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SIMPLE GEAR SET DYNAMIC TRANSMISSION ERROR MEASUREMENTS

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Abstract

The paper deals with methods, which is used for the dynamic transmission error measurements of a simple gear set at gearbox operational conditions that means under load and during rotation. The methods are aimed at the processing of impulse signals generated by incremental rotary encoders attached to the shaft with the gears in mesh. The measurement technique benefits from demodulation of phase-modulated signals using the Hilbert transform. Two signal processing methods for phase demodulation are analyzed. The first one is based on using FFT and the second on using digital filters. The theory is illustrated by experiments with TE measurements on truck gearboxes. To verify experimental results the incremental rotary encoder accuracy is tested.

INTRODUCTION

Noise and vibration problems in gearing are mainly concerned with the smoothness of the drive. The teeth contact stiffness is not a constant value but it is oscillating in synchronism with tooth meshing frequency due to the changing of the number of tooth pairs in contact and moving the teeth contact point along the tooth flank. The oscillation of the tooth contact stiffness causes the self excited angular vibration of the driven gear. A dynamic force, which is acting between meshing gears, results in the time varying force, which is acting at the shaft support, what excites the vibration of the gearbox housing and consequently noise emitting. Solving gearbox noise problem at the very source is based on the gear angular vibration analysis. The parameter that is employed to assess drive smoothness is the Transmission Error (TE) [1]. This parameter can be expressed as a linear displacement at a base circle radius defined by the difference of the output gear's position from where it would be if the gear teeth were perfect and infinitely stiff.

ANGULAR VIBRATION MEASUREMENTS

TE results from the gear angular vibration. There are many possible approaches to measuring TE as well, but, as Derek Smith points out in his book [2], in practice, measurements, which are based on the use of incremental rotary encoders, dominates. The simplest method for calculation of the instantaneous rotational speed is the reciprocal value of the time interval between two consecutive impulses. This method is not suitable if a few samples between adjacent impulses are recorded. If the string of encoder impulses as an analogue signal controls a gate for the high frequency clock signal (100 MHz or 1 GHz), which is an input of an impulse counter, then this method works properly. The method, which is preferred in this paper, is based on using the Hilbert transform (HT) for demodulation of the phase-modulated signals. There are two approaches to evaluate the Hilbert transform of a sampled signal as an approximation satisfying to practical purposes and they are

- ❑ Fast Fourier Transform (FFT)
- ❑ Digital filters.

Figure 1 demonstrates the principle of using FFT while the approach based on fhe digital filters is shown in figure 2.

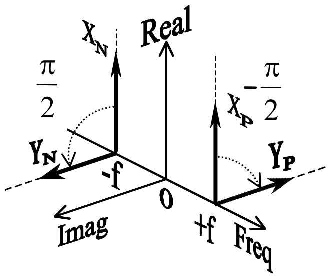

Figure 1 HT using FFT

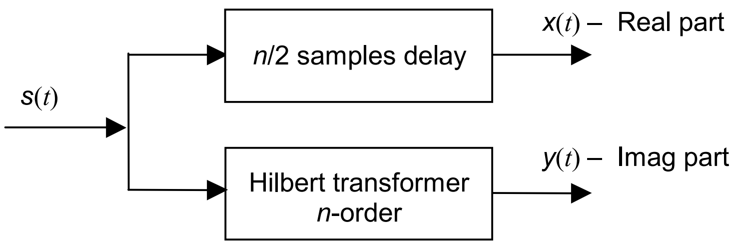

Figure 2 HT using digital filters

Concerning the Fourier transform [3], let the sampled signal be designated by x_n, $n = 0,1,...,N-1$. The relationship between the components X_k and Y_k, where $k = 0,1,...,N-1$, which are corresponding to the Fourier transform of the sampled signal x_k and its Hilbert transform y_k respectively, is given by the phase shift shown in figure 1. The inverse Fourier transform of Y_k, $k = 0,1,...,N-1$ results in the time domain signal y_n, $n = 0,1,...,N-1$, which is the Hilbert transform of the input signal.

The digital filter, which is performing HT, is called the Hilbert transformer. It is a high-pass or band-pass filter of the FIR or IIR filter type. The phase modulated harmonic signal $s(t)$ is an input for the Hilbert transformer to obtain the Hilbert transform $y(t)$ equaling to the imaginary part of the analytic signal. The group delay of the Hilbert transformer, using the FIR filter, is equal to half the digital filter order; therefore the real part of the analytic signal $x(t)$ is delayed by $n/2$ samples.

