + \frac{\sqrt{2}k_p d}{4t}\right] + \frac{|v_n|}{\sigma^2 c}, \tag{7}$$

$$\mathrm{X} = \frac{\omega t}{\sigma\ c}\left[1 + \left(9 + \frac{k_p^2}{2}\right)^{-1/2} + 0.85\frac{d}{t}\left(1 + \frac{|v_n|}{\sigma^2 c}\right)^{-1}\right], \tag{8}$$

$$\text{where } k_p = \frac{dK}{2\sqrt{-j}}. \tag{9}$$

Additional comparisons will also be performed using an equation suggested by Maa for slit shaped holes.

$$\theta = real\left(\frac{j\omega t}{\sigma\ c}\left[1 - \frac{\tanh(k_p\sqrt{j})}{k_p\sqrt{j}}\right]^{-1}\right) + \frac{\sqrt{2\nu\omega}}{\sigma c} + \frac{|v_n|}{\sigma c} \tag{10}$$

$$\mathrm{X} = imag\left(\frac{j\omega t}{\sigma\ c}\left[1 - \frac{\tanh(k_p\sqrt{j})}{k_p\sqrt{j}}\right]^{-1}\right) \tag{11}$$

where the perforate constant k_p in this case is based on the width (w) of the slit

$$k_p = \frac{wK}{2\sqrt{-j}}. \tag{12}$$

3. TEST SETUP

The standard in-duct two-microphone method [16-17] was used to produce the main part of the results. Two different impedance tubes were used. Sketches of the test rigs are shown in Figure 1.

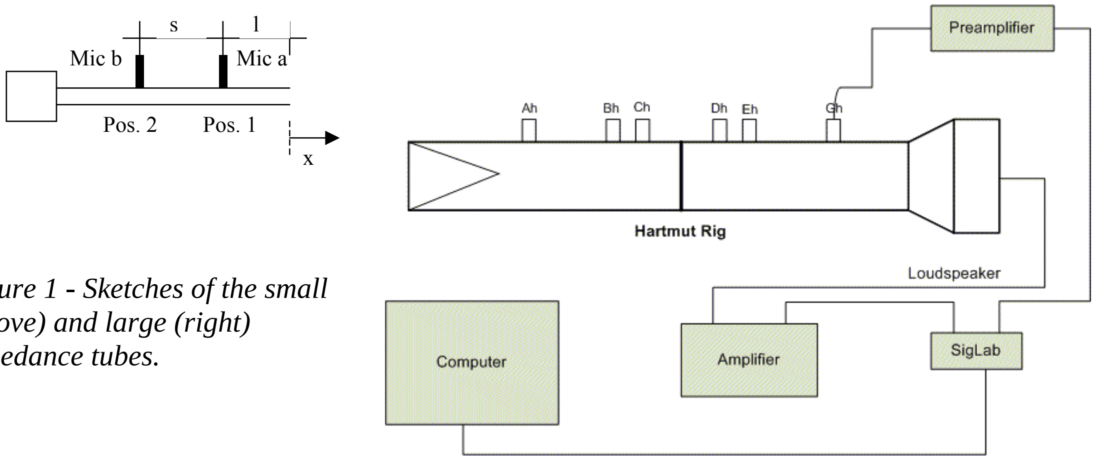

Figure 1 - Sketches of the small (above) and large (right) impedance tubes.

The distance between the microphones was, for the small impedance tube, 30 cm, giving a frequency range of 55 to 455 Hz, and the distance from microphone a to the sample was 15 cm. The sample was mounted in a holder at the end of the measurement pipe. The measurement pipe has an inner diameter of about 40 mm and the opening in the holder has a diameter of 38.5 mm. The impedance at the sample plane was first measured for the empty holder. The sample was then mounted and the impedance was measured again. The perforate impedance was calculated by taking the difference between these two impedances. For the large impedance tube two microphone separations were used 256mm and 32mm giving the frequency ranges 67 to 535 Hz and 535 to 4280 Hz. Measurements were however only made up to the upper limit of the plane wave range around 2000 Hz. The duct diameter and sample size was around 10 cm. The sample was in this case mounted in the middle of the pipe with an absorbing termination on the downstream side. Measurements were again made with and without the sample.

