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VERIFICATION OF THE SIMULTANEOUS EQUATIONS METHOD BY AN EXPERIMENTAL SYSTEM

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Abstract

This paper verifies the performance of the simultaneous equations method by an experimental active noise control system. The experimental results demonstrate that the method can automatically recover the noise reduction effect degraded by path changes and successfully works in practical systems. The simultaneous equations method is based on a principle different from the filtered-x algorithm requiring a filter modeled on a secondary path from a loudspeaker to an error microphone. Instead of the filter, called "secondary path filter," this method uses an auxiliary filter identifying the overall path consisting of a primary path, a noise control filter and the secondary path. As inferred from the configuration of the overall path, the auxiliary filter can provide two independent equations when two different coefficient vectors are given to the noise control filter. The method thereby estimates the coefficient vector of the noise control filter minimizing the output of the error microphone.

INTRODUCTION

The filtered-x algorithm [1] is widely applied to feedforward type active noise control (ANC) systems [2]. This algorithm, however, involves a well-known drawback to be solved. Actually, the algorithm requires a filter, called "secondary path filter," exactly modelled on the secondary path from a loudspeaker to an error microphone, whereas the secondary path in practical systems continuously changes. This path change inevitably increases the modelling error, and at worst, the ANC system thereby falls into an uncontrollable state [3].

As a means for repeatedly identifying the secondary path, [5] presents a way of feeding an extra noise to the loudspeaker. In practical systems, avoiding such feeding is desirable. Hence, a few methods capable of automatically recovering the noise reduction effect without feeding the extra noise have been proposed [6-9]. However, [6] and [7] neglect the feedback path from the loudspeaker to the noise detection microphone. In addition, the noise reduction speed of them is slower than that of the filtered-x algorithm, and the processing cost of [7, 8] is higher.

The simultaneous equations method proposed in [8, 9] can successfully work under the condition that the feedback path generates no howling [11]. Especially, [9] shows that the processing cost is lower than that of the filtered-x algorithm and also the noise reduction speed is higher. However, it has not yet been verified whether the simultaneous equations method can reduce the noise in practical systems. This paper verifies it by using an experimental system.

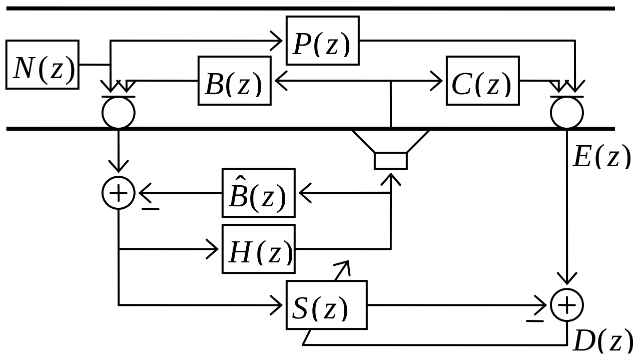

Figure 1 - Configuration of the feedforward type active noise control system using the simultaneous equations method

SIMULTANEOUS EQUATIONS METHOD

The simultaneous equations method [8] is characterized by an auxiliary filter substituted for the secondary path filter forming the core of the filtered-x algorithm [1]. Figure 1 shows the configuration of the feedforward type active noise control system using the simultaneous equations method, where these *z*-transforms designate the following signals, filters and paths.

