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Design of MEMS Resonator Array for Minimization of Mode Localization Factor Subject to Random Fabrication Error

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Abstract

This paper presents a robust optimal design method for a periodic structure type of MEMS resonator that is vulnerable to mode localization. The robust configuration of such a MEMS resonator to fabrication error is implemented by changing the regularity of periodic structure. For the mathematical convenience, the MEMS resonator is first modeled as a multi-pendulum system. The index representing the measure of mode variation is then introduced using the perturbation method and the concept of modal assurance criterion. Finally, the optimal intentional mistuning, minimizing the expectation of the irregularity measure for each substructure, is determined for the normal distributed fabrication error and its robustness in the design of MEMS resonator to the fabrication error is demonstrated with numerical examples.

INTRODUCTION

Vibrating micro-electro-mechanical (MEMS) resonators are a device widely used in wireless communication systems that acts as a narrow band-pass filter by selecting a wanted frequency component from the arbitrary input signal. Figure 1 shows the typical three-square-plate resonator in a bias and excitation configuration that specifically selects the first filter mode, where the ground plane, used for the output, connects to all structures. In theory, using the orthogonality property of mode shapes, the polarity of oscillating voltage inputs to the electrodes can be properly arranged so as to excite only a single mode of the multi-square-plate resonator. In practice, however, mode localization is likely to take place, which degrades the performance of the filter. Mode localization is a phenomenon that vibration is concentrated on a small area or part of a periodic structure with identical shaped substructures, when the periodic nature of the structure is locally and slightly defected due to manufacturing error, installation error, unexpected damage and so on [5,6,7].

For MEMS resonators, the probability of mode localization is considered very high because the fabrication process for MEMS devices is, in nature, exposed to much larger dimensional inaccuracy than conventional macro-scale machining processes. One of the most effective ways to resolve the mode localization is to introduce intentional mistuning in the periodic structural design[8,9].

In this paper, the mode localization in multi-square-plate resonators is investigated using the equivalent multi-pendulum model and the measure of mode localization is defined based on the concept of modal assurance criterion. And the intentionally mistuned resonator design is proposed, so that the expected measure of mode localization for the resonator is statistically minimized, considering the probability distribution of MEMS fabrication error. Finally, the robustness of the design to normal distributed fabrication error is demonstrated with numerical examples.

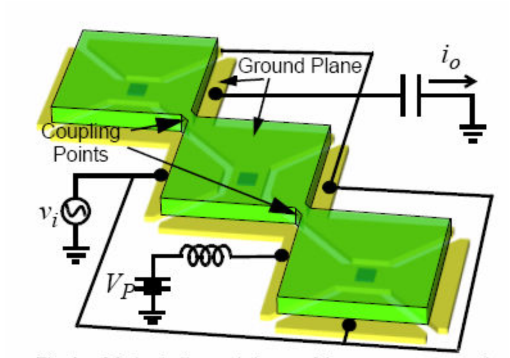

Figure 1 - Structure of three-square-plate MEMS resonator [1,2]

2. DYNAMIC ANALYSIS OF MEMS RESONATOR

2.1 Mathematical modeling

The three lower vibration modes of the three-square-plate resonator in Figure 1 are equivalent in mode shapes of the corresponding simplified three-pendulum model [1,2]. In this section, the multi-pendulum model of the multi-square-plate resonator is developed to analyze the dynamic characteristics with the system parameters varied.

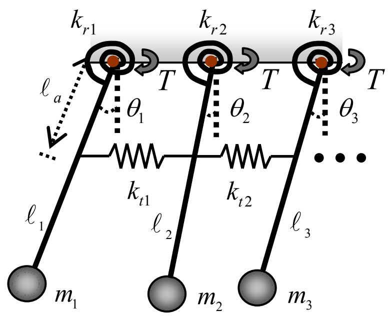

Figure 2 - Multi-pendulum system model

For the sake of equivalency in natural frequencies between the MEMS resonator and the multi-pendulum model, the parameters of the multi-pendulum model are to be properly tuned as[3,4]