Measurements in the small impedance tube were made for four different circular hole samples, as specified in Table 1, plus some perforates with slit-shaped holes.

Table 1- Specification of circular hole perforate samples tested in the small impedance tube..

Sample	d [mm]	t [mm]	σ
P1	3	1	0.28
P2	2	1.5	0.032
P3	2	2	0.086
P4	1	2	0.020

Measurements were first made with pure tone excitation at 110 Hz. In the second step a two tone excitation was used where the level and frequency of the second pure tone was varied. Then three four and five tones were added. The following excitation frequencies were used in combination with the 110 Hz excitation: 60, 120, 220, 440, Hz. Specifications for the test samples used in the large impedance tube are shown in Table 2 and 3.

Table -. Specification of circular hole perforate samples tested in the large impedance tube.

Sample	d [mm]	t [mm]	σ
C1	0.5	0.5	0.04
C2	0.5	0.5	0.0134
C3	0.4	0.6	0.0056

Table 3- Specification of slit-shaped hole perforate samples tested in the large impedance tube.

Sample	Average width	Thickness	Length	Distance	σ
S1	0.240mm	1mm	4.4mm	1.25mm	0.065
S2	0.095mm	1mm	4.0mm	1mm	0.043

4. EXPERIMENTAL RESULTS AND DISCUSSION

Compared to the results presented in [2] further measurements have been made using multi-tone excitation and with higher level of excitation. Another difference compared to results presented in [2] is that plots have been made against peak amplitude rather than r.m.s.-value of the particle velocity more in accordance with the models presented in section 2. Figure 2 shows the results at 110 Hz for sample P4 with 2% porosity compared to the models according to equations (1), (2), (7) and (8). It can be seen that the resistance model according to equation (7) agrees quite well with the single tone excitation results. With multi-tone excitation both equation (1) and (7) under-estimates the resistance and the slope of the curve caused by non-linearity. Both equation (2) and (8) under-estimates the initial slope of the measured reactance. When the reactance approaches zero it does not continue to decrease. On the average it instead takes a value around zero for higher particle velocities even if the variance in the data is high. This could be interpreted as the disappearance because of flow separation of the "mass plug" existing in the linear regime. Regarding the variation in resistance and reactance for higher particle velocities some further investigations were made. Each point in Figure 2 represents a measurement with a certain amplitude setting for a certain frequency. If the experiment is repeated with the same settings the same result is obtained. This means that the variation is not random. One possible interpretation is that since the non-linear effects are caused by flow separation in the pulsating flow through the holes the result will depend on the time variation of the flow through the hole. Two different pulsating flows with the same total particle velocity but with different frequency content, different amplitude settings or different phase between the tonal components, will give a different result.

