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ACCOUNT FOR ELASTO-INERTIAL PROPERTIES OF FRAMES AT EVALUATION OF NOISE INSIDE AIRCRAFT CABIN

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Abstract

The acoustic field in a closed cylindrical shell of finite length with a regular orthogonal system of stiffeners (stringers and frames) which models the propeller aircraft fuselage section is investigated. An analytical model accounting for frame discreteness and for their elasto-inertial properties is examined. The stringers in this case are “smeared”. The eigen-frequency density corresponding to this analytical model has its maxima not only in the vicinity of the ring frequency, but also in the vicinity of the frequencies characteristic of regular structures. Shell excitation by the aero-acoustic field of propeller and the sound field formation in the shell are described with account for the effects of a elasto-acoustic interaction between its oscillations and the sound-transmitting layers, the ambience and the medium in the limited volume. The analysis of the acoustic field inside aircraft within the limits of this analytical model permitted determining the conditions at which the framed shell of the fuselage can be treated as orthotropic one and the conditions at which the frame discreteness and their elasto-inertial properties, are to be taken into account.

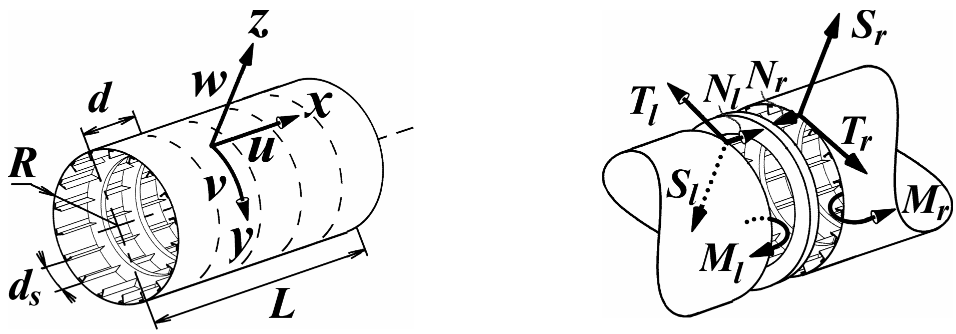
INTRODUCTION

The noise inside propeller aircraft is determined in many respects by elasto-inertial properties of the framed fuselage shell. To predict the noise at low frequencies corresponding to the blade passage frequency of the propeller and to its lower harmonics, a structurally orthotropic model of the shell can be used [1,2]. However the orthotropic shell model works well only in the cases and at the frequencies at which its long-wave modes dominate the short-wave ones in the process of the sound field excitation and formation. As the prediction frequency, stiffener rigidity or the requirements to prediction accuracy increase, more precise models accounting for the stiffener discreteness and, primarily the frames discreteness, are to be used.

The shell with discrete frames was considered in works [3,4] where the lowest modes received primary attention. The present work considers a shell model accounting for frames discreteness and their elasto-inertial properties and with stringers are "smeared". In other words, a milti-span closed cylindrical shell consisting of orthotropic spans and supported by mobile frames is considered. The task related to excitation of the shell, the sound-insulation structure layers, the closed volume and the surroundings is solved in this case as a coupled elasto-acoustic one. There is an analytical solution for such a model and it is obtained and presented in this work. The similar task was solved by the authors [5] for a shell with motionless frames.

PREDICTION RELATIONS

Consider a thin closed cylindrical shell of L in length and of R in radius free supported at butt-ends and stiffened by a regular system of stiffeners (stringers and frames).



a) Coordinate directions *b) Force directions*

Figure 1 – Scheme of stiffened shell, coordinate and force directions.

