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EVALUATION OF MODEL UNCERTAINTY IN MODAL ACOUSTIC RESPONSE ANALYSIS

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Abstract

The effect of model parameter variability on the low-frequency vibro-acoustic response of structures is discussed, and a statistical approach, based on first-order variance analysis presented for stochastic optimization. The modal correction method (a fast re-analysis, in which the effects of structural modifications are estimated by adjustment and re-solution of the modal equations) is reviewed in the context of uncertainty analysis. Subsequently, the accuracy of predicted response statistics is established for a representative structure by comparison with Monte Carlo simulation.

INTRODUCTION

The properties of real structures differ in service from nominal design conditions, as a result of normal variations in manufacture and the influence of environmental factors [1][2]. These variations can have a marked impact on the NVH response of both automobile and aerospace structures [3][4]. Low frequency vibro-acoustic prediction is usually performed using finite element models in which the properties are assumed known with certainty. However many properties are not deterministic but vary randomly. For example, nominally identical vehicle bodies and engine mounts can exhibit large variations from specification values owing to the effects of manufacturing tolerances and scatter in polymer properties. This can result in significant scatter in the NVH response of nominally identical vehicles, making design decisions difficult when based entirely on deterministic analysis [5]. In the car industry, there is current interest in statistical NVH analysis methods that account for the potential scatter in actual responses. In this paper we outline such an approach.

RELIABILITY-BASED NVH PREDICTION

Methods for predicting the effects of model parameter uncertainty on structural response belong to the theory of structural reliability. The general term “failure” is used to describe a condition in which the response exceeds some limit. In the context of vehicle NVH analysis the limit condition could for example, be the maximum allowable acoustic FRF response in decibels (at driver ear position) for some specified engine excitation frequency. Commonly used reliability methods include first-order reliability (FORM), Monte Carlo simulation, and second-order reliability (SORM) [2][5]. The response mean and variance may also be directly approximated by a Taylor series [7]. By considering the first-order terms only, an approximation to the actual mean and variance of response can be obtained as follows:

$$E(z) = h\left[E(x_1), E(x_2), ...E(x_n) \right] \tag{1}$$

$$\mathrm{var}(z) = \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x} \right)^2 \mathrm{var}(x_i) \tag{2}$$

The simplification obtained by equations (1) and (2) avoids the often prohibitively expensive calculation of higher derivatives. In frequency response analysis the response and derivatives are complex quantities. It has been established that for low parameter variance, the first-order sensitivity method, denoted here as FOSM, can give excellent results. Note that it is assumed here that the mean response is unchanged by the presence of variability. FOSM forms the basis of an approach to optimization discussed later in the paper.

FAST RE-ANALYSIS USING MODAL CORRECTION

The modal correction method (MCM) is an approximation performed at the level of modal equations. It belongs to the general class of basis-vector re-analysis methods and leads to a reduction in the computational effort for repeated dynamic response analyses as required in sensitivity and optimization calculations or Monte Carlo simulation. The original system eigenvectors are used to calculate a modification to the matrices in the modal equation of motion. The modified structural stiffness corresponding to the N variables is:

$$\bar{K}_s = K_s + \sum_{i=1}^{N} \Phi_s^T \Delta k_{si} \Phi_s \tag{3}$$

Analogous modifications to the modal mass and damping are performed. Physical modifications lead to a final modal equation of motion [4] as follows:

$$\left[-\omega^2\begin{bmatrix}\bar{M}_S & 0\\ 0 & M_F\end{bmatrix}+i\omega\begin{bmatrix}\bar{D}_S & -C\omega_F^{-1}\\ \omega_F^{-1}C^T & \bar{D}_{FN}+\bar{D}_{FB}\end{bmatrix}+\begin{bmatrix}\bar{K}_S & 0\\ 0 & K_F\end{bmatrix}\right]\begin{Bmatrix}q_S\\ q_F\end{Bmatrix}=\begin{Bmatrix}Q_S\\ Q_F\end{Bmatrix} \tag{4}$$

A random variable can represent a group of finite elements with the same property. For each variable, correction matrices are calculated once, and then scaled in an element-dependent manner. Physical responses are recovered from the modal solutions. But engineering accuracy is only achieved if the number of basis eigenvectors is sufficient.

