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ACTIVE VIBRATION CONTROL OF A ROTOR USING A NOVEL ADAPTIVE REPETITIVE CONTROL ALGORITHM

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Abstract

The paper investigates the possibility of using repetitive learning control to attenuate rotor vibrations on a rotor test rig. The test rig has a 3-kg rotor supported by journal bearings and a critical speed of approximately 50 Hz. The objective is to control the radial response at the rotor midpoint by using an actuator located outside the bearing span. Control forces are generated with an electro-magnetic actuator. The actual control scheme comprises of two algorithms, an inner loop collocated feedback system and an outer loop repetitive controller. The inner loop controller is utilised to provide favourable characteristics for the repetitive control loop, whereas the repetitive control algorithm compensates for a periodic excitation at the rotor midpoint.

A novel aspect of the controller design is that the length of the control output vector of the repetitive controller is updated as a function of the rotational speed. This approach results in a new adaptive repetitive control algorithm that can be applied to variable-speed machines. The repetitive control method uses a time-reversed FIR model of the control path to make the loop-gain function positive real in the frequency band of interest. A non-causal band-pass filter is applied on the control output vector to restrict high-frequency control actions. The main objective of this filter is to prevent the excitation of high-frequency non-linearities in the control path.

The experimental results illustrate that the repetitive controller successfully attenuates vibrations at the critical speed of the rotor, increasing the operating range of the rotor beyond the critical speed. The attenuation results obtained are comparable to those achieved in earlier studies with feedforward compensation methods. The best results are achieved when the frequency of rotation enables an integer ratio between disturbance period and sample rate.

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INTRODUCTION

Active vibration control solutions for rotors have been studied widely in the research literature in order to mitigate the effects of excitations due to rotation. The objective is either to control the deflection of the rotor or the transmission of vibration into the surroundings [2], [11]. The present paper continues earlier works [16], [17], where feedforward algorithms were used to control vibration on the test rig shown in Fig.1 and Fig. 2. The main focus of the paper is to investigate whether or not a novel adaptive repetitive learning control algorithm can be used to achieve similar attenuation levels as standard feedforward algorithms. The main contributions of this paper are the derivation of the novel adaptive repetitive control algorithm and subsequent experimental results on the test rig.

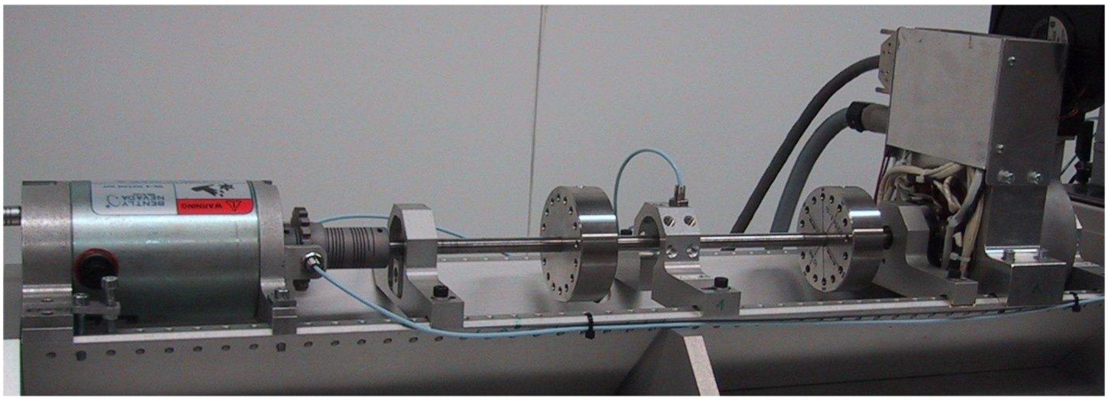

Figure 1 – The rotor test rig: the driving motor on left, the actuator on right.