The real and imaginary parts form an analytic signal $x_n + j\,y_n$. This complex signal is an input for calculation of the signal phase in radians using the *atan* function and resulting in values from the interval of $\left(-\pi/2,\pi/2\right\rangle$. The signs of the real and imaginary parts extend calculation of the phase to the interval of $\left(-\pi,\pi\right\rangle$. The angle as a time function contains jumps. The true phase φ_n of the analytical signal as the time function must be unwrapped. The unwrapping algorithm is based on the fact that the absolute value of the phase difference $\Delta\varphi_n = \varphi_n - \varphi_{n-1}$ between two consecutive phase samples is less than the value of π radians.

TRANSMISSION ERROR MEASUREMENTS

The basic equation for calculation of TE, concerning a simple gear set, is given as

$$TE = \left(\Theta_1 - \frac{n_2}{n_1}\Theta_2\right) r_1 , \qquad (1)$$

where n_1, n_2 are teeth numbers of the pinion and wheel respectively, Θ_1, Θ_2 are the angles of rotation of the mentioned gears and r_1 is the pinion radius.

The simple gear train to be tested consists of the 27 and 44-tooth gears and it is a part of a truck gearbox. The 27-tooth gear is mounted on the gearbox countershaft, where the 39-tooth gear is under load together with the mentioned 27-tooth gear. The 44-tooth gear is mounted on the gearbox output shaft, where the 20-tooth gear is under load together with the 44-tooth gear as well. The sketch of the tested gear set and the configuration of two incremental rotary encoders, designated by E_1 and E_2, are shown in figure 3. Both the encoders generate a string of impulses. As a consequence of Shannon's sampling theorem a few impulses must be recorded during each mesh cycle. It means that the number of impulses produced per encoder complete revolution must be a multiple of the tooth number. If five harmonics of toothmeshing frequency are required to be traced then the number of impulses per gear revolution must be at least ten times greater than the number of the gear teeth. The encoder generating 500 impulses per revolution seems to be an optimum.

As it was mentioned, a perfectly uniform rotation of gear produces an encoder signal having in its frequency spectrum a single component at the frequency that is a product of the gear rotational frequency and the number of the encoder impulses generated per revolution. The phase of this carrying component is a linear function of time. As the phase is the argument of the cosine function it can be associated with the gear rotation angle multiplied by the mentioned impulse number. The non-uniform rotation results in small variation of the phase around the mentioned linear function of time. In this case the basic frequency of the impulse signal is modulated, which gives rise to sidebands around the carrying component in the spectrum.

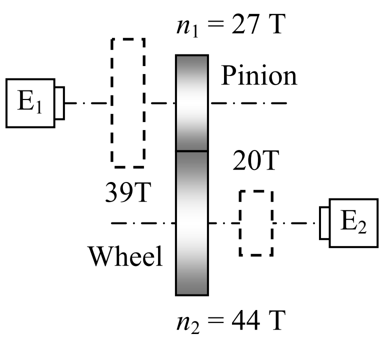

Figure 3 Gears under load and TE measurement arrangement

USING FOURIER TRANSFORM TO CALCULATE HILBERT TRANSFORM

As it was mentioned, the Hilbert Transform can be calculated by using either FFT or the digital filter (Hilbert transformer). This paper section is focused at using the FFT method. Before evaluating FFT it is recommended to resample the measured signal according to the gear rotational frequency in such a way that the length of the resampled time record equals to a power of two, namely to at least 2048 samples per gear revolution for the 500-impulse encoder. The order spectrum of the 2048-sample record ranges to the value of 800 orders. There is a space for ±300 sideband components around the mentioned carrying component with the frequency of the 500 order. Resulting records are presented as a function of dimensionless revolutions rather than seconds and the corresponding FFT spectra are presented with the frequency axis in dimensionless orders rather than in Hz.

The order spectra of the impulse string signal, produced by encoders in figure 3, are shown in figure 4 and 5. The pinion is rotating at 1040 RPM and under load of 1300 Nm. Both these measurements were done by the same order analyzer and not simultaneously. For the first measurement the basic frequency for resampling was equal to the rotational speed of the pinion while resampling of the second measurement was done according to the rotation of the wheel.

