

STRIATION PROCESSING OF SPECTROGRAM DATA

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Abstract

The determination of seabed properties and their incorporation in sound propagation models for estimating the effectiveness of sonar and underwater communications is generally required in shallow water regions in the vicinity of commonly used shipping and fishing routes. It has been suggested by others that the extraction of acoustic data from interference patterns observable in ambient noise spectrograms can be used as an effective means of determining seafloor properties in these situations. Within this paper, the problem of obtaining information from a frequency vs. range spectrogram and relating this to the governing modal model is discussed. An existing method for estimating modal group and phase velocities (and hence seabed properties) from spectrogram information, based upon a dual mode analysis, is critically reviewed and the need for a more complete description of the modal model is outlined.

INTRODUCTION

Seabed properties affect the propagation of sound in shallow-water environments and hence accurate modelling of the sea floor is necessary to predict sonar performance. Direct estimation is time-consuming, expensive and only accurate at the specific measurement location. Geoacoustic inversion techniques using acoustic measurements do not tend to suffer from these limitations and have become the preferred means of seabed characterisation [1].

Underwater activities requiring sonar often occur in shallow water regions in the vicinity of commonly used shipping and fishing routes, so noise generated by passing surface ships is frequent. The use of ship noise in geoacoustic inversions is therefore a potentially viable means of determining sea floor parameters. The benefits of using existing sound sources generated at remote locations are threefold: no additional sound, which has the potential to damage sea creatures [2], is emitted into the underwater environment; no physical means of producing sonar sound, such as an air-gun, is needed; and the array, and ultimately any sea-vessel to which it is attached, becomes more difficult for a potentially hostile third party to detect. As a result of this, the development of inversion techniques using ambient ship noise as a source, has been of considerable interest in recent years ([3], [4], [5]).

D'Spain and Kuperman [6] developed an analytical model, based upon the waveguide

invariant, to predict the frequency dependence of broadband interference patterns in shallow water. Heaney [3],[7],[8] provides a method to extract three acoustic observables from a spectrogram and relates these to the sediment geoacoustic properties.

Within this paper, the features observable on a spectrogram are highlighted. Subsequently, the underlying problem of obtaining information from a spectrogram and relating this to the governing modal model is discussed. An existing dual-mode method of relating information obtained from spectrograms to modal group and phase velocities, and subsequently to seabed properties, is critically reviewed and conclusions regarding the accuracy of this method are drawn.

SPECTROGRAM FEATURES

A spectrogram, such as is shown in Figure 1 is a visual representation of sound pressure as a function of frequency and range (or time). The most notable features of the plot are the striations (interference patterns), which are lines of constant (sound pressure) magnitude.



Figure 1 – Example of a spectrogram generated from simulated data.

The interference patterns are a function of the propagation characteristics of the environment and so can be used to determine the environmental properties. In the near-field the striations are mainly due to the Lloyd mirror effect [9]; however, at ranges much further from the source, interaction between various modes and with the seafloor become more dominant.

INTERPRETATION OF THE WAVEGUIDE INVARIANT

The structure of the striations is dependent upon the group and phase velocities of the modes governing the sound propagation within the environment and hence can be related to the seafloor properties. The waveguide invariant [10], β , describes the dispersive characteristics of the field in a waveguide. It is defined as the ratio between the differential phase slowness, S_p , and the differential group slowness, S_q , where the slowness is the reciprocal of the velocity:

$$\beta = -\frac{\partial S_p}{\partial S_q}.\tag{1}$$

Here an isovelocity waveguide (a waveguide with a constant wave speed irrespective of position) is considered. Such a waveguide will have a waveguide invariant equal to square of the cosine of the grazing angles of the propagating modes. For the lowerorder modes with small grazing angles, the waveguide invariant is approximately unity [10]. A waveguide with a sediment layer will exhibit dispersive characteristics and its waveguide invariant will generally not be unity. The waveguide invariants are generally positive for bottom-interacting propagation and negative for refracting sound paths. For bottom-interacting propagation, the exact value of β will depend heavily upon the geoacoustic parameters of the sediment.