$$J_i = \rho h_i L_i^2 \sigma, \quad k_{ri} = \frac{\lambda^2 h_i^3 E}{12 L_i^2 (1-\nu^2)} \kappa, \quad k_{c_i} = \frac{\varsigma_i \, \mathrm{t}_i \mathrm{b}_i^3 \, G}{\ell_{si}} \tag{1}$$

where $J_i = m_i \ell_i^2$, $k_{ci} = k_{ti} \ell_a^2$, and L_i, h_i are the side length and thickness of the i-th square plate. The equation of motion of the multi-pendulum system can then be written by introducing the dimensionless terms, with $\omega_n^2 = k_r / J$

$$\tau = \omega_n t , \quad \omega = \frac{\Omega}{\omega_n} , \quad \gamma = \frac{k_c}{k_r} , \quad f_{i0} = \frac{T_{i0}}{J\omega_n^2} \tag{2}$$

$$\mathbf{M}\ddot{\mathbf{\Theta}} + \mathbf{C}\dot{\mathbf{\Theta}} + \mathbf{K}\mathbf{\Theta} = \mathbf{F}e^{j\omega\tau} \tag{3}$$

where $\mathbf{M}$,**C, K** is the dimensionless mass, viscosity damping, stiffness matrix respectively and

$$\mathbf{\Theta} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{bmatrix}^T , \quad \mathbf{F} = \begin{bmatrix} f_{10} & f_{20} & \cdots & f_{n0} \end{bmatrix}^T \tag{4}$$

$T_i(t)$ is the harmonic torque of frequency Ω and amplitude T_{i0}, exerted to the *i*-th pendulum.

Table 1 shows the validity of the equivalent analysis model, natural frequencies calculated from the three-pendulum model were compared with those calculated from the finite element model of the three-square-plate resonator, as the side lengths, L_1 and L_2, are varied. The maximum discrepancy in natural frequency calculation is found to be far less than 0.3%, confirming that the simple multi-pendulum model can well represent the vibration behavior of the actual resonator.

Table 1 - Comparison of natural frequencies between the actual resonator and the equivalent multi-pendulum model

$(L_2/L_0)^2$-1 \ $(L_1/L_0)^2$-1		0		0.025	
		FEM	Multi-Pendulum	FEM	Multi-Pendulum
0	1st	1	1	0.987	0.987
	2nd	1.026	1.026	1.017	1.017
	3rd	1.079	1.075	1.075	1.072
0.025	1st	0.990	0.990	0.980	0.980
	2nd	1.025	1.026	1.014	1.015
	3rd	1.063	1.060	1.059	1.056
0.05	1st	0.976	0.976	0.970	0.970
	2nd	1.025	1.026	1.012	1.012
	3rd	1.052	1.050	1.048	1.045

2.2 Performance degradation of the MEMS resonator due to mode variation

The forced harmonic response is given, by letting $\theta_i = X_i e^{j\omega\tau}$, as

$$\mathbf{X} = [X_1 \quad X_2 \quad \cdots \quad X_n]^T = [-\omega^2\mathbf{M} + \mathbf{K} + j\omega\mathbf{C}]^{-1}\mathbf{F} \tag{5}$$

Now, consider the case when the input torques to all pendulums exert in phase with the same amplitude, i.e. $f_{i0} = f_0$, so that only the first mode of the multi-square-plate resonator is excited. From Eq.(5), we can then define the dynamic magnification factor[5] η as

$$\eta = \sum_{i=1}^{n} \eta_i = \sum_{i=1}^{n} \left| \frac{X_i}{f_0} \right| \tag{6}$$

where η_i is the dynamic magnification factor for the *i*-th square plate. Note that the peak amplitude of the dynamic magnification factor indicates the quality factor (Q-factor) of the resonator. Thus, the performance of the multi-square-plate resonator can be evaluated by η. Figures 3 compares the dynamic magnification factors for the original and perturbed seven-square-plate resonators. For the original resonator, since only the first mode is excited, the value of η is greatly magnified with the Q factor of about 1400 at the first modal frequency. On the other hand, for the perturbed resonator, the value of η at the first modal frequency leaks to the second and third modal frequency regions, having the reduced Q-factor of about 1000.