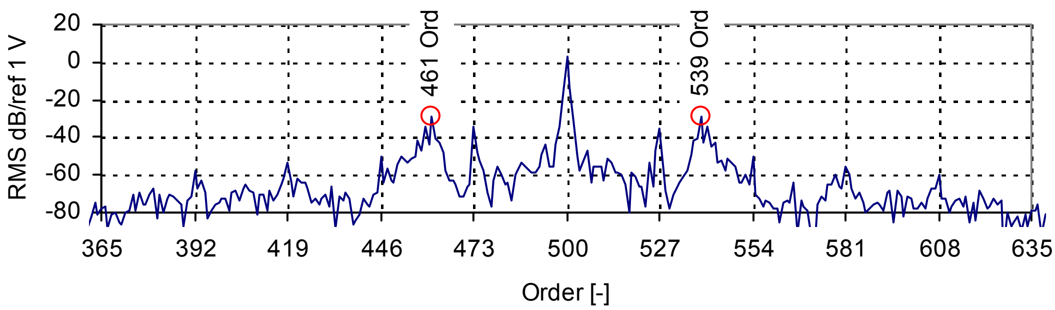

Figure 4 Frequency spectrum of phase modulated signal generated by the E2 encoder

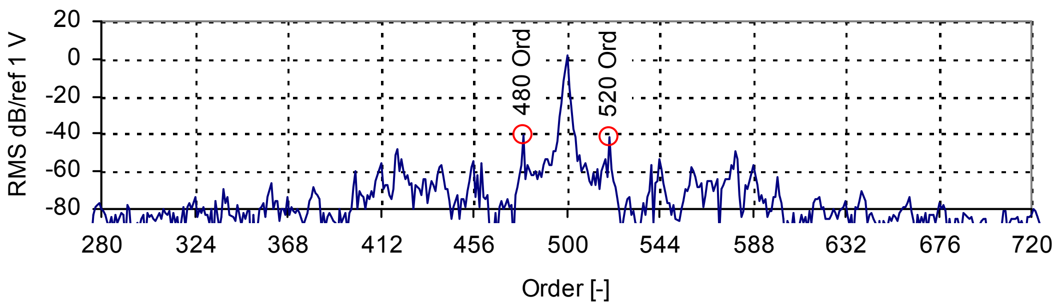

Figure 5 Frequency spectrum of phase modulated signal generated by the E2 encoder

The gear speed variation at the toothmeshing frequency results in the phase modulation of the impulse signal base frequency. As noted above the phase-modulated signal contains sideband components around the carrying component situated in figure 4 at 500 ± 27*k* order for the 27-tooth pinion and in figure 5 at 500 ± 44*k* order for the 44-tooth wheel, where k = 1,2,... The first spectrum contains a response of the 37-tooth gear while the second spectrum contains a response of the 20-tooth gear, both these gears are under load. Take notice of the fact that the dominating components in both the sidebands exceed the background noise level at least by 20 dB or even more. Both the spectra were calculated from time signals that are a result of the synchronized averaging of 100 revolutions of the tested gears.

Records, corresponding to the complete rotation of the pinion and wheel, are inputs for unwrapping the phase using the procedure described in the section dealing with angular vibration measurements. The diagram time axis is in revolutions, which is in fact the dimensionless quantity $0 \le \tau \le 1$ related to the length of the time interval of one complete gear revolution. In other words, this dimensionless quantity τ is a relative nominal rotation angle at the uniform rotational speed. The encoder signal phase variation requires to be normalized because the phase change corresponding to one gear complete revolution is 500 times greater than the angle of rotation. The phase modulating signal is calculated as the difference of the normalized unwrapped phase Θ_1 and the linear term as the nominal rotation

$$\Delta\Theta_1 = \Theta_1 - 2\pi\,\tau \ , \qquad (2)$$

where τ is the normalized time corresponding to the pinion complete revolution.

The signals, which are shown in figure 6 and 7, are synthesized from five tooth meshing frequency components including their several sidebands (3 for the 27-tooth gear and 4 for the 44-tooth gear).

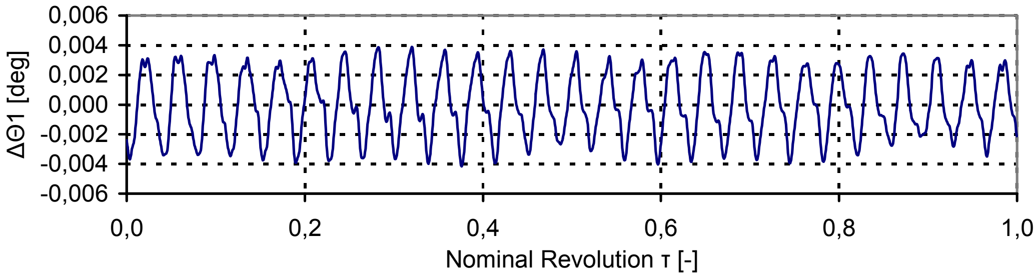

Figure 6 Angular vibration of the 27-tooth gear during the pinion complete revolution

