ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

MODAL ANALYSIS OF A WOODEN TEST BED DESIGNED TO SOUND SOURCE IDENTIFICATION BY INVERSE FEM

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Abstract

The detection of sound sources in aircraft interiors is a time consuming procedure. At present intensity measurements conducted over the complete surface, to detect sound sources where sound is transmitted into the cabin. Since this procedure is highly ineffective the Inverse Finite Element Method has been applied to this problem, because this approach is capable of detecting acoustic hot-spots at the surface by using measurement data of the interior as boundary conditions for the inverse analysis. It is obvious that the verification of this method requires experimental data of a well-defined enclosure. Therefore, a wooden test bed has been designed and its modal response has been analyzed.

INTRODUCTION

Nowadays, several noise source localization techniques have been established to detect acoustic hot spots. The selection of the right method depends on the information required, as well as on the application. The methods range from simple sound pressure measurements to more advanced methods as Acoustic Holography [5], [7], [8], [11], Beamforming [1], [6], and Inverse Boundary Element Method (IBEM), see [10]. However, many localization techniques have the drawback, that they are only applicable in free-field conditions. For this reason, measurements in interiors are often conducted by creating artificial free-field conditions.

In order to avoid this problem – caused by standing waves – a novel approach has been introduced in order to detect acoustic hot spots using the Inverse Finite Element Method (IFEM), see [2] – [4], [9]. This method has the advantage, that it can be applied to identify sound sources in interiors without creating artificial free-field conditions. It is therefore well suited for interior noise problems.

The approach – presented in the paper – is based on a FE-Model of the interior sound field in the frequency domain. This model has been applied to predict the sound pressure distribution in the interior by a standard FE-Simulation. Data taken from a certain subspace of the interior – called measurement area – have been used afterwards as boundary conditions for the IFEM that have been applied to recalculate the original sound pressure distribution at the interior boundary.

In order to verify this numerical procedure, an experimental setup consisting of a barrel with nine loudspeakers mounted at the circumference has been build up. This setup is capable of creating various sound fields within the barrel by activating different sets of speakers. A vibro-acoustical modal analysis has been performed to verify the system dynamic, predicted by FE-Simulations. In contrast to previous publications this has been the first step to apply the IFEM to lifelike interiors.

Both, the IFEM approach and the results of the experimental modal analysis will be discussed in the following sections.

SOUND SOURCE IDENTIFICATION BY INVERSE FEM

The inverse finite element formulation

The FE-Method for the time-harmonic analysis of nearly undamped interior noise problems is based on the Helmholtz equation

$$\Delta p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0; \qquad [k] = [m^{-1}] \tag{1}$$

where Δ represents the Laplace-Operator and $k = 2\pi f / c$ the wave number that is determined by the excitation frequency f and the speed of sound c. The corresponding boundary conditions (BC) are given by the Dirichlet BC for the acoustic pressure

$$p = \bar{p} \qquad on \qquad R_p , \tag{2}$$

and the Neumann BC for the normal component of particle velocity

$$-\mathbf{n} \cdot \frac{1}{\rho} \nabla p = i(2\pi f)\bar{v}_n \qquad on \qquad R_v . \tag{3}$$

As stated in [5], discretization of (1) using the FE-Method leads to a set of algebraic equations for the sound pressure that can be summarized as:

$$\mathbf{Kp} = \mathbf{v} . \tag{4}$$

$\mathbf{K}$ is the stiffness matrix, $\mathbf{p}$ the vector of the excess pressure and $\mathbf{v}$ a vector that is proportional to the particle velocity in the sound field, and therefore called generalized velocity vector. The solution of (4) with respect to the BC leads the unknown pressure field $\mathbf{p}$. This process is called forward calculation (FC).