$P(z)$: Primary path from noise detection microphone, Md, to error microphone, Me

$C(z)$: Secondary path from loudspeaker, Sp, to error microphone, Me

$B(z)$: Feedback path from loudspeaker, Sp, to noise detection microphone, Md

$B(z)$: Feedback control filter

$N(z)$: Primary noise

$H(z)$: Noise control filter

$S(z)$: Auxiliary filter

$X(z)$: Input signal of noise control filter

$D(z)$: Identification error

$E(z)$: Output signal of error microphone

In this configuration, the auxiliary filter $S(z)$ is used for identifying the overall path from the input of the noise control filter to the output of the error microphone. This overall path is moreover rearranged as shown in Fig. 2, where

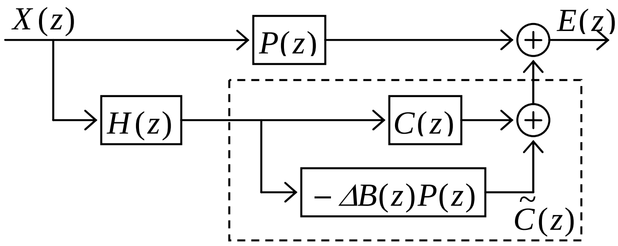

Figure 2 - Block diagram of the overall path identified by the auxiliary filter

$$\Delta B(z) = B(z) - \hat{B}(z) \tag{1}$$

$$\tilde{C}(z) = C(z) - \Delta B(z)P(z) \tag{2}$$

Naturally, $\tilde{C}(z)$ is equal to $C(z)$ when the feedback control filter perfectly cancels the feedback path: $\Delta B(z) = 0$. In any case, the auxiliary filter gives

$$S(z) = P(z) + H(z)\tilde{C}(z) \tag{3}$$

after the identification. In this system, our purpose is to derive the optimum noise control filter, $H_{opt}(z)$, satisfying

$$P(z) + H_{opt}(z)\tilde{C}(z) = 0\,. \tag{4}$$

(4) states that the estimation of $P(z)$ and $\tilde{C}(z)$ is necessary for the derivation of $H_{opt}(z)$. However, the available signals for this estimation are only $X(z)$ and $E(z)$; thus, this system can provide only (3). As it stands, estimating $P(z)$ and $\tilde{C}(z)$ from only (3) is impossible. For the estimation, two independent equations are requisite.

To obtain the two equations, the simultaneous equations method exploits the fact that the system can give arbitrary coefficient vectors to the noise control filter if we accept the degradation of the noise reduction effect. Such acceptance can provide us two relations:

$$S_1(z) = P(z) + H_1(z)\tilde{C}(z) \tag{5}$$

$$S_2(z) = P(z) + H_2(z)\tilde{C}(z) \tag{6}$$

after the identification of the overall path. Clearly, (5) and (6) are valid when the two different coefficient vectors given to the noise control filter satisfy $H_2(z) \neq H_1(z)$. However, the degradation of the noise reduction effect is unacceptable in practical use. To prevent the degradation, the simultaneous equations method exploits the error involved in the estimated coefficient vector of the noise control filter.

Under this condition, the eliminations of $\tilde{C}(z)$ and $P(z)$ provide

$$P(z) = \{S_1(z)H_2(z) - S_2(z)H_1(z)\} / \{H_2(z) - H_1(z)\} \tag{8}$$

$$\tilde{C}(z) = \{S_1(z) - S_2(z)\} / \{H_1(z) - H_2(z)\}\,. \tag{9}$$

As mentioned above, our purpose is to estimate $H_{opt}(z)$ satisfying (4). In addition, $P(z)$ and $\tilde{C}(z)$ necessary for calculating $H_{opt}(z)$ are obtained as (8) and (9). Then, remaining operation is only the substitution of (8) and (9) into (4). The substitution yields

$$\{S_1(z)H_2(z) - S_2(z)H_1(z)\} - H_{opt}(z)\{S_2(z) - S_1(z)\} = 0 . \tag{10}$$

Moreover, as inferred from Fig. 2, $H_2(z) \neq H_1(z)$ satisfies $S_2(z) - S_1(z) \neq 0$. Therefore, (10) gives

$$H_{opt}(z) = \{S_1(z)H_2(z) - S_2(z)H_1(z)\} / \{S_2(z) - S_1(z)\} \tag{12}$$

consisting of the known components.