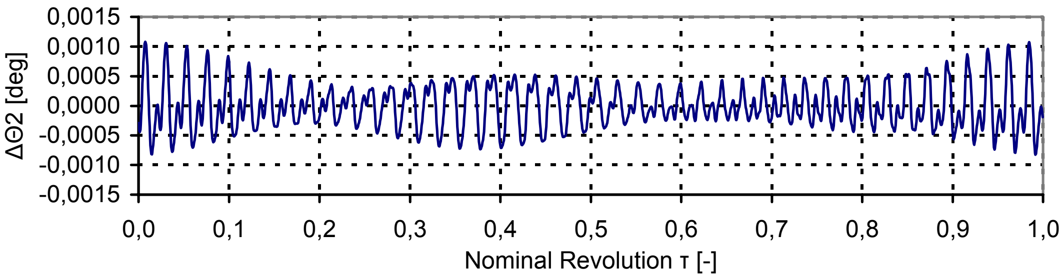

Figure 7 Angular vibration of the 27-tooth gear during the wheel complete revolution

As both the encoder signals are recorded separately the true phase delay between these signals in relation to the gear rotation angle is to be detected. This problem is solved thanks to the fact that the average toothmesh responses, for example in acceleration of some point on the gear case, to dynamic forces acting between meshing teeth are theoretically the same for both the measurements. In this way synthesized signals are called the average toothmesh signal [4]. Therefore, both the encoder impulse signals are sampled together with the mentioned acceleration signal. The two-stage averaging of the twice-measured acceleration signal gives average toothmesh responses that are delayed against each other. The lag for the maximum correlation gives the value of relative delay.

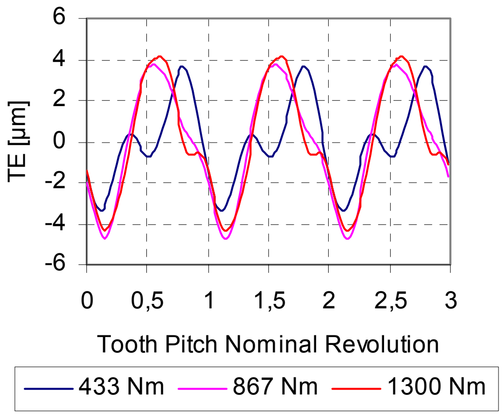

Figure 8 Transmission error against tooth pitch nominal rotation

Angle variations can be easily transformed into arc length variations and then the difference between these signals gives TE. The result for 3 times repeating tooth pitch rotations and three levels of gear loading is shown in figure 8. As it is evident the peak value of TE is not depending strongly on load. The tooth surface of the gears under test is modified properly to reduce emitted noise at operational conditions.

The method based on using FFT requires signal resampling as a part of the order analysis. The signal analyzer (LabShop Pulse) with the 25,6 kHz frequency range (65536 Hz sampling frequency) limits the RPM range of measurements approximately to 1900 RPM when using the above described method.

USING FIR FILER TO CALCULATE HILBERT TRANSFORM

To demonstrate the phase demodulation using the FIR filter, the frequency spectrum of an encoder signal segment, which are sampled simultaneously at the frequency of 65536 Hz during the time interval of a second, is shown in figure 9. The gear set operational conditions are the same as at the measurement presented above.

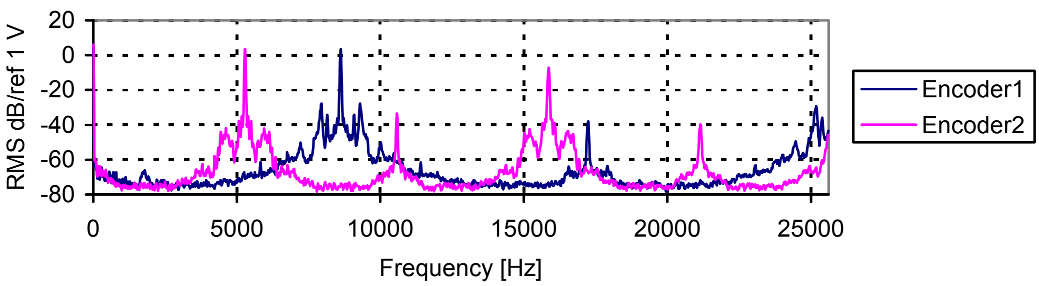

Figure 9 Encoder signal spectra

The impulse signal is composed of several harmonics of the basic frequency due to the rectangular shape of the impulses. The first step is to filter out the higher harmonics using a pass-band filter. It is possible to employ the digital pass-band filter or filtration in the frequency domain using FFT and then using the inverse FFT. The filtered signals are processed by using the procedure described in the section dealing with angular vibration measurements. The result is the unwrapped phase time history. Both the phases are translated into the arc length using the pitch circle radius of the appropriate gear (r_1, r_2). The arc length difference as the transmission error (TE) is given by the formula

$$TE = r_1\Theta_1 - r_2\Theta_2 = \left(\Theta_1 - \frac{n_2}{n_1}\Theta_2\right)r_1 = \Delta\Theta\, r_1 \ , \qquad (3)$$