AN APPROACH TO STOCHASTIC OPTIMIZATION

Stochastic optimization involves the search for robust solutions to problems with random variables [6]. The approach described here uses equation (2) (FOSM) to approximate the response variance. Maximum likely frequency response amplitudes are then derived by adding some probable deviation to the mean response. The mean values of the model parameters and the variance of each parameter (expressed as a standard deviation or coefficient of variance) can be defined as independent design variables for the purposes of optimization. Reflecting the relationship with product quality, COV variables are termed “quality variables”. The optimization problem can then be posed in the form of an objective function in which we wish to minimize:

$$F = f(X, COV_x) \tag{5}$$

subject to inequalities: $U \le U_{\max}$, $X_{lower} \le X \le X_{upper}$, $COV_{X\,lower} \le COV_X \le COV_{X\,upper}$. Using equations (2) and (3), and assuming a Gaussian response distribution, we have at probability level p:

$$U = E(U) + Z \cdot \left(\sum_{i=1}^{n}\left(\frac{\partial U}{\partial X_i}\right)^2 \sigma^2{}_{X_i}\right)^{\frac{1}{2}} \tag{6}$$

where $Z(p) = \Phi^{-1}(p)$. When quality variables are defined, it is useful to assign a “cost” penalty to low variance. Optimization that includes a cost penalty for variance model parameters, makes possible optimum selection of both mean and coefficient of variance of model parameters, and may provide new economic solutions to NVH problems. To illustrate the stochastic optimization capability, Figure 1 has been taken from reference [3]. This figure, generated at the end of optimization, shows that the upper bound of response has been reduced to the target response. In this example, the “cost” was also reduced. The final values of the mean and quality variables demonstrate that a cost-optimal NVH design solution may require that the quality of response-critical components be improved while being lowered elsewhere.

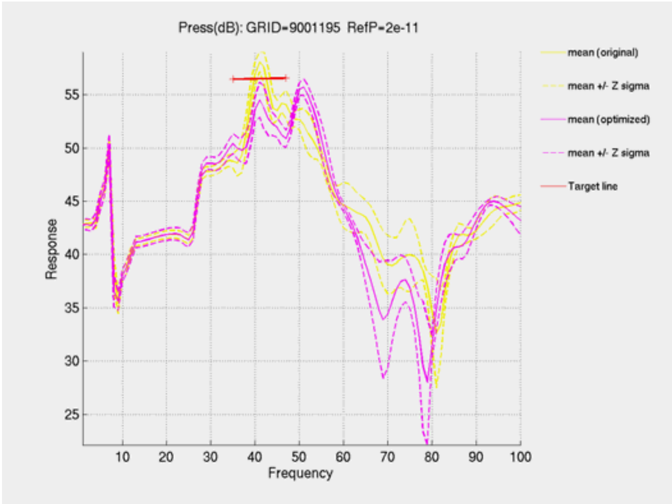

Figure 1. Frequency response functions obtained via stochastic optimization

The validity of the above procedure depends on the FOSM approximation for variance, the assumption that the mean response is independent of variance in the parameters and the accuracy of the modal correction method. The interest in the accuracy of MCM and FOSM motivates the numerical experiments described in the next section.