The shell consists of N identical orthotropic spans of d in length and the stringers are "smeared". The equation for such span vibrations can be described by the matrix expression:

$$\left(\begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{pmatrix} + \frac{\partial^2}{\partial \bar{t}^2}\begin{pmatrix} 0 \\ 0 \\ 1+\overline{m}_s \end{pmatrix}\right)\begin{pmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \overline{q}_w \end{pmatrix} \tag{1}$$

$$L_{11} = (1+\overline{E}_s\overline{A}_s)\frac{\partial^2}{\partial \overline{x}^2} + \frac{1-\mu}{2}\frac{\partial^2}{\partial \overline{y}^2}, L_{12} = \frac{1+\mu}{2}\frac{\partial^2}{\partial \overline{x}\partial \overline{y}}, L_{13} = \mu\frac{\partial}{\partial \overline{x}} + \overline{z}_s\overline{E}_s\overline{A}_s\frac{\partial^3}{\partial \overline{x}^3}, L_{22} = \frac{1-\mu}{2}\frac{\partial^2}{\partial \overline{x}^2} + \frac{\partial^2}{\partial \overline{y}^2},$$

$$L_{23} = \frac{\partial}{\partial \overline{y}},\; L_{33} = 1 + \frac{\overline{h}^2}{12}\left(\frac{\partial^2}{\partial \overline{x}^2} + \frac{\partial^2}{\partial \overline{y}^2}\right)^2 + \frac{\partial^2}{\partial \overline{x}^2}\left(\overline{E}_s(\,\overline{I}_s + \overline{z}_s^{\,2}\overline{A}_s)\frac{\partial^2}{\partial \overline{x}^2} + \overline{G}_s\overline{J}_s\frac{\partial^2}{\partial \overline{y}^2}\right),$$

$$K = Eh/(1-\mu^2),\; \overline{E}_s\overline{A}_s = E_sA_s/(d_sK),\; \overline{E}_s\overline{I}_s = E_sI_s/(d_sKR^2),\; \overline{G}_s\overline{J}_s = G_sJ_s/(d_sKR^2),$$

$$\bar{t} = t\omega_{Ring},\; \omega_{Ring}^2 = K/(\rho hR^2),\; \overline{m}_s = \rho_sA_s/(\rho hd_s),\; \overline{z}_s = z_s/R,\; \overline{q}_w = q_wR/K.$$

Here all the coordinates and displacements are related to radius R and their directions are shown in figure 1a). The following specifications are used: μ, E, ρ, h are the Poisson coefficient, Young's module, density, shell thickness; $\rho_s, d_s, A_s, I_s, J_s, z_s$ are the stringer density, their placement step, cross-section area, momentum, turning momentum, eccentricity (positive for the internal location); q_w is the overall normal action force; ω_{Ring} is the ring frequency.

The frame vibrations can be described by the following expressions

$$\left(\begin{pmatrix} L_{22r} & L_{23r} \\ L_{23r} & L_{33r} \end{pmatrix} + \frac{\partial^2}{\partial \bar{t}^2} \begin{pmatrix} 0 \\ \bar{m}_r \end{pmatrix} \right) \begin{pmatrix} \bar{v} \\ \bar{w} \end{pmatrix} = \begin{pmatrix} -\bar{T} \\ \bar{S} \end{pmatrix}, \quad \begin{matrix} \bar{T} = TR/(Kd), \\ \bar{S} = SR/(Kd), \end{matrix}$$

$$L_{22r} = \bar{E}_r \bar{A}_r (1 - \bar{z}_r)^2 \partial^2/\partial \bar{y}^2, L_{23r} = \bar{E}_r \bar{A}_r (1 - \bar{z}_r) \partial/\partial \bar{y} (1 + \bar{z}_r \partial^2/\partial \bar{y}^2), \quad (2)$$

$$L_{33r} = \bar{E}_r \bar{I}_r (\partial^2/\partial \bar{y}^2 + 1)^2 + \bar{E}_r \bar{A}_r (1 + \bar{z}_r \partial^2/\partial \bar{y}^2)^2,$$

$$\bar{m}_r = \rho_r A_r /(\rho h d), \quad \bar{E}_r \bar{A}_r = E_r A_r /(dK), \bar{E}_r \bar{I}_r = E_r I_r /(dKR^2), \quad \bar{z}_r = z_r / R.$$

Here ρ_r, A_r, I_r, z_r are the frame density, cross-section area, its momentum, eccentricity (positive for the internal location); S is the cutting-off force, T is the tangential force directed as shown in figure 1b). We neglect in eq. (1,2) all the inertial terms except the radial (although it is right only for $n > 4$ [4]) and the terms of the h^2 / R^2 order.