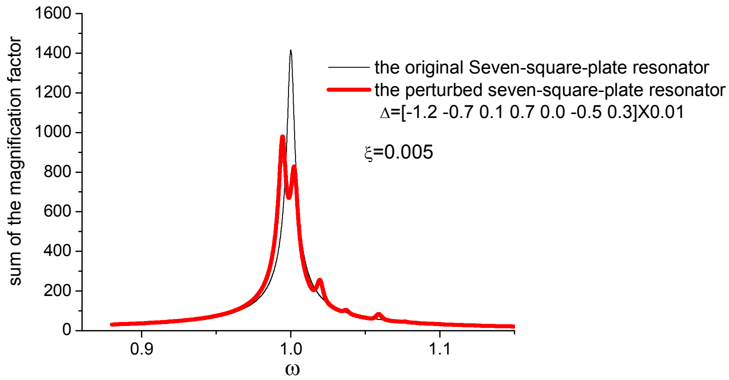

Figure 3 - Dynamic magnification factor

3 MODE LOCALIZATON FACTOR

3.1 Mode localization factor

The mode localization factor representing the amount of mode variation is defined by the perturbation method and the concept of modal assurance criteria (MAC) [11]. The mode variation is normally well predicted by the conventional perturbation method from the original system characteristics. However, for the weakly coupled periodic system with almost repeated modal frequencies, it fails, requiring Liu's perturbation method [10]. According to Liu's method, the eigenvectors of the irregular system is approximated by the linear composition of the eigenvectors of the original system, as given by

$$\varphi = \mathbf{X}_c^o \boldsymbol{\beta} \rightarrow \varphi_i = \sum_{j=1}^{l} \beta_{ji} x_j^o \tag{7}$$

$$\mathbf{X}_c^\mathbf{o} = [x_1^o, x_2^o, \cdots, x_l^o] \tag{8}$$

where, l is the number of close modes, φ_i is the approximated i^{th} eigenvector of the irregular system, $\mathbf{X}_c^\mathbf{o}$ is the matrix composed of close eigenvectors of the original system and β_{ji} is the influence coefficient that represents how much the j^{th} mode affects the change of the i^{th} mode. Here, β_{ji} can be obtained by solving the eigenvalue problem given by

$$(\mathbf{X}_c^{\mathbf{o}T} \mathbf{K} \mathbf{X}_c^\mathbf{o} - \boldsymbol{\mu} \mathbf{X}_c^{\mathbf{o}T} \mathbf{M} \mathbf{X}_c^\mathbf{o}) \boldsymbol{\beta} = 0 \tag{9}$$

In structural dynamics area, MAC has been widely used to represent the correlation between the calculated and measured eigenvectors. Now, MAC_i can be defined, to quantify the similarity between the i-th eigenvectors of the original and varied systems, as

$$MAC_i = \frac{\left| x_i^{oT} \varphi_i \right|^2}{(x_i^{oT} x_i^o)(\varphi_i^T \varphi_i)} \tag{10}$$

Note that MAC_i becomes close to 1, when the *i*-th mode shape of the irregular system still remains almost regular, not much varied from the corresponding regular original mode shape. The reverse also holds true. The *i*-th mode localization factor, that represents the degree of dissimilarity between the *i*-th eigenvectors of the original and varied systems, can then be naturally defined as

$$S_{Li} = 1 - MAC_i = 1 - \frac{\beta_{ii}^2}{\sum_{j=1}^{l} \beta_{ji}^2} \tag{11}$$

4. OPTIMAL CONFIGURATION ROBUST TO MODE LOCALIZATION

In this section, the optimal design of the multi-square-plate resonator is pursued, introducing intentional irregularities into the nominal size of each square plate, the substructure and accounting for the random fabrication errors associated with the primary design parameter, the size of the square-plate. Statistical method is used to decide the optimal system configuration because random error with irregular pattern occurs in the periodic structure.