This paper next applies a frequency domain technique to transforming $H_{opt}(z)$ into a filter coefficient vector [9]. In the frequency domain, $H_{opt}(z)$ is expressed as

$$H_{opt}(k) = \{S_1(k)H_2(k) - S_2(k)H_1(k)\} / \{S_2(k) - S_1(k)\} \tag{13}$$

where $H_1(k)$, $H_2(k)$, $S_1(k)$ and $S_2(k)$ are the frequency responses of the noise control filter and auxiliary filter, and k is the element number of them calculated by the fast Fourier transform (FFT). The transformation using FFT moreover requires the estimation of the frequency response of the auxiliary filter, $S(k)$. For this estimation, this paper uses the following frequency domain adaptive algorithm:

$$S_{j+1}(k) = S_j(k) + \mu \sum_{i=jI+1}^{(j+1)I} D_i(k) X_i^*(k) \Bigg/ \sum_{i=jI+1}^{(j+1)I} X_i(k) X_i^*(k) , \tag{14}$$

where j is the block number, μ is the step size, i is the FFT duration number, I is the number of blocks, $D_i(k)$ is the k th spectrum element of the identification error $D(z)$ shown in Fig. 1, $X_i(k)$ is the k th spectrum element of the noise control filter input, and $\{*\}$ designates the complex conjugate. The calculation of (14) is repeated J times, and its result is used as $S_1(k)$ and $S_2(k)$. In (14), the summation is applied to reducing the probability that the denominator of the second term becomes zero.

VERIFICATION BY EXPERIMENTAL SYSTEM

Using an experimental system shown in Fig. 3, this paper verifies the performance of the simultaneous equations method. The experimental system is constructed with a vinyl chloride pipe of 83 mm diameter and controlled by a personal computer. Table 1 shows the main equipment used in the experimental system.

Figure 4 shows the decreasing properties of the error microphone output obtained working the experimental system under the following conditions:

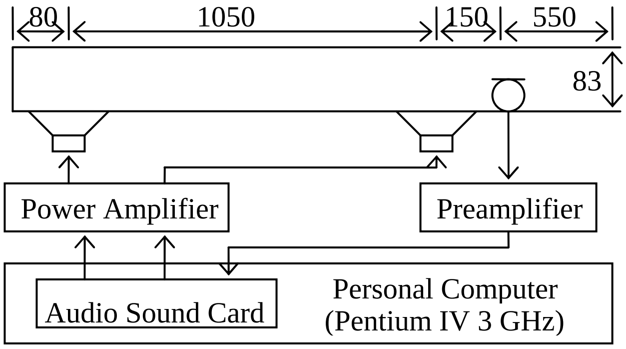

Figure 3 - Configuration of the experimental system used for verifying the performance of the proposed method (The unit of scale is millimeter)

Table 1 Equipment used in the experimental system

Personal Computer	Dell, Dimension 8300 (Pentium IV 3 GHz)
Power Amplifier	Yamaha HC-2700
Pre-amplifier	Audio Technica, AT-MA2
Loudspeaker	Pioneer, TS-E1076
Microphone	Audio Technica, AT-805F
Audio Sound Card	M Audio, Delta 44

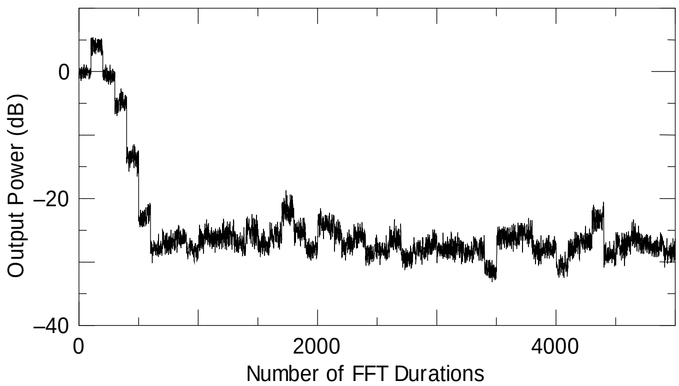

Figure 4 - Decreasing properties of the output of the error microphone

(1) The number of taps of the auxiliary filter is 1,024.
(2) Accordingly, the length of the FFT duration is 2,048.
(3) The number of taps of the noise control filter is 512.
(4) $\mu = 0.25$, $I = 5$, $J = 20$.
(5) A recorded diesel engine generator exhaust gas noise is used as the primary noise.
In Fig. 4 , the horizontal axis designates the number of FFT durations, and the output power shown in the vertical axis is calculated as