The interference patterns that appear on a spectrogram are a function of the cosine of the difference in mode wavenumber, k_{mn} . Due to this, the striations have a periodicity of $2\pi k_{mn}(\omega)r$, where r is the range. Using a dual mode analysis, D'Spain and Kuperman [6] show that for two adjacent striations separated by a distance $r_2 - r_1 = \rho$, at a frequency $\omega = \omega_0$, the difference in range between the adjacent striations can be directly related to the difference in phase slowness of the modes ΔS_p^{mn} :

$$\Delta S_p^{mn}(\omega_o) = \frac{2\pi}{\omega_o \rho} = \frac{1}{f_o \rho}.$$
(2)

Similarly, the difference in frequency, Ω , between striations at a range $r = r_0$ is a function of the difference in group slowness of the modes ΔS_q^{mn} :

$$\Delta S_g^{mn}(\omega_c) = \frac{\partial \Delta k_{mn}(\omega_c)}{\partial \omega_c} = \frac{1}{\Omega_c r_o},\tag{3}$$

where Δk_{mn} is the difference between the m^{th} and n^{th} modal wavenumbers. The center frequency between the striations is ω_c . The ratio between the phase slowness and group slowness is therefore:

$$\frac{\Delta S_p^{mn}}{\Delta S_q^{mn}} = \frac{\Omega_c r_o}{f_o \rho}.$$
(4)

D'Spain and Kuperman also show that the equation governing striation behaviour in a range independent environment is:

$$\omega = \omega_0 \left(\frac{r}{r_0}\right)^{\beta}.$$
(5)

Since β is constant for a range independent environment and ω_0 and r_0 are striation specific constants, Eq. (5) can be rewritten as:

$$\omega = C r^{\beta},\tag{6}$$

where C is a striation specific constant. Heaney states that the waveguide invariant is equal to the normalised striation slope [3]:

$$\frac{\Delta f}{f} / \frac{\Delta r}{r} = \beta. \tag{7}$$

Assume that (r_2, ω_2) is a point midway, in both range and frequency, between two adjacent striations governed by $\omega = C_2 r^{\beta}$ and $\omega = C_3 r^{\beta}$. The midpoint frequency, ω_2 , meets C_3 at r_1 and C_2 at r_3 , whilst the midpoint range, r_2 , meets C_3 at ω_3 and C_2 at ω_1 , as depicted in Figure 2.



Figure 2 – Geometry for the determination of the waveguide invariant.

Hence, using the relationships $\omega_3 = C_3 r_2^{\beta}$, $\omega_1 = C_2 r_2^{\beta}$ and $\omega_2 = C_3 r_1^{\beta} = C_2 r_3^{\beta}$, the normalised striation slope at (r_2, ω_2) is:

$$\frac{\Delta\omega}{\omega} / \frac{\Delta r}{r} = \frac{\omega_3 - \omega_1}{\omega_2} \frac{r_2}{r_3 - r_1}$$
(8)

$$= \frac{r_3^{\beta} - r_1^{\beta}}{r_3 - r_1} \frac{\left(\frac{r_3 + r_1}{2}\right)^{\beta + 1}}{r_1^{\beta} r_3^{\beta}}.$$
(9)

Taking the limit $\Delta r/r \rightarrow 0$ yields:

$$\lim_{\frac{\Delta r}{r} \to 0} \frac{\Delta \omega}{\omega} / \frac{\Delta r}{r} = \lim_{r_3 \to r_1} \frac{r_3^{\beta} - r_1^{\beta}}{r_3 - r_1} \frac{\left(\frac{r_3 + r_1}{2}\right)^{\beta + 1}}{r_1^{\beta} r_3^{\beta}} = \beta.$$
(10)

The initial assumption that the waveguide invariant is the normalised striation slope is therefore only an approximation; however at sufficiently high ranges this assumption is valid. For example, if $r_2 = 3000$ m, $\Delta r = 100$ m, and $\beta = 0.9$, the normalised striation slope given by Eq. (8) evaluates to 0.9002, which is sufficiently close to the waveguide invariant for computational purposes. If the range of interest were only 100 m but the striation spacing and waveguide invariant were the same as for the previous example, the normalised striation slope would be 0.9549, a 6% error. The normalised striation slope approximates the waveguide invariant to a reasonable accuracy only at sufficiently large ranges from the source.

Heaney [3] uses Fourier transform theory to relate the range to the maximum spread in horizontal wavenumber and to define the time spread as the reciprocal of the minimum frequency spacing of striations. He proposes that the waveguide invariant can be defined in terms of the differential phase slowness and differential group slowness of two propagating modes, namely the lowest and highest excited propagating modes:

$$\beta = \frac{\frac{\Delta f}{f}}{\frac{\Delta r}{r}} = -\frac{S_{p0} - S_{pmax}}{S_{g0} - S_{gmax}} = -\frac{\left(\frac{1}{v_0} - \frac{1}{v_{max}}\right)}{\left(\frac{1}{u_0} - \frac{1}{u_{max}}\right)},\tag{11}$$

where v_0 and v_{max} are the phase velocities of the slowest and fastest propagating modes respectively and u_0 and u_{max} are the respective group velocities of these two modes.