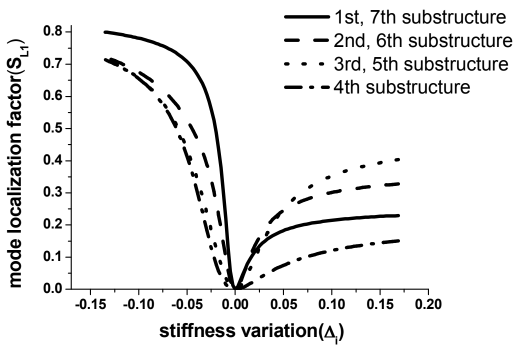

Figure 6 - Sensitivity of mode localization factor to stiffness variation of substructure: seven-square-plate resonator

Figure 6 shows the sensitivity of mode localization factor, associated with the fundamental mode, to the variation of the *i*-th substructure stiffness, Δ_i, from the nominal value for the seven-square-plate resonator. The substructure stiffness variation is incurred by the change in the side length of the square plate. Note that the mode localization factor is far more sensitive to the decrease than the increase in the plate stiffness and it is less sensitive to the stiffness change of the center plate, substructure #4 than the other substructures. In particular, the decrease in the stiffness of substructures #1 and 7, the outmost plates, most affects on the mode localization factor. The results in Figure 6 suggests that the intentional irregularity introduced to the nominal design value for the side length of each square plate may reduce the probable mode variation of multi-square-plate resonators when they are subject to random fabrication error.

The stiffness variation in the *i*-th substructure (Δ_i) due to random fabrication error is assumed to be a normal distributed random variable, whose probability density function is given by

$$p(\Delta_i) = \frac{1}{\sqrt{2\pi}\sigma_{\Delta i}} \exp\left(-\frac{(\Delta_i - \mu_{\Delta i})^2}{2\sigma_{\Delta i}^2} \right) \tag{12}$$

where μ_Δ and σ_Δ^2 are the mean and variance of Δ. Note that the mean value is related to the intentional irregularity. Now, the optimal design problem becomes such that the mode localization factor is to be minimized for the design variable μ_Δ when the fabrication error variance σ_Δ^2 is given.

The expected mode localization factor for substructure #*i* associated with the fundamental mode is defined as

$$\mu_{Li}(\mu_{\Delta i}) = E[S_{Li}(\Delta_i)] = \int_{-\infty}^{\infty} S_{Li}(\Delta_i) p(\Delta_i, \mu_{\Delta i}) d\Delta_i \qquad (13)$$

Note that the mode localization factor is a function of the stiffness error mean, while its variance is fixed. Thus, optimization should be carried out for $\mu_{\Delta i}$ by minimizing the mode localization factor. The optimized design value determines the intentional irregularity for the nominal side length of structure # *i*.

The expected mode localization factor for substructure #1 is plotted against the error mean for error standard deviation of 0.02 in Figure 7. The minimum expected mode localization factor is found at $\mu_{\Delta 1}$ of 0.021 (marked by broken line). It implies that the stiffness of substructure #1 should be increased by 2.1 % from its nominal value, in order to minimize the expected mode localization factor. The similar optimization procedure applies to other substructures.

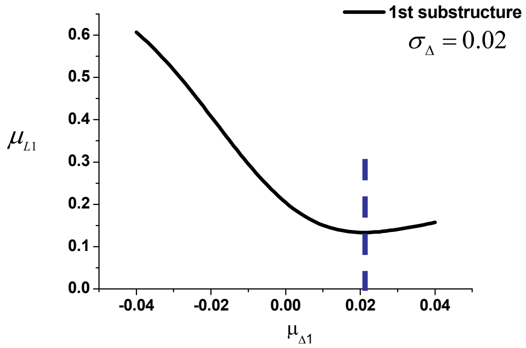

Figure 7 - Expected mode localization factor vs. intentional mistuning $\mu_{\Delta 1}$ *: substructure # i*

The optimized configuration of the seven-square-plate resonator are determined to be $1+\mu_\Delta^o$, with $\mu_\Delta^o = [2.1\ 1.0\ 0.5\ 1.4\ 0.5\ 1.0\ 2.1]\%$, as shown in Figure 8. Since the values of μ_Δ^o increase almost in proportion to the standard deviation σ_Δ, the W-shaped optimal configuration is retained, irrespective of σ_Δ. The W-shaped configuration, which should be symmetric about the center substructure, suggests that the stiffness of the outmost end substructures should be largest.