$$Pe_i = 10\log_{10} \sum_{k=0}^{1023} E_i(k)E_i^*(k) \Big/ Pe_0 \qquad (32)$$

where $E_i(k)$ is the k th spectrum component calculated using the error microphone output samples in the i th FFT duration, and Pe_0 is calculated as

$$Pe_0 = \sum_{i=0}^{IJ-1}\sum_{k=0}^{1023} E_i(k)E_i^*(k) \Big/ IJ \tag{33}$$

which is approximated to the average power of the error microphone output detected previous to feeding the secondary noise to the loudspeaker.

In this experiment, the first 200 ($= IJ \times 2$) FFT durations are used for only setting up the simultaneous equations, and the operation of updating the coefficient vector of the noise control filter starts from the 200th duration. According to the results, the output powers of the error microphone decrease to less than -20 dB after two or four updating operations, and subsequently, this system keeps the output less than 20 dB. On the other hand, the output power fluctuates in the durations after has decreased to less than -20 dB, although inaudible.

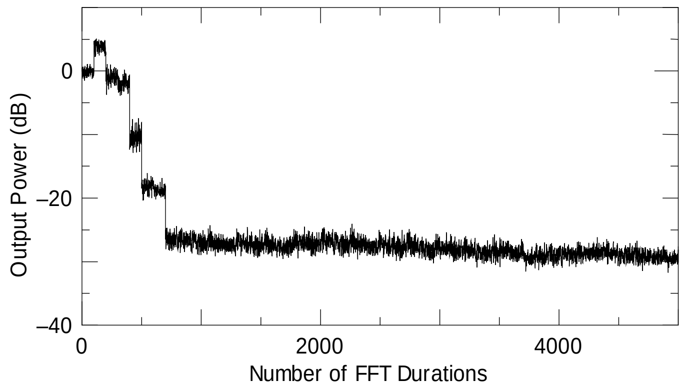

Figure 5 - Decreasing properties of the output of the error microphone built in the experimental system with the averaging operation

To reduce this fluctuation, this paper adds an operation of averaging the coefficient vector of the noise control filter to the simultaneous equations method. Figure 5 is a decreasing property obtained by applying the averaging operation,

$$\hat{H}_{opt}(k) = H_{opt(k)} \times 0.1 + \hat{H}_{opt}(k) \times 0.9\,, \tag{34}$$

to the simultaneous equations method only when the output power is less than -20 dB. Apparently, the averaging operation firmly maintains the output power.

Here, let's confirm the frequency characteristics of the noise reduction effect provided by the simultaneous equations method. Figure 6 shows the average of the power spectrums calculated using the error microphone output samples detected in five FFT durations, which is calculated as