It is proposed here that this assumed relationship needs to be used with caution. This relationship is only a reasonable approximation when the grazing angle of the highest propagating mode is small. As an example of when the relationship does not hold, the propagation of sound within the isovelocity duct depicted in Figure 3 is considered.



Figure 3 – Geometric model of an isovelocity duct. **D** is the depth of the waveguide, z_s and z_r are the locations of the source and receiver respectively and **c** is the speed of sound within the medium.

The phase velocity of each mode, v_m , is simply the ratio between the frequency, ω , and the horizontal modal wavenumber, k_{rm} :

$$v_m = \frac{\omega}{k_{rm}},\tag{12}$$

and the group velocity, u_m , is the differential of the frequency with respect to the horizontal modal wavenumber:

$$u_m = \frac{d\omega}{dk_{rm}}.$$
(13)

In an isovelocity waveguide the horizontal modal wavenumber of the m^{th} mode is defined as:

$$k_{rm} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left[\left(m - \frac{1}{2}\right)\frac{\pi}{D}\right]^2},\tag{14}$$

where c is the speed of sound propagation within the medium and D is the depth of the waveguide. The group velocity within an isovelocity duct can therefore be derived as:

$$u_m = \frac{d\omega}{dk_{rm}} = \frac{c^2}{\omega} k_{rm}.$$
(15)

The ratio of differential phase to group slowness for two modes, m and n, is:

$$\alpha = \frac{\Delta S_p^{mn}}{\Delta S_g^{mn}} = \frac{S_p^m - S_p^n}{S_g^m - S_g^n} = \frac{\left(\frac{1}{v_m} - \frac{1}{v_n}\right)}{\left(\frac{1}{u_m} - \frac{1}{u_n}\right)},$$
(16)

where S_p is the modal phase slowness and S_q is the modal group slowness.

Substituting the group and phase velocities into Eq. (16) yields an expression for the ratio of phase to group slowness, α :

$$\alpha = -\frac{c^2}{\omega^2} k_{rm} k_{rn}.$$
(17)

By assuming that n = 1, (the lowest excited mode):

$$\alpha = \sqrt{1 - \left(\frac{c}{4fD}\right)^2 \left(1 + (2m-1)^2\right) + \left(\frac{c}{4fD}\right)^4 (2m-1)^2}.$$
 (18)

As an example, consider the environment shown in Figure 3. A source is located at z_s and the frequency of interest is 100 Hz. The highest order propagating mode is m = 13, which cuts on at 93.75 Hz. For this case $\alpha = 0.3477$. This is significantly smaller than the approximately unity value which would be obtained from the normalised striation slope of the full-modal model isovelocity spectrogram.

The highest order propagating mode is cut on at a frequency very close to the frequency of interest, and hence the interaction between it and the lowest propagating mode has not fully developed.

Figure 4 shows a sketch of the striation patterns due to interactions between the highest and lowest order propagating modes. At frequencies just above the m^{th} cut-on frequency, the normalised slope of the striations is significantly less than the far-field value. It is only at much higher frequencies and ranges that the normalised striation slope approaches the expected isovelocity waveguide value. For this reason, determination of the waveguide invariant by considering only the slowest and fastest propagating modes may result in large errors.



Figure 4 – Concept sketch of striations resulting from interaction between lowest and m^{th} propagating modes.

RANGE AND FREQUENCY SPACINGS

The rapid characterisation method developed by Heaney [7] relates range to the maximum spread in horizontal wavenumber and defines the time spread as the reciprocal of the minimum frequency spacing of striations. Either the range or the frequency spacing of the striations is determined. The method assumes that at any point on the spectrogram the striation spacing is the minimum possible and can therefore be determined by considering interaction between the slowest and fastest propagating modes only.

In general the actual range and frequency spacing at any given point will be greater than these values. This is due in part to the mutual interference of modes between the lowest and highest orders.

Consider the case when a receiving hydrophone is located along a nodal plane of the highest order mode. The received signal, and therefore the resulting range and frequency spacings in the spectrogram, will be completely independent of the highest order mode, and will depend only upon the interaction between the other propagating modes. The resulting striation spacing will therefore be greater than that calculated. A more detailed theoretical explanation of how the interaction of intermediate modes affects the spacing has been omitted from the current discussion due to space limitations.

CONCLUSION

A published method [7] of analysing the spectrogram waveguide invariant and range and frequency spacings using a two mode model has been discussed, and errors associated with the method have been outlined. Limits on the validity of such an approach have been proposed.

The two-mode model is unable to fully describe the striation patterns of a spectrogram and so more complex multi-mode methods must be considered.

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