The solution for the finite shell is obtained from the solution for the infinite one, leaving the frame "halves" on the shell butt-ends. The span eigen-functions for the harmonic oscillations of the infinite shell are sought in the form of expansion in terms of functions:

$$\begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} = \bar{W}_j \exp(\bar{\lambda}_j \bar{x}) \begin{pmatrix} \bar{U}_j \cos(n\bar{y} + \pi s/2) \\ \bar{V}_j \sin(n\bar{y} + \pi s/2) \\ \cos(n\bar{y} + \pi s/2) \end{pmatrix}, \quad s = \{0,1\}. \quad (3)$$

Here and below the time factor $\exp(i\omega t)$ is omitted. The substitution of functions (3) into the left-side part of (1) permits detecting eight wave numbers ($\bar{\lambda}_i, i = 1,..,8$) for any frequency. The radial displacements in the running wave can be expressed as a sum of

$$w_{nms} = f_1 \cos(n\bar{y} + \pi s/2), \quad f_1(x_1) = \sum_{j=1}^{8} \bar{W}_j e^{\bar{\lambda}_j \bar{x}_1}. \quad (4)$$

The frame vibrations are presented as follows:

$$v_r = V_r \sin(n\bar{y} + \pi s/2), w_r = W_r \cos(n\bar{y} + \pi s/2), \; V_r = \sum \bar{V}_j W_j, W_r = \sum W_j. \quad (5)$$

The substitution of (5) into (2) gives the following expression:

$$\begin{pmatrix} l_{22r} & l_{23r} \\ l_{23r} & l_{33r} \end{pmatrix}\begin{pmatrix} V_r \\ W_r \end{pmatrix} = \begin{pmatrix} \overline{T} \\ \overline{S} \end{pmatrix}, \begin{matrix} l_{22r} = \overline{E}_r\overline{A}_r(1-\overline{z}_r)^2 n^2,\ l_{23r} = \overline{E}_r\overline{A}_r(1-\overline{z}_r)n(1-\overline{z}_r n^2), \\ l_{33r} = \overline{E}_r\overline{I}_r(n^2-1)^2 + \overline{E}_r\overline{A}_r(1-\overline{z}_r n^2)^2 - \overline{m}_r\overline{\omega}^2. \end{matrix} \quad (6)$$

The eigen-functions for the wave running over the infinite shell with discrete frames have the repeatability property ($f(x) = e^{i\alpha} f(x-d)$). It means that the eigen-functions of the adjacent spans differ only in the phase step. This phase step is constant and determined by the longitudinal mode index m ($\alpha = \pi m / N$) and this corresponds to the condition of node generation on the shell butt-ends in the case of two contrary wave superposition.

In two span joints three conditions of continuity are realized as well as the condition of equality of angle inclinations. With account for the phase step constancy, they all are expressed as follows:

$$\sum W_j e_j = 0,\ \sum W_j \overline{\lambda}_j e_j = 0, \sum W_j \overline{U}_j e_j = 0,\ \sum W_j \overline{V}_j e_j = 0,$$
$$e_j = (1 - exp(\overline{\lambda}_j \overline{d} - i\alpha)),$$
$$\text{or}\ \sum (e_j,\ \overline{\lambda}_j e_j,\ \overline{U}_j e_j,\ \overline{V}_j e_j)^T W_j = 0. \quad (7)$$