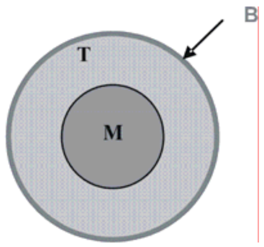


Figure 1 – Spatial domain-decomposition

In order to derive the IFEM the calculation domain is split into three regions, an inner measurement sub-domain (M), a transition sub-domain (T), and an outer boundary (B) as illustrated in Fig 1. In the absence of unknown volume-sources in the transition sub-domain it is possible to decompose Eqn. (4) as follows

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{K}_{23} \\ \mathbf{K}_{31} & \mathbf{K}_{32} & \mathbf{K}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{MK} \\ \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{MK} \\ \mathbf{v}_{TK} \\ \mathbf{v}_{BU} \end{bmatrix}. \tag{5}$$

The first index of the three sound pressure sub-matrices $\mathbf{p}_{ij}$ and the three sub-matrices $\mathbf{v}_{ij}$ of the generalized velocity vector $\mathbf{v}$ denotes the sub-domain of the decomposed calculation domain, whereas the second index denotes whether the variable is known (K) or unknown (U).

As described in [9] the unknown parts of the sound pressure vector $\mathbf{p}$ can be computed by the solution of a reduced problem that is given by

$$\begin{bmatrix} \mathbf{K}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{22} & \mathbf{K}_{23} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{11}\mathbf{p}_{MK} \\ -\mathbf{K}_{21}\mathbf{p}_{MK} \end{bmatrix}. \tag{6}$$

The solution of

$$\begin{bmatrix} \mathbf{K}_{32} & \mathbf{K}_{33} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \\ \mathbf{v}_{BU} \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{31}\mathbf{p}_{MK} \end{bmatrix} \tag{7}$$

leads to the unknown velocities at the outer boundary. This procedure is called inverse calculation (IC).

Remarks on the solution of the reduced problem

Eqn. (6) can be solved to determine the complete sound field in the interior using only the measurements in the inner region of the sound field. For simplicity it is rewritten as follows

$$\mathbf{A}\mathbf{x} = \mathbf{b}; \qquad \mathbf{A} \equiv \begin{bmatrix} \mathbf{K}_{12} & \mathbf{K}_{13} \\ \mathbf{K}_{22} & \mathbf{K}_{23} \end{bmatrix}, \ \mathbf{x} \equiv \begin{bmatrix} \mathbf{p}_{TU} \\ \mathbf{p}_{BU} \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} -\mathbf{K}_{11}\mathbf{p}_{MK} \\ -\mathbf{K}_{21}\mathbf{p}_{MK} \end{bmatrix}. \qquad (8)$$

As stated in [2] it has been found that the condition number of the new portioned stiffness matrix $\mathbf{A}$ is very high, especially if realistic measurement errors are taken into account. Therefore, several regularization techniques, e.g. Truncated Singular Value Decomposition (TSVD) or Tikhonov Regularization (TR) have been applied to solve the ill-posed problem that is given by (8).

More details about the regularization algorithms that have been used to solve (8) can be found in previous publications. The implementation of the TSDV and TR is described in [2]-[3], and [9] whereas the effect of smoothing to the IFEM is explained in [4].

VIBRO-ACOUSTICAL MODAL ANALYSIS

Experimental setup

A wooden test bed, see [9], has been used to simulate a cylindrical enclosure with sound hard boundaries. Only one loudspeaker (LS 1) has been used to excite the system with frequency-banded white noise. As shown in Fig. 1, twelve microphones (six on each rope) have been used to measure the sound pressure.