A NUMERICAL CASE STUDY OF THE EFFECTS OF VARIABILTY

Figure 2 shows a finite element (FE) model of a stiffened-steel box structure (2m x 1.5m x 1m) enclosing an air cavity and resting on spring mounts. The structural FE model has around 150000 degrees of freedom (DOF). The box comprises 2mm thick plate components that are connecting by spot and seam welds. This structure exhibits many of the dynamic characteristics of a car body enclosing a passenger compartment air cavity. Unit harmonic forcing (of magnitude 1 Newton) is applied at a point on the base. The frequency response of some 30 structural DOF and 2 internal air pressures is calculated in 1 Hz steps across the frequency range 10-150 Hz. These calculations were performed in fluid-structure modal frequency response analysis in NASTRAN. Structural modes up to 350Hz (554 modes for nominal design) and 87 fluid modes up to 600 Hz were used in forming the modal equations. A modal damping value of 2% was used for both structure and fluid. (To save calculation time, the physical coupling matrix was calculated once and read from an external file during subsequent calculations). In addition, approximate calculations were carried out using MCM and FOSM in the program CDH/VAO.

Monte Carlo analyses - A total of 42 uncertain plate thicknesses were defined as uncorrelated random variables. Gaussian distributions respectively with 2.5% coefficient of variance, 5% COV and 10% COV were used. A uniform distribution with variance corresponding to 5% COV was also used. For each of the 4 levels of variability, 2000 sets of realizations were generated. Thus in total, 8000 NASTRAN runs were performed. Computations were performed on HP Itanium, IBM Power 5, and NEC Itanium servers, taking approximately 2500 CPU hours in total.

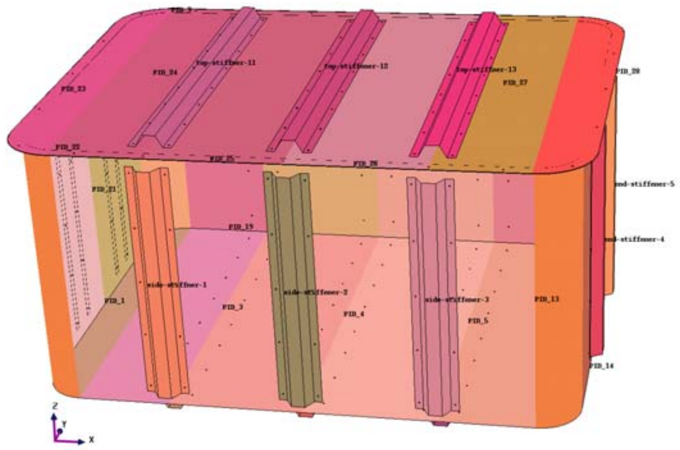

Figure 2. Stiffened-steel box model for comparison of Monte Carlo simulations and FOSM

Approximate MCM and FOSM analyses - For all distributions, a FOSM calculation of variance was performed. An approximate Monte Carlo analysis was carried out using MCM in CDH/VAO for 150 realizations of the 5% COV distribution.

Summary of Results **-** Figure 3 shows good-fit distributions of box mass and, as an indicator of stiffness, the number of modes below 350Hz generated from results of the Monte Carlo analysis. Figure 4 presents the range over all analyses and mean response amplitude of a selected acoustic response. Figure 5 shows the actual mean and the nominal response. In Figure 6 the standard deviation, calculated for Monte Carlo analysis, is presented together with the standard deviation calculated in FOSM. Figure 7 shows the MCM results calculated for 5% COV in CDH/VAO. The results show that: i) in the presence of variability in the model parameters, large deviations from nominal responses may result; ii) the variability in the response increases nonlinearly with variability in the input parameters; iii) with increasing parameter variability the mean response calculated in Monte Carlo smoothens and can markedly differ from nominal response; iv) variance calculated in the FOSM approximation is valid only for low levels of variability in the input parameters; and v) the results for Gaussian and uniform distributions with the same variance are practically identical.