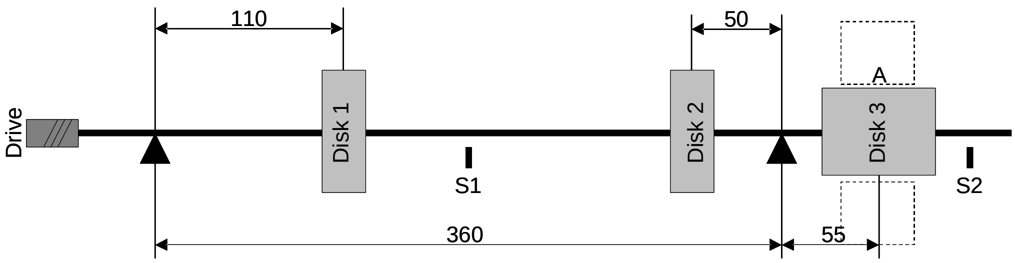

Figure 2 – The test rig layout: the displacement sensors locate at points "S1" and "S2", the actuator locates at point "A" (the dimensions are in millimetres).

The subject plant is a rotor test rig with a slim shaft supported by journal bearings. The diameter of the rotor shaft is 10 mm and the bearing span is 360 mm. The shaft has three disks attached to it making the rotor weigh about 3 kg. The rotor is driven by an electrical motor at one end. An electro-magnetic actuator, modified from a magnetic bearing unit, is fixed at the non-driven end and exerts radial forces on the rotor armature through an air gap of about 0.3 mm. The actuator is operated using a separate programmable control unit. The foundation, to which the actuator and the journal bearings are fixed, is assumed to be rigid compared with the flexibility of the rotor. The rotor has its first bending resonance mode (*i.e.* the critical speed) at approximately 50 Hz. The natural frequencies are slightly different in the horizontal and vertical directions.

Because the first bending mode on this particular rig is the most dominant one, attenuation of vibration at the frequency of this mode is the primary focus of this paper. Because the load disturbance generated by the first bending mode is a sinusoid, the *Internal Model Principle* [4] can be used to generate a compensating signal at the frequency of the first bending mode. The principle has been utilised in the compensation of rotation harmonics in rotor vibration control in different forms of feedforward compensation algorithms, see *e.g.* [6], [13], and [15]. The fundamental idea is to generate an infinite feedback gain at the frequency of the load disturbance. Feedforward algorithms often use a sinusoidal reference signal generated by a rotation speed measurement. This reference signal is then filtered through an adaptive filter and fed into the actuator to make a counter-acting force at frequencies of interest. This paper considers an alternative feedback approach called repetitive control. Initially, repetitive controller algorithms were developed to track, or to compensate, periodic signals; the principle was first presented in [10]. Since this publication, the repetitive control methodology has been developed further for example in [18], [12], [14], and [7]. The key idea behind the method is to continuously refine (learn) the control output by using the old outputs and the error data, the simplest repetitive control algorithm being

$$u(t) = u(t-T) + e(t) \tag{1}$$

where $u(t)$ are the control outputs, $e(t)$ is the control error and T is the delay time to be set according to the period of the signal to be tracked or compensated. Positive feedback of the delayed control signal leads to infinite feedback gain at the frequencies matching with the period stated by the delay time. Subsequent analysis shows that due to this infinite feedback gain, the loop gain of the controlled system must be positive real [8]. The requirement for positive-realness can be stringent for mechanical systems, often consisting of sharp resonances and anti-resonances with rapid phase changes. Moreover, the change into the discrete time domain may lead to instability, since sampled systems are rarely positive real [8]. These facts motivate the use of a more advanced repetitive control law with filters

$$u(t) = Q(q)u(t-T) + K(q)e(t) \tag{2}$$

where $Q(q)$ and $K(q)$ are filters, and q is the forward shift operator. Filter $Q(q)$ will later be referred as the Q-filter; it has been developed to restrict the control actions to a desired frequency band [18]. The delay line allows a non-causal digital implementation of the Q-filter, and therefore the filter can be designed to have zero-phase characteristics [3]. Filter $K(q)$ is used to make the loop gain positive real at the frequencies of interest where $Q(q) \approx 1$ [8], [18].

