ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

# MINIMISING THE NUMBER OF MICROPHONES REQUIRED FOR CHARACTERISATION OF DISTRIBUTED SOURCE REGIONS

Keith R Holland and Philip A Nelson

Institute of Sound and Vibration Research, University of Southampton, SO17 1BJ, UK
krh@isvr.soton.ac.uk

## Abstract

The use of arrays of microphones to locate and characterise acoustic sources is becoming more commonplace. Beamforming algorithms are applied to the microphone outputs to focus the array at a point, or to scan an area or volume for sources. The number and position of the microphones in a beamforming array are determined only by the frequency range of interest and the required spatial resolution. However, these beamforming algorithms cannot yield realistic estimates of source strength when more than one source is present within the 'beam' of the array. If realistic estimates of source strength distribution are required, more advanced signal processing algorithms, such as the inverse method, are required. In these cases, it is accepted wisdom that the number of microphones must equal or exceed the number of sources present. For many potential applications of these inverse methods, the size and complexity of the source regions of interest can lead to the requirement for prohibitively large numbers of microphones. It is the purpose of this paper to demonstrate how, through re-formulation of the inverse method, the requirements for the minimum number of microphones can be relaxed when certain assumptions can be made concerning the correlation structure of the source region.

## INTRODUCTION

Many distributed acoustic sources consist of regions of source strength that are well correlated over a certain length scale but which are not well correlated with neighboring regions. This paper deals with the application of inverse methods to yield reliable estimates of the source strength distribution of such sources from a knowledge of the radiated sound field at a finite number of sensor points. In particular, studies of the minimum number of sensor points required are carried out via novel formulations of the inverse problem which exploit prior knowledge of the source spatial correlation structure.

It is demonstrated that, for the inverse problem to have a unique solution, the number of measured pressure cross-spectra must equal or exceed the number of unknown source cross-spectra. Somewhat crucially for the current work, it should be noted that this criterion differs from the need for the number of measurement positions to exceed the number of source elements. The source cross-spectral matrix associated with the type of source distribution described above has a block-like structure and a number of zero elements; the number of required cross-spectra are therefore in general less than the square of the number of source elements, as would be the case for a fully-populated matrix. Exploitation of this source matrix structure leads to a significant relaxation in the minimum number of required pressure measurement sensors.

## THEORY

Consider a number of pressure sensors $M$ used to detect the sound field radiated by a number of sources $N$. The vector $\mathbf{p}$ of complex pressures is related to the vector $\mathbf{q}$ of source strengths by the $M \times N$ matrix of Green functions such that

$$\mathbf{p} = \mathbf{G}\mathbf{q} \tag{1}$$

The matrix $\mathbf{S}_{pp}$ of pressure cross-spectra is then given by

$$\mathbf{S}_{pp} = E\left[\mathbf{p}\mathbf{p}^{\mathrm{H}}\right] = E\left[\mathbf{G}\mathbf{q}\mathbf{q}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\right] = \mathbf{G}\mathbf{S}_{qq}\mathbf{G}^{\mathrm{H}} \tag{2}$$

where the matrix $\mathbf{S}_{qq}$ of source strength cross-spectra is given by $E[\mathbf{q}\mathbf{q}^{\mathrm{H}}]$ where $E[\ ]$ denotes the expectation operator and H denotes the Hermetian transpose [1]. When the source distribution consists of separate correlated regions, as described above, Sqq will have a block-like structure, thus:

$$\mathbf{S}_{qq} = \begin{bmatrix} \mathbf{S}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{S}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{S}_L \end{bmatrix} \tag{3}$$

where $\mathbf{S}_1$, $\mathbf{S}_2$ … $\mathbf{S}_L$ are sub-matrices of source cross-spectra each of which represents a region of the total source strength distribution containing correlated source strength elements. The source sub-matrices are each square with dimensions $(N_1 \times N_1)$, $(N_2 \times N_2)$ … $(N_L \times N_L)$, such that the total number of source elements $N = N_1 + N_2 + \ldots N_L$. Similarly, one may partition the Green function matrix such that

$$\mathbf{G} = \left[\mathbf{G}_1 \vdots \mathbf{G}_2 \vdots \cdots \vdots \mathbf{G}_L\right] \tag{4}$$