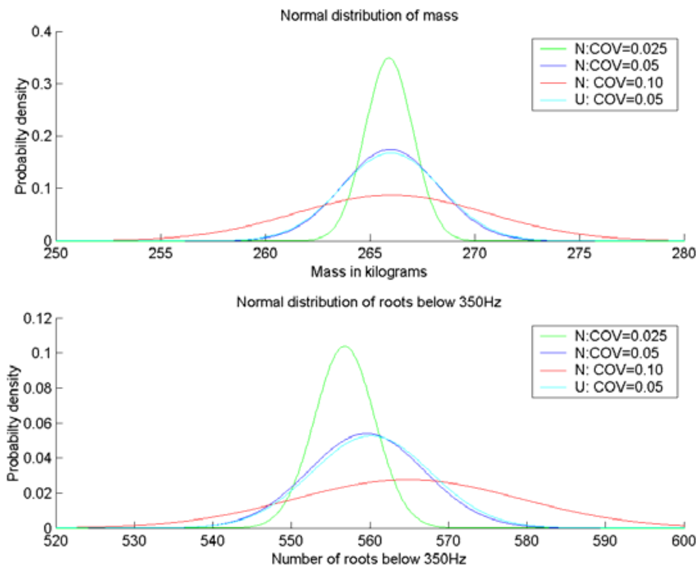

Figure 3. Probability distributions of mass and roots below 350Hz

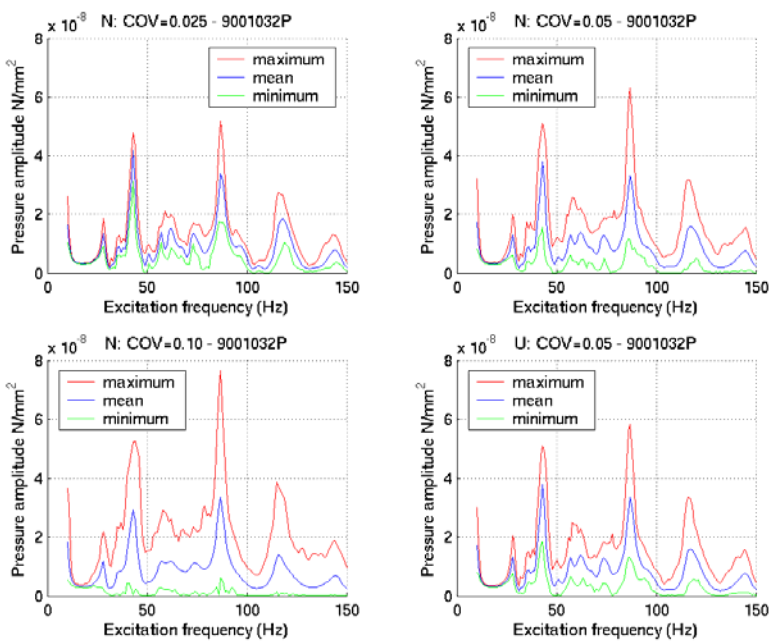

Figure 4. Bounds for Monte Carlo-derived acoustic responses, also showing corresponding mean values.