The ring is subjected to the forces at the left side (l) and at the right side (r) (figure 1b):

$$\begin{aligned} S_{r,l} &= -K\left((\overline{w}_{\overline{xxx}} + (2-\mu)\overline{w}_{\overline{xyy}})\overline{h}^2/12 + \overline{E}_s(\overline{I}_s + \overline{z}_s^2\overline{A}_s)\overline{w}_{\overline{xxx}} + \overline{G}_s\overline{J}_s\overline{w}_{\overline{xxy}} + \overline{z}_s\overline{E}_s\overline{A}_s\overline{u}_{\overline{xx}}\right), \\ T_{r,l} &= -Eh/(2(1+\mu))(\overline{u}_{\overline{y}} + \overline{v}_{\overline{x}}). \end{aligned} \quad (8)$$

Add up the left forces and the right forces of the opposite direction:

$$\begin{aligned} \overline{S} &= \sum g_j^S W_j,\ g_j^S = -(\overline{D}_x\overline{\lambda}_j^3 + \overline{z}_s\overline{E}_s\overline{A}_s\overline{\lambda}_j^2\overline{U}_i)e_j/\overline{d}, \\ \overline{T} &= \sum g_j^T W_j,\ g_j^T = (1-\mu)\overline{\lambda}_j\overline{V}_j e_j/(2\overline{d}), \overline{D}_x = \overline{h}^2/12 + \overline{E}_s(\overline{I}_s + \overline{z}_s^2\overline{A}_s). \end{aligned} \quad (9)$$

Write down the conditions of continuity on the frame of longitudinal tension and momentum as follows

$$\begin{aligned} &\sum g_j^M \overline{W}_j = 0, \quad g_j^M = (\overline{D}_x\overline{\lambda}_j^2 + \overline{z}_s\overline{E}_s\overline{A}_s\overline{\lambda}_j\overline{U}_i)e_j/\overline{d}, \\ &\sum g_j^N \overline{W}_j = 0, \quad g_j^N = ((1+\overline{E}_s\overline{A}_s)\overline{U}_j\overline{\lambda}_j + \overline{z}_s\overline{E}_s\overline{A}_s\overline{\lambda}_j^2)e_j/\overline{d}. \end{aligned} \quad (10)$$

Combine relations (6,7,9,10) into a unified resolving matrix relation, on the basis of which eigen-frequences and coefficients W_j are found:

$$\sum_{j=1}^{8}(l_{23r}\overline{V}_j + l_{33r} - g_j^S,\ l_{22r}\overline{V}_j + l_{23r} - g_j^T,\ g_j^N,\ g_j^M, e_j, \lambda_i e_j, \overline{U}_i e_j, \overline{V}_j e_j)^T W_j = 0. \quad (11)$$

The complex conjunction of the oscillation form of the wave running along the span f_1^* makes the form of the wave running to meet it and this explains the symmetry of function $f_1(x_1)/f_1(d/2) = f_1^*(d-x_1)/f_1^*(d/2)$ relative to the span middle. Making use of this symmetry property, one can write down an expression for eigen-functions of the N-spanned shell as follows

$$F_n(x) = Re(\bar{f}_1(x_1)e^{i(\alpha(n-1/2)-\pi/2)}),\ n = 1,..,N,$$
$$x_1 = x - d(n-1),\ \bar{f}_1(x_1) = f_1(x_1)/f_1(d/2). \qquad (12)$$

The sound field inside the shell can be presented in the form of an expansion:

$$p = \sum_{n,m',s} P_{nm's} J_n(\mu_{m'}r)\chi_{m'}(k_{m'}x)cos(n\theta + \frac{\pi s}{2}),\ k_{m'} = \frac{\pi m'}{L},\ \mu_{m'}^2 = \frac{\omega^2}{c^2} - k_{m'}^2. \qquad (13)$$