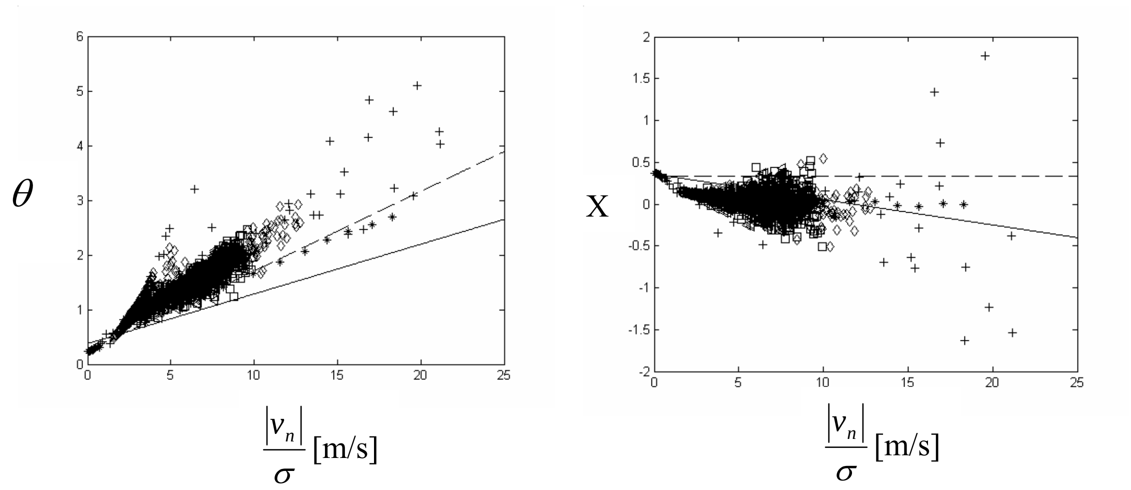

Figure 2 - Normalized perforate impedance for sample P4 at 110 Hz as a function of peak particle velocity in the holes; left resistance, right reactance, measured using different number of tones; stars – single tone, + two tones, diamonds – three tones, squares – four tones, < - five tones, straight line- theory according to equations (1) and (2), dashed line - theory according to equations (7) and (8).

Figure 3 shows the normalized perforate impedance at 400 Hz for sample C1 with 4% porosity obtained using single tone excitation and random noise excitation. The resistance obtained using single tone excitation seems to agree with theory according to equation (1) for higher particle velocities and with equation (7) for low particle velocities. For random excitation the opposite apply, equation (7) gives the best fit at low particle velocities and equation (1) gives the best fit at high particle velocities. For the reactance equation (2) gives a reasonable fit with measured data for single tone excitation while the reactance obtained using random excitation is over-estimated. Comparing the results presented in Figure 2 and Figure 3 one can observe a difference in how well experimental and model results agree. There is a difference in hole size and porosity but this difference is not that large. Another possible explanation is that the difference in downstream boundary condition, free space for Figure 2 and a pipe with absorbing termination for Figure 3, may influence the non-linear impedance results.

Figure 4 shows the normalized perforate impedance at 300 Hz for sample S1 with 6.5% porosity obtained using single tone excitation and random noise excitation. The resistance obtained using single tone excitation seems to agree with theory according to equation (10). For random excitation equation (10) over-estimates the resistance slightly. The reactance according to equation (11) gives an over-estimation of the measured reactance and the difference grows with increasing particle velocity.