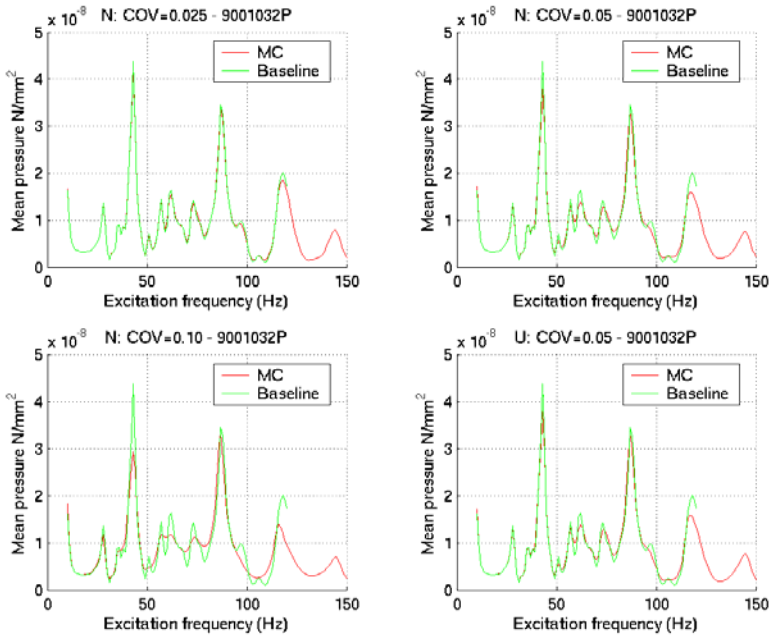

Figure 5. Monte Carlo-derived and nominal mean values

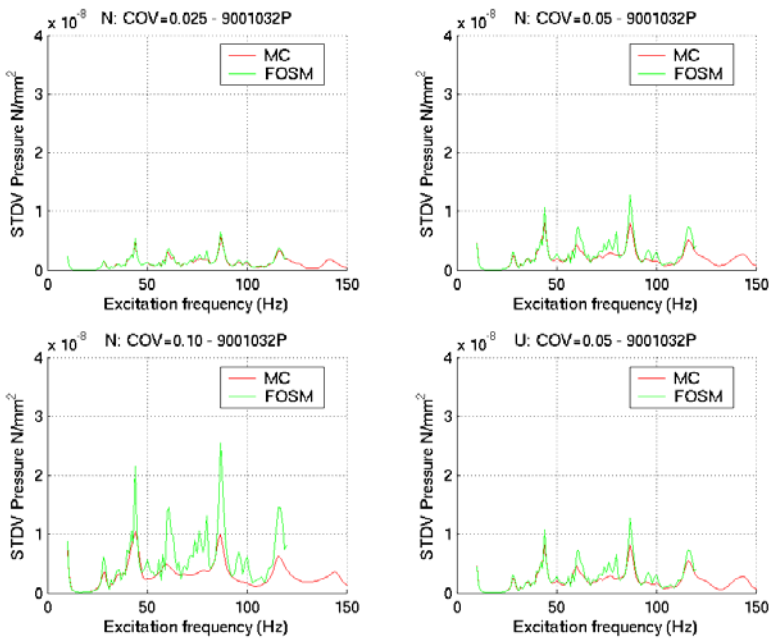

Figure 6. Monte Carlo-derived and FOSM standard deviations

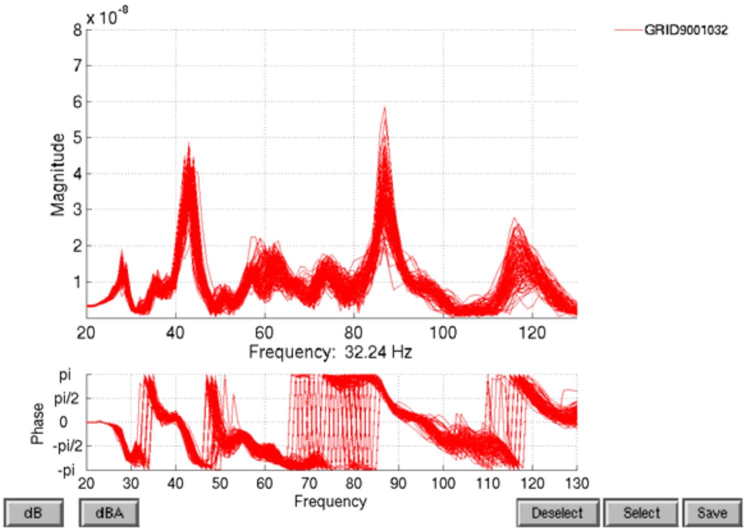

Figure 7. FRF's and phase responses for approximate Monte Carlo using MCM

CONCLUSIONS

With reference to low frequency acoustic analysis, this paper discussed the effects of FE model parameter uncertainty. An approach to stochastic optimization based on a first-order estimate of variance is discussed. The modal correction method, a fast re-analysis method, has been reviewed in the context of uncertainty analysis. The accuracy of MCM and FOSM has been assessed by comparison with Monte Carlo analysis. FOSM based stochastic optimization is shown to be valid only for very low parameter variability.

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