Where J_n is Bessel function, $\chi_{m'} = cos(k_{m'}x)$ for acoustically rigid butt-ends and $\chi_{m'} = sin(k_{m'}x)$ for acoustically soft ones. To solve the associated task related to the interior sound field with account for the interaction with sound insulation layers and the ambience, we use the relations from work [5]. Eventually we shall get a connection between the sound pressure amplitudes in the shell $P_{nm's}$ and the generalized forces Q_{nms} of the external field:

$$Q_{nms} = \sum_{m'} P_{nm's} J_{nm'} C_{nmm'} \left(Z_{nm}^{cyl}(a_{21} + a_{11}/Z_{nm'}^{ac}) + (1+\Sigma)(a_{22} + a_{12}/Z_{nm'}^{ac}) \right),$$
$$Z_{nm}^{cyl} = i\omega m_{sp}\left(1 - \omega_{nm}^2(1+i\eta)/\omega^2\right) + Z_{nm}^{out},\ Z_{nm'}^{ac} = i\omega\rho_{ac} J_{nm'}/(\mu_{m'}J'_{nm'}),$$
$$M_{nm} = m_{sp} + \rho_r A_r \sum F^2(pd)/\nu_F,\ \Sigma = \rho_r A_r \sum F(pd)\chi(pd)/(\nu_F m_{sp}),$$
$$C_{nmm'} = \int F\chi dx/(\nu_F M_{nm}),\ \nu_F = \int F^2 dx,\ m_{sp} = \rho h(1+\overline{m}_s),\ J_{nm'} = J_n(\mu_{m'}R_a). \qquad (14)$$

Here $C_{nmm'}$ are the coefficients of the expansion of longitudinal components of acoustical modes in terms of elastic modes with account for the shell mass non-uniformity, R_a, ρ_{ac} are the acoustic volume radius and density, Z_{nm}^{cyl} is the modal impedance of the shell with account for the ambience response.

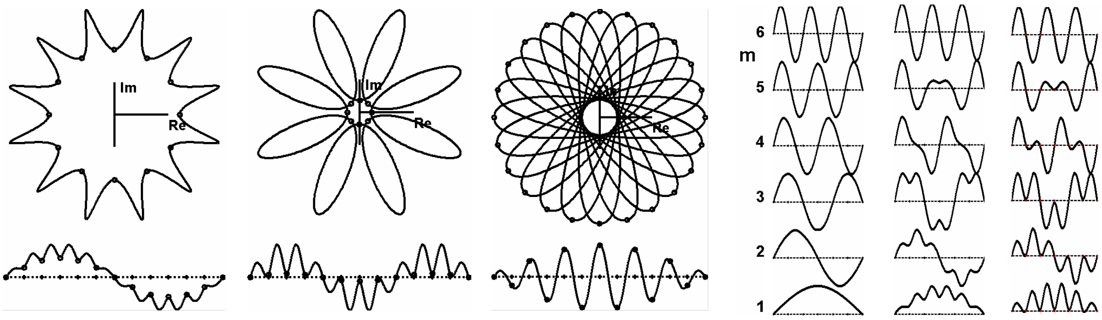
The efficiency of reducing the sound level by a board structure can be characterized through the ratio of the pressure squared of the exciting field at the point of its intensity maximum to the sound pressure square averaged over the volume:

$$NR(\omega) = 10lg\left(q_{max}^2(\omega) \big/ \left\langle p^2(\omega) \right\rangle \right) \qquad (15)$$

INVESTIGATION RESULTS

For test calculations the parameters of models corresponding to the parameters of light aircraft were chosen. The shell length is $L = 5.4\,\mathrm{m}$; radius is $R = 1.2\,\mathrm{m}$; step of arrangement of frames is $d = 0.45\,\mathrm{m}$. The sound insulation structure consisted of a trim panel (2.5kg/m^2), an air gap (0.03m) and an insulating layer (0.07m). The shell butt-ends were assumed to be acoustically soft. The shell was excited by the field described by the function:

$$q(x, y) = q_{max}\, exp\left(-(x - x_0)^2 / \Lambda_x^2(\omega) - y^2 / \Lambda_y^2(\omega) - i\pi y / \Lambda_{yi}(\omega)\right). \quad (16)$$



a) *n=6, m=2, 396Hz* b) *n=9, m=3, 504Hz* c) *n=6,m=13, 605Hz* *d) n=1* *e) n=6* *f) n=10*

Figure 2 – Examples of the eigen- functions of shell.