where the dimensions of these Green function sub-matrices are $(M \times N_1)$, $(M \times N_2)$ … $(M \times N_L)$ respectively. Equation (2) can now be written as

$$\mathbf{S}_{pp} = \mathbf{G}_1\mathbf{S}_1\mathbf{G}_1^{\mathrm{H}} + \mathbf{G}_2\mathbf{S}_2\mathbf{G}_2^{\mathrm{H}} + \cdots \mathbf{G}_L\mathbf{S}_L\mathbf{G}_L^{\mathrm{H}} \tag{5}$$

which, by using the identity

$$\nu\left(\mathbf{A}\mathbf{X}\mathbf{B}^{\mathrm{T}}\right) = \left(\mathbf{A} \otimes \mathbf{B}\right)\nu\left(\mathbf{X}\right) \tag{6}$$

can be rearranged to yield

$$\nu(\mathbf{S}_{pp}) = (\mathbf{G}_1 \otimes \mathbf{G}_1^*)\nu(\mathbf{S}_1) + (\mathbf{G}_2 \otimes \mathbf{G}_2^*)\nu(\mathbf{S}_2) + \cdots (\mathbf{G}_L \otimes \mathbf{G}_L^*)\nu(\mathbf{S}_L) \tag{7}$$

where the operator $\nu(\ )$ orders the elements of a matrix into a column vector consisting of stacked rows of that matrix, and the operator ⊗ denotes the Kronecker product. Finally, equation (7) can be arranged into a block matrix-vector multiplication such that

$$\nu(\mathbf{S}_{pp}) = \mathbf{G}_K \mathbf{s} \tag{8}$$

where

$$\mathbf{G}_K = [\mathbf{G}_1 \otimes \mathbf{G}_1^* \vdots \mathbf{G}_2 \otimes \mathbf{G}_2^* \vdots \cdots \vdots \mathbf{G}_L \otimes \mathbf{G}_L^*] \tag{9}$$

and

$$\mathbf{s} = [\nu(\mathbf{S}_1) \vdots \nu(\mathbf{S}_2) \vdots \cdots \vdots \nu(\mathbf{S}_L)]^{\mathrm{T}} \tag{10}$$

The dimensions of $\mathbf{G}_K$ are thus $M^2 \times (N_1^2 + N_2^2 + \cdots N_L^2)$ and those of **s** are $(N_1^2 + N_2^2 + \cdots N_L^2)$. Equation (8) is now in a form suitable for inversion to yield the unknown source cross-spectra **s**. For a solution to exist, the number of measured pressure cross-spectra must be equal to, or greater than, the number of unknown source cross spectra; that is we require $M^2 \geq (N_1^2 + N_2^2 + \cdots N_L^2)$. For all cases where the largest $N_i$ is less than $M$, the formulation of the inverse problem in equation (8) shows that the minimum number of required sensors is always less than the number of source elements. The least squares estimate of the solution of equation (8) is given by

$$\mathbf{s} = [\mathbf{G}_K^{\mathrm{H}} \mathbf{G}_K]^{-1} \mathbf{G}_K^{\mathrm{H}} \nu(\mathbf{S}_{pp}) \tag{11}$$

the accuracy of which is determined by the conditioning of the Green function matrix $\mathbf{G}_K$ and hence the geometry of the problem. If more assumptions are made about the structure of the source distribution, the minimum number of required sensors can be further reduced. The following section describes some source distributions that are commonly found in aeroacoustic noise problems, along with ways in which the inverse method could be used to yield reliable estimates of the source strength distributions, and the radiated sound field, using 'reduced' numbers of pressure sensors.

## EXAMPLE SOURCE DISTRIBUTIONS

### The Line Source with Short Correlation Length

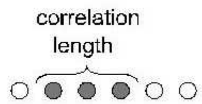


*Figure 1 A Line Source with Short Correlation Length*

Figure 1 shows a linear array of 6 sources where each source is correlated with its nearest neighboring sources but uncorrelated with all others. The source cross-spectral matrix takes the form

$$\mathbf{S}_{qq} = \begin{bmatrix} S_{11} & S_{12} & 0 & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ 0 & S_{32} & S_{33} & S_{34} & 0 & 0 \\ 0 & 0 & S_{43} & S_{44} & S_{45} & 0 \\ 0 & 0 & 0 & S_{54} & S_{55} & S_{56} \\ 0 & 0 & 0 & 0 & S_{65} & S_{66} \end{bmatrix} \tag{12}$$