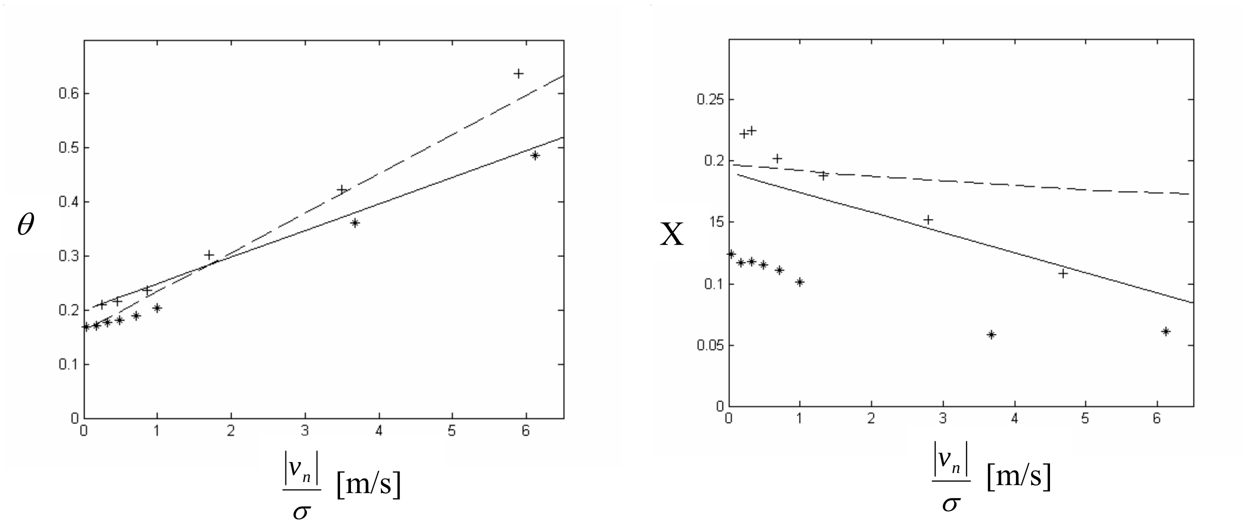

Figure 3 - Normalized perforate impedance for sample C1 at 400 Hz as a function of peak particle velocity; left resistance, right reactance, stars – random noise excitation, + - single tone excitation, straight line- theory according to equations (1) and (2), dashed line - theory according to equations (7) and (8).

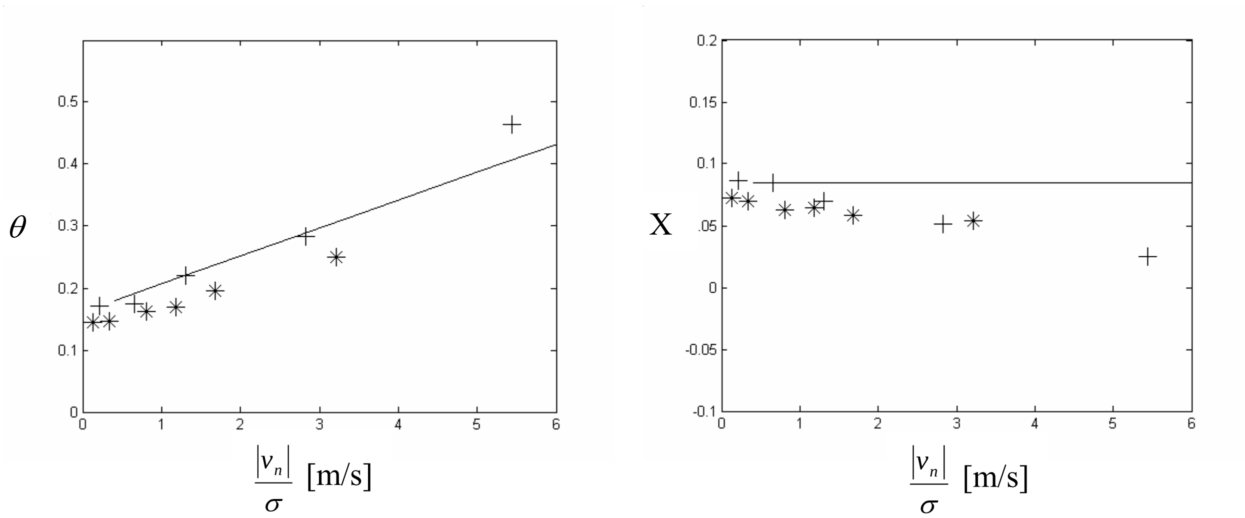

Figure 4 - Normalized perforate impedance for sample S1 at 300 Hz as a function of peak particle velocity; left resistance, right reactance, stars – random noise excitation, + - single tone excitation, straight line- theory according to equations (10) and (11).

5. SUMMARY AND CONCLUSIONS

Results of an experimental study of nonlinear harmonic interaction effects for perforates and micro-perforates has been presented. By using multi-tone excitation and random noise excitation a study of the measured perforate impedance at discrete frequencies could be made while varying the excitation level at other frequencies. The following conclusions could be drawn: There is a nonlinear dependence of both resistance and reactance at the measurement frequency when the excitation is changed at other frequencies. This could have implications for solving duct propagation problems through perforated pipes. There is a reasonable agreement between the results of theoretical models and single-tone excitation measurements. When multi-tone excitation or random noise excitation is used different perforate impedance results are obtained compared to single-tone excitation results.

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