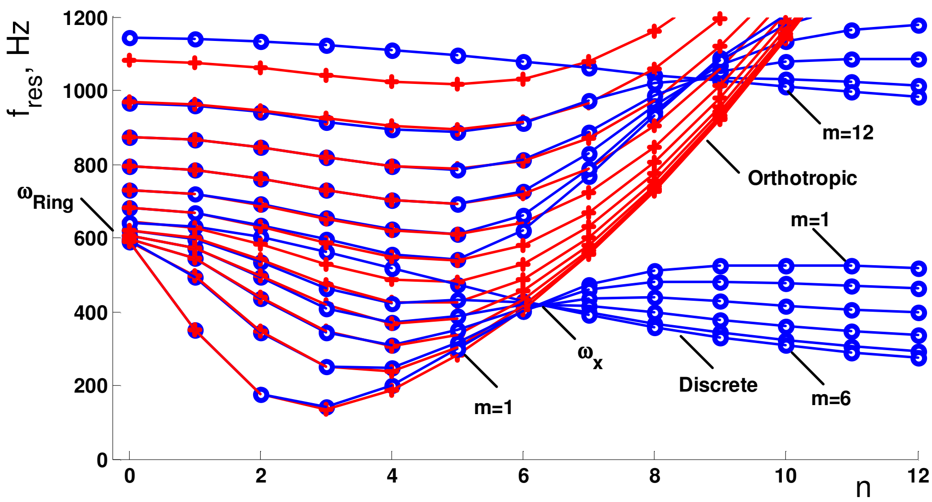


Figure 3 – Eigen-frequencies for a 6-spanned shell.

Figures 2(a,b,c) present examples of the form of vibration of 12-spanned shell (at the bottom) and of the respective form of the running wave in the complex plane (at the top). Note, that it directly follows from the phase step constancy in the running wave that the longitudinal component of the shell eigen-functions, similarly to any other regular system, can be expanded into a series as follows:

$$F(x) = A_0 \sin(k_0 x) + \sum_{p=1}^{\infty} (A_p \sin(k_p x) + A_{-p} \sin(k_{-p} x)),\ k_p = k_0 + 2\pi p / d. \quad (17)$$

Figures 2(d,e,f) present longitudinal shell eigen-functions for the first group at different values of circumferential index *n*. At small values of indices *m* and *n* eigen-functions are slightly different from the sinusoidal ones. As the indices increase, the form of modes becomes more distorted. As the index *n* increases, the frame rigidity increases and their vibrations attenuate, in comparison with the span vibrations.

Figure 3 presents the eigen-frequencies calculated for a 6-spanned shell. For comparison the prediction results obtained with the use of the orthotropic model are given. The frequencies for each index *n* are broken up into groups with *N* modes in each one, thus manifesting stop-band and pass-band behavior. At a certain value of the circumferential index $n = n_{x1}$ the frequencies thicken, similarly to frequency thickening close to the ring frequency at *n*=0,1, when the value of longitudinal index *m* has practically no effect on the eigen-frequency. This frequency thickening is associated with a transfer from the modes with positive dependency of the eigen-frequency on *m* to the modes with motionless frames with negative dependency. A similar interaction at some certain its own *n* is observed for each group of modes.