where $S_{ij}$ denotes the cross-spectrum between source strengths $q_i$ and $q_j$. Applying the identity in equation (7) to equation (2) and deleting the zero elements of $\nu(\mathbf{S}_{qq})$ and the corresponding columns of $(\mathbf{G} \otimes \mathbf{G}^*)$, gives the reduced order system

$$\nu_R(\mathbf{S}_{pp}) = (\mathbf{G} \otimes \mathbf{G}^*)_R \nu_R(\mathbf{S}_{qq}) \tag{13}$$

where the R subscript denotes the reduced-size resulting from removal of the zero elements of $\nu(\mathbf{S}_{qq})$. Estimates of the 16 remaining non-zero unknown source cross-spectra via the inverse method follows from a least squares solution similar to equation (11) using 16 measured pressure cross-spectra; thus the outputs of the 6 sources can be fully determined using only 4 suitably placed pressure sensors.

**The Two-Dimensional Source with Short Correlation Length**

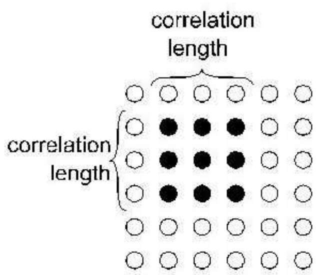


*Figure 2 A Two-Dimensional Source with Short Correlation Length*

Figure 2 shows a two-dimensional array of sources where each source is correlated with its nearest neighboring source in two dimensions. The source cross-spectral matrix for this source takes the form

$$\mathbf{S}_{qq} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} & 0 & 0 & 0 & 0 \\ \mathbf{S}_{21} & \mathbf{S}_{22} & \mathbf{S}_{23} & 0 & 0 & 0 \\ 0 & \mathbf{S}_{32} & \mathbf{S}_{33} & \mathbf{S}_{34} & 0 & 0 \\ 0 & 0 & \mathbf{S}_{43} & \mathbf{S}_{44} & \mathbf{S}_{45} & 0 \\ 0 & 0 & 0 & \mathbf{S}_{54} & \mathbf{S}_{35} & \mathbf{S}_{56} \\ 0 & 0 & 0 & 0 & \mathbf{S}_{65} & \mathbf{S}_{66} \end{bmatrix} \tag{14}$$

Where each of the sub-matrices are themselves banded and take the form of equation (12). Solution of this problem is then similar to that for the line source above.

**The Circular, Axi-Symmetric Source Distribution**

Figure 3 shows the geometry for a simulation into the performance of the above technique when further simplifications of the source correlation structure are possible. In this case, the circular disc source has correlation along a radial line, but zero correlation in the circumferential direction; furthermore, the entire source is axi-symmetric such that the source strength distribution along any radial line is the same as that along any other. The simulated source is divided into 1197 source elements with 19 sources along each of 63 radial lines. In formulating the inverse problem, it is shown that the (axi-symmetric) field radiated by these 1197 sources can be fully determined using a far-field polar array of just 19 pressure sensors.

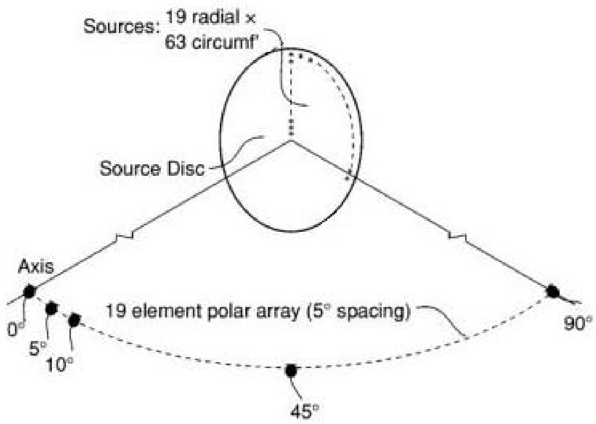


*Figure 3 Geometry for Circular Axi-Symmetric Source Simulations*

To illustrate the formulation of the inverse problem for this case, a similar but much simpler system is described. Consider an axi-symmetric source as described but with just two radial lines containing two source elements each as shown in Figure 4.