$$Pa(k) = 10\log_{10} \sum_{m=1}^{5} E_m(k)E_m^*(k) \Big/ I \tag{35}$$

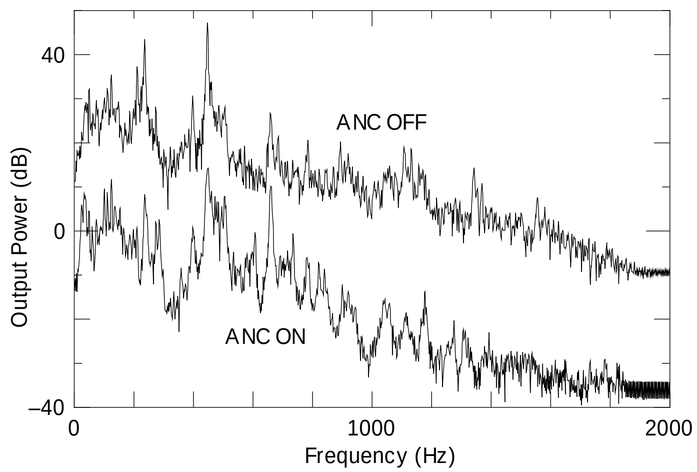

Figure 6 - Power spectrums of the error microphone outputs, where "ANC ON" and "ANC OFF" denote after and before the application of the active noise control

In this result, we can see that the noise reduction effect is obtained in the whole frequency range. Usually, the inversion of the effect is observed especially in the low and high frequency bands. In this experimental result, such inversion is not observed. This is an advantage of the simultaneous equations method.

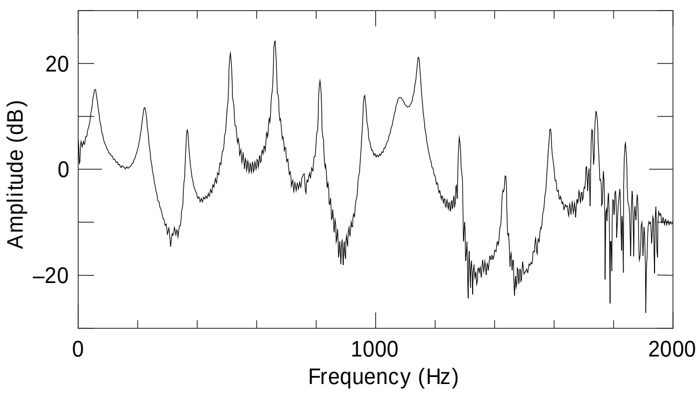

Figure 7 - Amplitude characteristics of the noise control filter

Figure 7 also shows the amplitude characteristics calculated from the coefficients of the noise control filter. These sharp resonances, observed in Fig. 7, indicate that the noise control filter with many taps is required for obtaining the sufficient noise reduction effect.

The strong point of the simultaneous equations method is that the noise reduction effect degraded by path changes can be automatically recovered. This paper finally verifies the point using the experimental system. Figure 8 shows the recovering property obtained by the experimental system, where the path change is substituted by multiplying the output of the noise control filter by -1 (change from $C(z)$ to $-C(z)$). This experimental result demonstrates that the simultaneous equations method successfully works in practical systems whose secondary path changes.

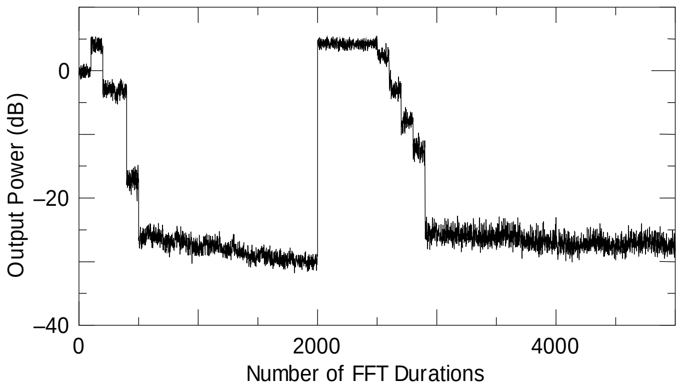

Figure 8 - Recovering properties provided by the proposed method

CONCLUSION

This paper has applied the simultaneous equations method to the experimental system and has verified that the method successfully works in practical systems. This simultaneous equations method can also be applied to updating the coefficient vector of the feedback control filter canceling the feedback path from the secondary source to the noise detection sensor [11]. Our subsequent studies will hence focus on the verification of the feedback path identification method using the simultaneous equations method by an experimental system.

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