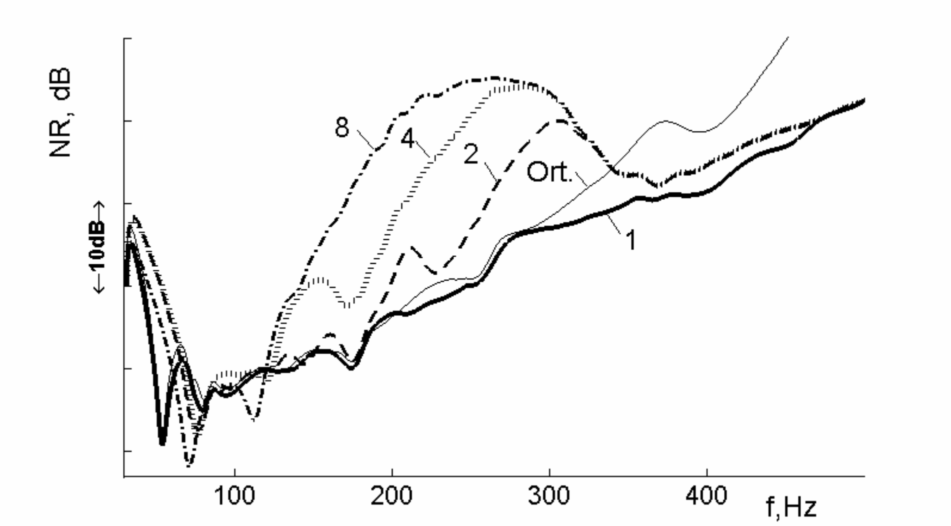
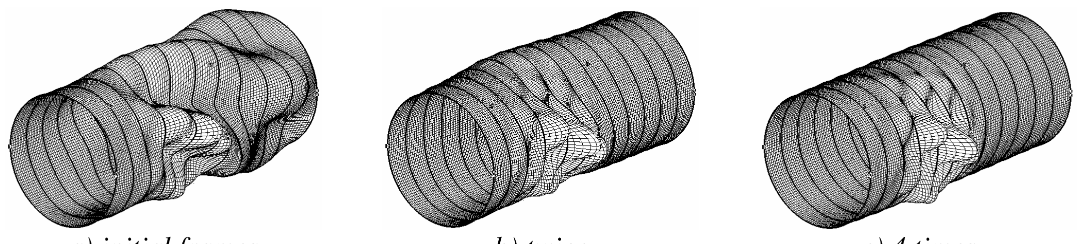


Figure 4 – Dependency of NR on frames rigidity. 1 – initial frames, 2,4,8 – enlarged frames in 2,4,8 times, Ort. – orthotropic model for initial frames .



a) initial frames *b) twice* *c) 4 times*

Figure 5 – Influence of frames rigidity on the shell deformation. f = 250 Hz, x_0 = 2m.
a) *initial frames b) frames with twice profile enlargement c) 4 times.*

Figure 4 presents a NR-variation for the shells with different frames rigidity due to enlargement of their Z-like profile length in 2,4,8 times. It is seen that the NR enlargement is high only in a certain frequency range. The frames rigidity enlargement results in the decrease of the lower margin of this range. Figure 5 presents the shell oscillation deformation for three types of frames with account for the interactions with the insulation layers, the ambience and the medium in the limited volume. The shell is excited by the propeller aero-acoustic field at 250 Hz (the third harmonic) with the intensity maximum at the distance of 2m from the edge.

On the basis of this investigations carried out the boundaries of the structurally orthotropic model application were determined. They can be determined without resorting to a discrete model. The orthotropic model can be applied in the following cases: 1) if the frame response is small i.e. the frequency difference obtained according to the orthotropic model for the shell with frames and without them is small; 2) if the frame response is large, but several spans are accommodated on the longitudinal wave length ($m < N/3$) and the circumferential index value is less than the value, at which the frequency thickening of the first group takes place ($n<n_{x1}$). The index of thickening n_{x1} can be approximately determined as the index value, at which the frequency of a separate freely supported span is equal to the frequency of the whole shell with m=1 obtained according to the orthotropic model.

This model with discrete frames is valid only on condition that the stringers can be "smeared", i.e. if the circumferential index is not large ($n < \pi R / 3d_s$) and if the stringers are compliant enough, that is when the isolated shell cell frequency exceeds the frequencies obtained when they are "smeared"

SUMMARY

The exact analytical solution for predicting the sound field inside the shell is obtained for a model of the closed cylindrical orthotropic shell with discrete rings. It is found that the resonant frequency thickening can be revealed in the case of account for the frame discreteness and they can lead to a considerable sound level increase in the shell. The field of application of the model with discrete frames is indicated. The simple method of orthotropic model correctness limitations is proposed.

ACKNOWLEDGEMENTS

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