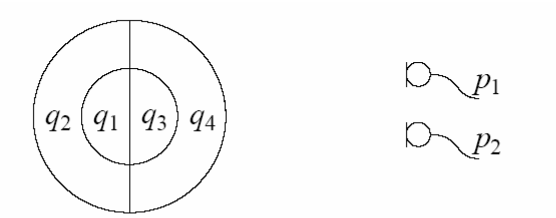


*Figure 4 Simpler Source Distribution to Illustrate Formulation of the Inverse Method*

Considering the source correlation structure above, we can make the following simplifications

$$\begin{aligned} &S_{11}\left(=q_1 q_1^*\right) = S_{33} = S_1 \\ &S_{22} = S_{44} = S_2 \\ &S_{12} = S_{34} = S_3 \\ &S_{21} = S_{43} = S_3^* \\ &S_{13} = S_{14} = S_{23} = S_{24} = 0 \\ &S_{31} = S_{41} = S_{32} = S_{42} = 0 \end{aligned} \qquad (15)$$

such that the source cross-spectral matrix becomes

$$\mathbf{S}_{qq} = \begin{bmatrix} S_1 & S_3 & 0 & 0 \\ S_3^* & S_2 & 0 & 0 \\ 0 & 0 & S_1 & S_3 \\ 0 & 0 & S_3^* & S_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \mathbf{S} \end{bmatrix} \qquad (16)$$

where **S** is a (2 × 2) sub-matrix. Splitting the Green function matrix **G** into two (2 × 2) sub-matrices $\mathbf{G}_1$ and $\mathbf{G}_2$, equation (2) can be written

$$\mathbf{S}_{pp} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{G}_1^{\mathrm{H}} \\ \mathbf{G}_2^{\mathrm{H}} \end{bmatrix} = \mathbf{G}_1 \mathbf{S} \mathbf{G}_1^{\mathrm{H}} + \mathbf{G}_2 \mathbf{S} \mathbf{G}_2^{\mathrm{H}} \qquad (17)$$

Applying the identity in equation (7) yields

$$\nu\left(\mathbf{S}_{pp}\right) = \left(\mathbf{G}_1 \otimes \mathbf{G}_1^*\right)\nu(\mathbf{S}) + \left(\mathbf{G}_2 \otimes \mathbf{G}_2^*\right)\nu(\mathbf{S}) \qquad (18)$$

From which the inverse problem can be written

$$\nu(\mathbf{S}) = \left[\left(\mathbf{G}_1 \otimes \mathbf{G}_1^*\right) + \left(\mathbf{G}_2 \otimes \mathbf{G}_2^*\right)\right]^{-1} \nu\left(\mathbf{S}_{pp}\right) \qquad (19)$$

Thus the 4 non-zero unknown source cross-spectra **S** can be determined from the outputs of just two pressure sensors.

Figures 5 to 7 shows the result of extending this technique to the 1197 element source distribution shown in Figure 3. For this simulation, the forward model (generation of the pressure sensor cross-spectra) involves the superposition of the outputs of 8192 monopoles distributed over the source disc. The forward source cross-spectral matrix is computed following the spatial correlation assumptions above and is then coupled to a full matrix of free-space Green functions to compute the pressure cross-spectral matrix for 19 pressure sensors arranged in an arc. To add a degree of realism to the simulation, and to test for robustness of the inversion process, 10% of random noise is added to the pressure cross-spectral matrix prior to inversion.

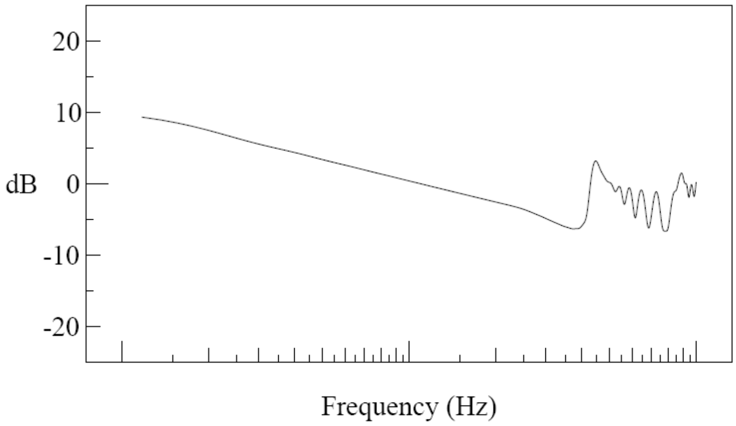


*Figure 5 True Far-Field Pressure Spectrum at 55° From Axis: 1197-Element Axi-Symmetric Source Distribution*