ALGORITHM DESCRIPTION

Initially, proportional-derivative (PD) controllers together with an averaging low-pass filter are applied as the basic feedback laws that tailor the rotor system characteristics to be more favourable for repetitive control. Two independent controllers are implemented, one in each radial direction. The transfer function from the displacement at the rotor endpoint to the force at the actuator is

$$H_{fb}(q) = \left(\frac{K_d}{T_S}\left(1 - q^{-1}\right) + K_p \right) \frac{1}{2}\left(q^{-1} + 1\right) \tag{3}$$

where K_d is the derivative gain, K_p is the proportional gain, T_S is the sample time. The PD controller uses the rotor endpoint displacement signals as its inputs (*S2* in Fig. 2). The control topology can be considered approximately collocated whereas the repetitive controller presented below is non-collocated; it uses the midpoint displacement signals (*S1*). In Fig. 3, $G'(q)$ denotes the transfer function from the actuator force to the rotor endpoint displacement and $G(q)$ stands for the transfer function from the actuator force to the midpoint under the assumption that the PD loops (*i.e.* $H_{fb}(q)$) are closed.

Tracking capability of any periodic signal makes repetitive control an attractive alternative solution for rotating machines. Repetitive control is particularly attractive when the frequencies or the waveform of the excitation are not known in advance. The delay time of the positive feedback loop is to be adjusted according to the speed of rotation that determines the fundamental period of the excitation. For variable-speed machines this implies that the delay time must be adjusted during the real-time execution of the algorithm. Another option is to use the rotation-phase-based delay [5]. Such a choice makes it possible to use the constant delay angle in the positive feedback loop, typically one revolution. However, the present work exploited time-varying delay in the feedback loop for two reasons: 1) the estimation of the rotor phase was not considered sufficiently reliable and accurate compared with the signal processor capability to maintain constant sampling intervals, 2) technical restrictions on the control unit made it difficult to trigger the signal processor based on the phase signal or the rotor revolution pulses.

The starting point for the development of a repetitive controller with adaptive delay time was a gradient based method where the feedback path consists of a truncated FIR filter working as the plant model. This approach is selected, because it is a computationally inexpensive method and its stability is guaranteed if the maximum phase error is ±90° with respect to the dynamics of the control path [9]. The update scheme for the gradient-based repetitive controller is

$$u(t) = q^{-N}(\gamma Q(q)u(t) - \alpha G_m(q^{-1})e(t)) \tag{4}$$

where the leak coefficient, $0 < \gamma < 1$, can be used to implement a forgetting factor into the integrator component of the algorithm, and $Q(q)$ is a symmetric filter with zero phase lag

$$Q(q) = (c_P q^{-P} + c_{P-1} q^{-P+1} + \ldots + c_0 + \ldots c_{P-1} q^{P-1} + c_P q^P), P \le N \tag{5}$$

where $[c_P \ldots c_0]$ are the FIR coefficients and P is the order of the filter. The coefficient $\alpha > 0$ determines the convergence rate of the algorithm. $G_m(q^{-1})$ is the model of the control path $G(q)$ represented as a truncated FIR approximation.

$$G_m(q^{-1}) = (a_M q^M + a_{M-1} q^{M-1} + \ldots + a_1 q + a_0), M \le N \tag{6}$$

where $[a_M \ldots a_0]$ are the FIR coefficients, determined from the time reversed impulse response of the plant $G(q)$, and M is the order of the filter. Note that both the plant model and the Q-filter are non-causal filters. However, the overall algorithm is causal, if the filter orders M and N do not exceed the delay length N, as q^{-N} is the common factor in the control law in Eq. (4). Maintaining the causality restricts the length of the FIR approximation and thus its accuracy. The FIR approximation may in fact cause destabilising effects that in some cases can be avoided by windowing techniques [1].