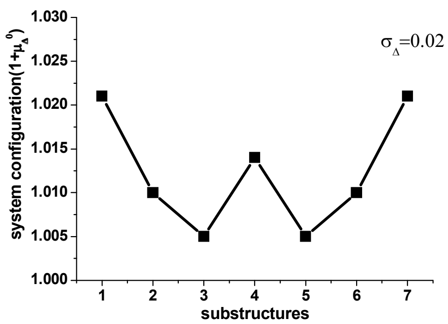

Figure 8 - Optimal resonator configuration

Figure 9 compares the probability distributions of the mode localization factor calculated for the seven-square-plate resonator with the nominal and modified configurations. Note that the probability of low mode localization factor is significantly improved by the optimal configuration. In other words, the optimized configuration is more robust to fabrication error than the original, nominal configuration for the seven-square-plate resonator, in the sense that the expected mode localization factor is minimized. It implies that the resonators with the optimal configuration yield statistically higher Q-factor than the original periodic structure with nominal, identical side length.

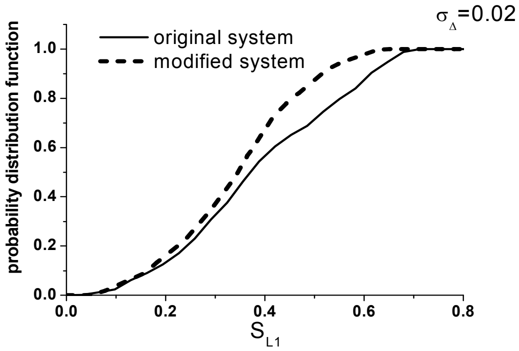

Figure 9 - Probability distribution function of L *for* $\sigma_{\Delta} = 0.02$

To evaluate the performance of the resonators with the original and modified configurations, the sum of dynamic magnification factors from the seven substructures was calculated for the fabrication error of $\Delta = [-1.2\ -0.7\ 0.1\ 0.7\ 0.0\ -0.5\ 0.3]$ as in Figure 3. Although the results in Figure 10 represent only one arbitrarily selected realization among infinitely many possible cases, the comparison of performance of two different configurations is informative. Note that the sum of dynamic magnification factors, the total Q-factor, has been significantly improved, at least for this case. For the nominal configuration, when it is subject to the specified fabrication error, the Q-factor is likely to leak to the neighboring modes, i.e. the second and third modes as shown in Figure 10.

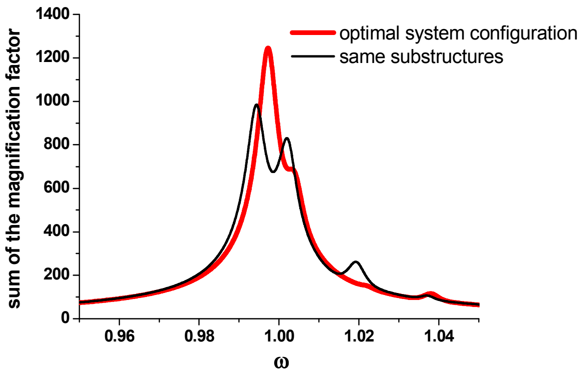

Figure 10 - Dynamic magnification factor; total Q-factor

4. CONCLUSION

In this paper, a multi-pendulum model has been introduced to effectively analyze the vibration characteristics of multi-square-plate MEMS resonator. Using the analysis model, the mode localization factor, that represents the mode variability due to system stiffness and mass change, was defined by using the perturbation method and MAC value. Then, assuming the fabrication error as a normal distributed random variable, the expected mode localization factor was calculated. An optimization problem was then formulated to find the optimal system configuration by minimizing the expected mode localization factor. It is confirmed that the optimally designed multi-square-plate resonator with irregular side lengths for each square-plate yields statistically higher Q-factor than the original resonator with identical side lengths. The effectiveness of the optimal design has been successfully demonstrated with a seven-square-plate resonator model throughout the paper.

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