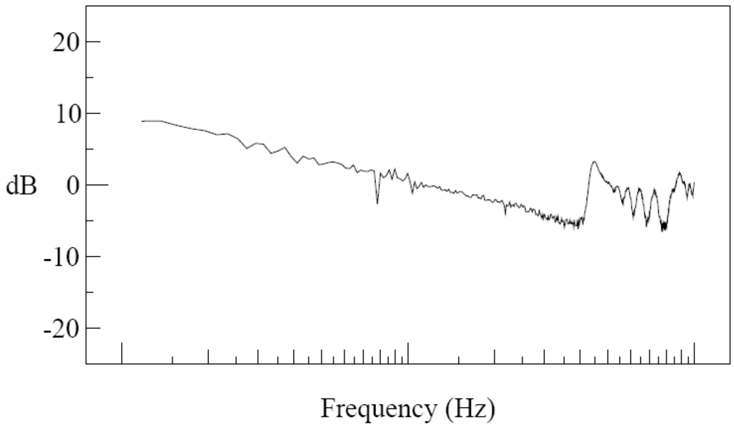


*Figure 6 Reconstructed Far-Field Pressure Spectrum at 55° From Axis using Outputs From 19 Pressure Sensors with 10% Random Noise Added : 1197-Element Axi-Symmetric Source Distribution*

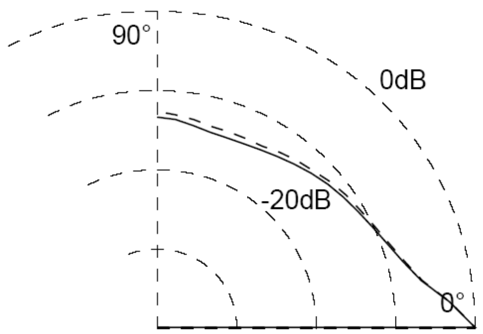


Figure 7 Far-Field Directivity of 1197-Element Distributed Source: True Directivity (dashed line) and Reconstructed Directivity (solid line) using Outputs From 19 Far-Field Pressure Sensors with 10% Random Noise Added

## DISCUSSION

The application of the techniques described in this paper may bring about considerable savings in the number of pressure sensors required for investigating distributed aeroacoustic sources, and hence the complexity and expense involved in such undertakings. The sheer numbers of sensors required when applying the traditional, at-least-one-microphone-per-source-element, criterion to the radiation of high frequencies by large distributed sources can render the inverse method unusable in many cases. Fortunately, many distributed sources encountered in the field of aeroacoustics have known spatial correlation structures within them that can be exploited, through suitable formulation of the inverse model, to drastically reduce the required number of sensors. The simulations presented here suggest that these benefits may be real, and that the resultant matrix inversion may be robust, but it should be borne in mind that very little experimental verification of the application of these techniques has been carried out to date, although some early experiments carried out by the authors do show promise.

## CONCLUSIONS

This paper describes a method for formulating the inverse problem associated with distributed aeroacoustic sources to exploit any known spatial correlation structures within the source. The following conclusions may be drawn from the analysis and simulations presented above.

- By exploiting any known correlation structures in distributed aeroacoustic sources, the excepted criterion for the minimum number of pressure sensors required for inverse source location and quantification may be relaxed.
- Many different correlation structures can be built into the inverse model; three examples are described.
- For large sources at high frequencies, when the source correlation lengths may be small, the potential saving in the required number of pressure sensors could be very significant.
- Experimental verification of these techniques is required before confidence can be had in there usefulness in real-world applications.

## REFERENCES

[1] P. A. Nelson and S. H. Yoon, “Estimation of acoustic source strength by inverse methods: Part 1, conditioning of the inverse problem”, Journal of Sound and Vibration 233(4), 643-668 (2000)

## ACKNOWLEDGEMENT

This work was carried out at the Institute of Sound and Vibration Research under the Rolls-Royce University Technology Centre (UTC) in Gas Turbine Noise