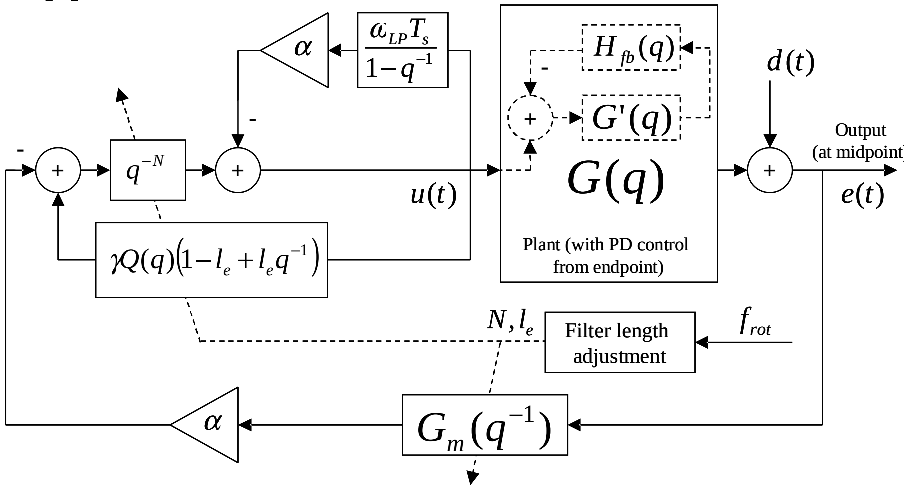

Figure 3 – The implemented feedback and the repetitive control systems.

Repetitive control law with adaptive delay time

In the algorithm developed in this section, the delay time is selected on-line according to the rotor speed measurement. First, the integer number of samples (N) required is determined. The number of samples is always rounded downwards. Second, the fact that the required delay time does not necessarily meet with the integer number of samples is taken into account by interpolation. For the interpolation, we define the length error ($0 \le l_e < 1$) that describes a relative error due to the rounding. The number of samples and the length error are defined as

$$N = \mathrm{floor}\left(\frac{1}{f_{rot}T_S}\right), \quad l_e = \frac{1}{f_{rot}T_S} - N \tag{7}$$

where N is the number of samples, f_{rot} is the measured speed of rotation in revolutions per second, and l_e is the length error. The both parameters are then used in the control law in Eq. (8).

The repetitive control method, being integrative, provides high feedback gain at zero frequency (DC). The band-pass filter was, however, impossible to implement in the Q-filter due to technical restrictions. Having a FIR filter with low DC gain and unity amplification at frequencies starting about 20 Hz was not realisable with the chosen filter lengths. This problem was avoided by implementing a separate DC removal integrator in the control law. The control law with interpolation and DC removal functions yields

$$u(t) = \gamma q^{-N} Q(q)\big((1-l_e) + l_e q^{-1}\big)u(t) - \alpha q^{-N} G_m(q^{-1}) e(t) - \alpha\left(\frac{\omega_{LP}T_S}{1-q^{-1}}\right)u(t) \tag{8}$$

where ω_{LP} is the integrator gain, used for the adjustment of low-pass corner frequency.

The interpolation and the DC removal integrator modify the phase of the feedback loops and have some destabilising effects on the algorithm. This feature contradicts the original idea having zero-phase feedback loops to ensure stability [18]. The stability analysis is, however, outside the scope of this paper and will be published later. The conclusion of the analysis was that a low-pass-type *Q*-filter was essential to maintain the stability at high frequencies.

EXPERIMENTAL RESULTS AND ANALYSIS

The algorithm presented in the previous section was implemented on the control unit. Fig. 4 shows the responses for repetitive control when the rotor speed is 30 rps; the attenuation provided by the repetitive controller is about 10 dB at the two first rotation harmonics. The maximum attenuation is about 15 dB when the rig runs at its critical speed. When compared with results obtained with *Convergent Control* method [17], the repetitive control method has similar performance when the frequency of rotation enables an integer ratio between disturbance period and sample rate. When the ratio is not close to an integer value, the performance of the repetitive control is slightly worse (Fig. 5).