The frequency spectrum of the phase difference $\Delta\Theta$ contains the low frequency components as a result of the encoder errors or non-uniform drive. To remove it the comb filter with the pass-bands centered at the first 5 harmonics of the toothmeshing frequency is employed. The TE time history is resampled in such a way that each data segment, corresponding to the complete revolution of the 27-tooth gear, contains the same number of samples. The result of the record resampling is shown in figure 10. All 10 records of the TE time history, shown in figure 10, are synchronously averaged according to the gear rotations. The averaged TE time history, related to the complete rotation of both the gears, is shown in figures 11 and 12.

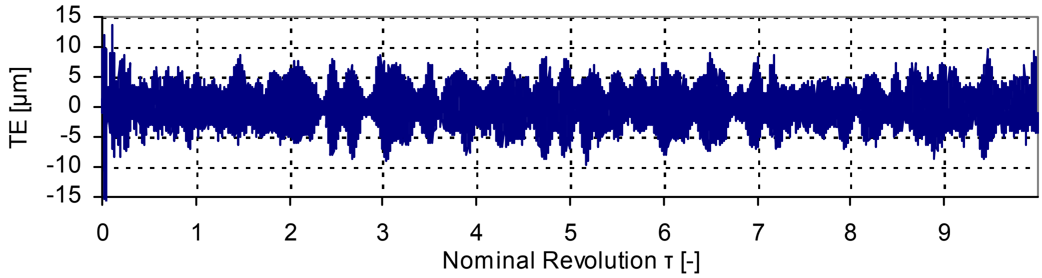

Figure 10 Phase difference versus the nominal revolution number of the 27-tooth gear

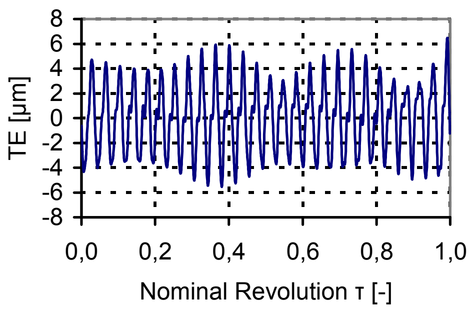

Figure 11 TE versus the nominal revolution number of the 27-tooth gear

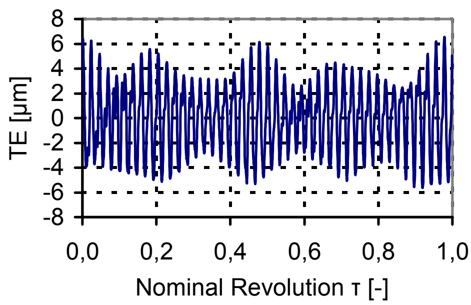

Figure 12 TE versus the nominal revolution number of the 44-tooth gear

The method based on using the digital filter as the Hilbert transformer can be employed to analyze the TE time history. The RPM range is twice larger comparing to the cases when using FFT. The records are without the phase shift, which means that only two encoder impulse signals are required for simultaneous measurements.

INCREMENTAL ROTARY ENCODER ACCURACY

The measurement system was instrumented by the Heidehain incremental rotary encoders of the ERN 460-500 type. To evaluate errors in distribution of impulses against the angle of the encoder rotation, both the encoders were mounted on a shaft what ensured that they are rotating at the same rotational speed. Using the analytical method described above, the difference between modulation signals gives the error in distribution of impulses. The error level corresponding to the tooth pitch rotation determines the final accuracy of the TE measurements, which is less than the value of the 0.0001 angular degrees.

SUMMARY

This paper is focused at the problem of the simple gear set transmission error (TE) measurement. Variation of TE is the cause of angular vibration of both the mating gears and consequently gearcase vibration and noise. This paper deals with the measurement method that is based on the use of encoders generating a string of 500 impulses per a gear revolution. The impulse signal is processed by the Hilbert transform using FFT or the digital filter as the Hilbert transformer. The advantages and disadvantages of both these methods are discussed. Employing FFT for calculation of the Hilbert transform gives the TE time history only for a tooth pitch rotation while employing the digital filter results in the TE time history of several gear complete rotations and simultaneously the RPM range is twice larger than for the previously mentioned method. The theory is illustrated by experimental data.

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