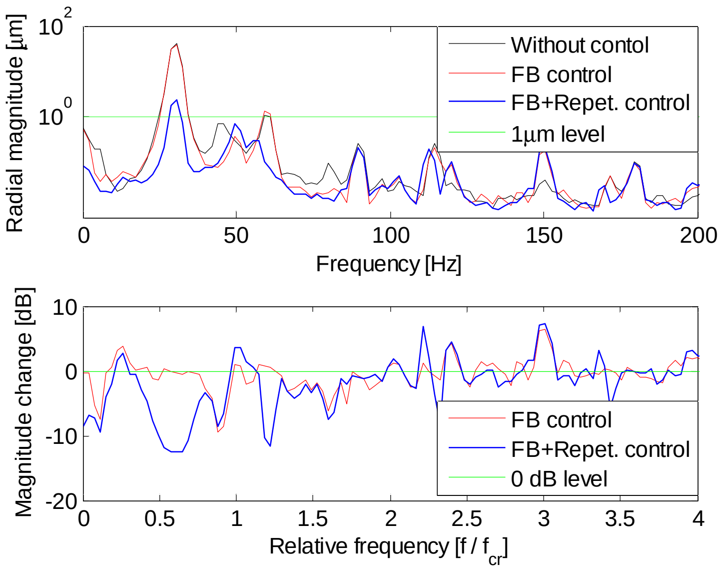

Figure 4 – The midpoint responses when running 30 rps without control, with feedback (FB) control, and with feedback and repetitive control together (f/f_{cr} = 1 at the critical speed).

For the both control systems concerned, the same topologies and the same parameters were used in both radial directions (except the system models used in the repetitive control were different in the horizontal and vertical directions). The following parameter values were used: K_d = 86 Ns/m, K_p = 7 N/mm, ω_{LP} = 6.2 rad/s and T_S = 0.1 ms. The repetitive control loop gain, $|\alpha G_m G|$, was varied from 0.2 to 0.8

depending on the rotor speed. The leak coefficient γ was mainly unity, but at 60 rps and above, it was reduced to 0.9999 to restrict the control force. The restriction was done in order to restrict the amplitude at the end of the rotor (reasons for this are explained in [17]).

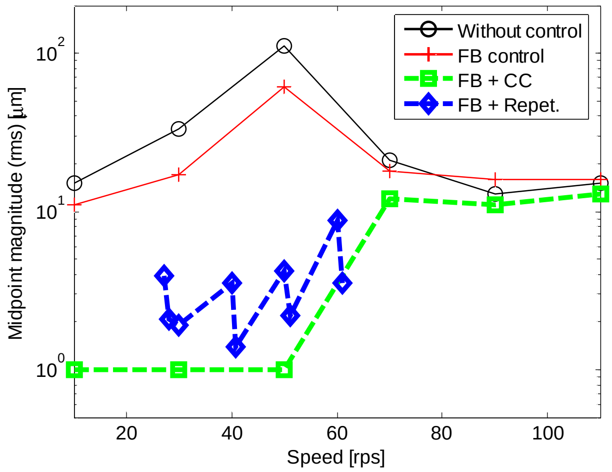

Figure 5 – The midpoint displacement without control, with feedback (FB) control, with feedback and Convergent Control (FB + CC) [17], and with feedback and repetitive control.

CONCLUSIONS

In this paper a gradient-based repetitive control law was modified for vibration control of a variable-speed rotor. This was achieved by applying an adaptive time delay on the control law. Due to issues with modelling uncertainty, interpolation between successive samples, and the implementation of a DC removal functionality required the use of a *Q*-filter in order to avoid high-frequency control actions. The achievable disturbance attenuation was highly dependent on the rotation speed; the largest attenuation was achieved when the rotor speed enabled an integer ratio between the disturbance period and the sample rate.

The performance of the repetitive control method was similar to the performance achieved previously with feedforward control methods when the delay time matched with the rotation frequency. Otherwise, the performance of repetitive control was slightly worse.

The presented experimental work did not make full justice for the repetitive control method's ability to track any periodic disturbance matching with the delay time (limited by the *Q*-filter) in the context of active control of vibrations In certain rotating machines, the frequencies of the most significant excitations may not be predictable. For example, rolls working in a nip contact may develop different barring vibration frequencies, and therefore cannot be approached with feedforward algorithms in a straightforward manner, whereas repetitive control algorithms can be still used without modifications due to the internal model.

For the future work, the next step is to study the developed control method under variable-speed conditions and to present stability analysis with respect to the interpolation and the DC removal functionality. The algorithm presented in this paper will also be tested on an electrical machine, where the objective is to control internal radial forces of the machine in order to reduce rotor vibrations.

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