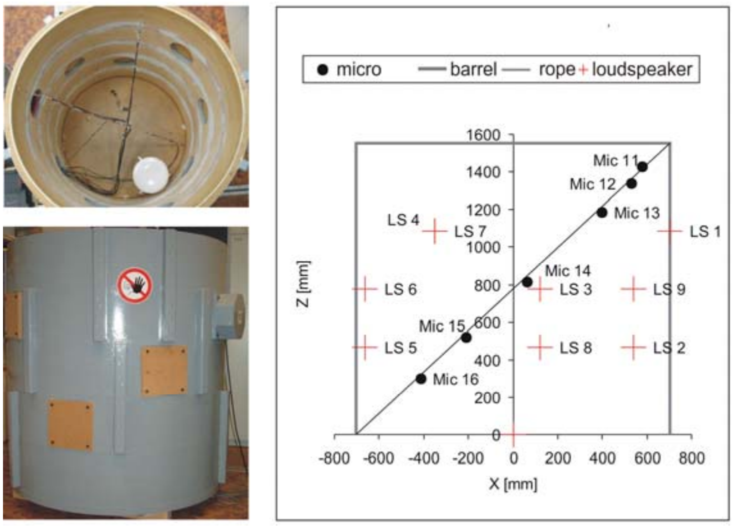
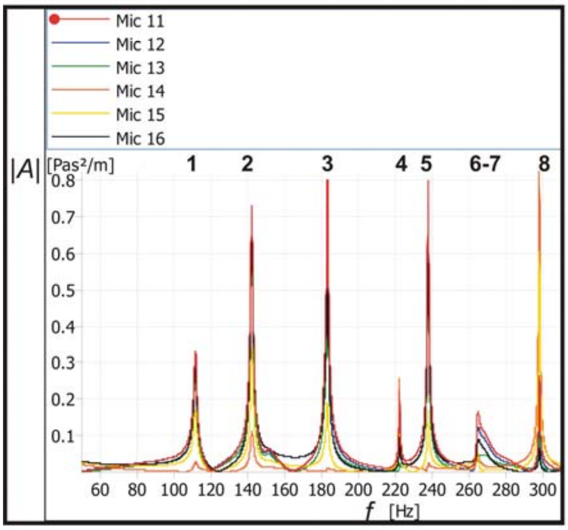
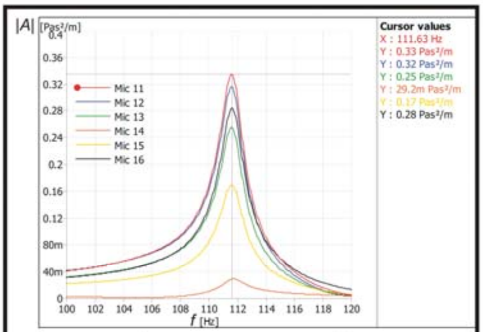


Figure 2 – Experimental setup and coordinate system for experimental modal analysis

Modal response of the test bed

Fig 3 presents the experimental results. Due to the small amount of damping the system response peaks at the resonant frequencies listed in Fig. 3. The spatial pressure-distribution is illustrated for the first mode (Fig. 3, right). As one expects the maximum values have been measured at the positions of microphone 11 and 16 whereas the minimum value has been found at the position of microphone 14. Hence, the characteristics of the first axial mode have been clearly identified.





Mode number	f_R [Hz]	a [m/s²]
1	111,69	0,2
2	142,94	0,27
3	183,19	0,36
4	222,25	0,42
5	237,81	0,2
6-7	-	-
8	298,06	0,26

Figure 3 – Frequency response, resonant frequencies, and modal response at 111,69 Hz

Numerical pre-examinations

In order to verify the numerical model that will be later on used for IC the experimental data has been compared with results of time-harmonic numerical simulations. The acoustic application mode of the commercial FE-program COMSOL® has been used for this purpose.

The acceleration that has been measured at the loudspeaker-surface for each resonant frequency has been applied to the numerical model. Therefore, the loudspeaker has been modelled using the Neumann BC, see Eqn. (3).

Fig. 4 presents the results of the numerical investigations. For each resonant frequency the modal response is shown for the xz-plane and the yz-plane. Fig 4 (A) clarifies that the first axial mode appears at 111,69 *Hz*.

The comparison between the calculated and measured data at the microphone positions is shown in Fig 5. It can be seen that the experimental setup is capable of verifying the numerical results for the 1st, 4th, 5th, and 8th mode. As illustrated by Fig. 4 (B)-(C) it has been necessary to change the sensor positions in order to verify the modal response at 142,94 *Hz* and 183,19 *Hz*.

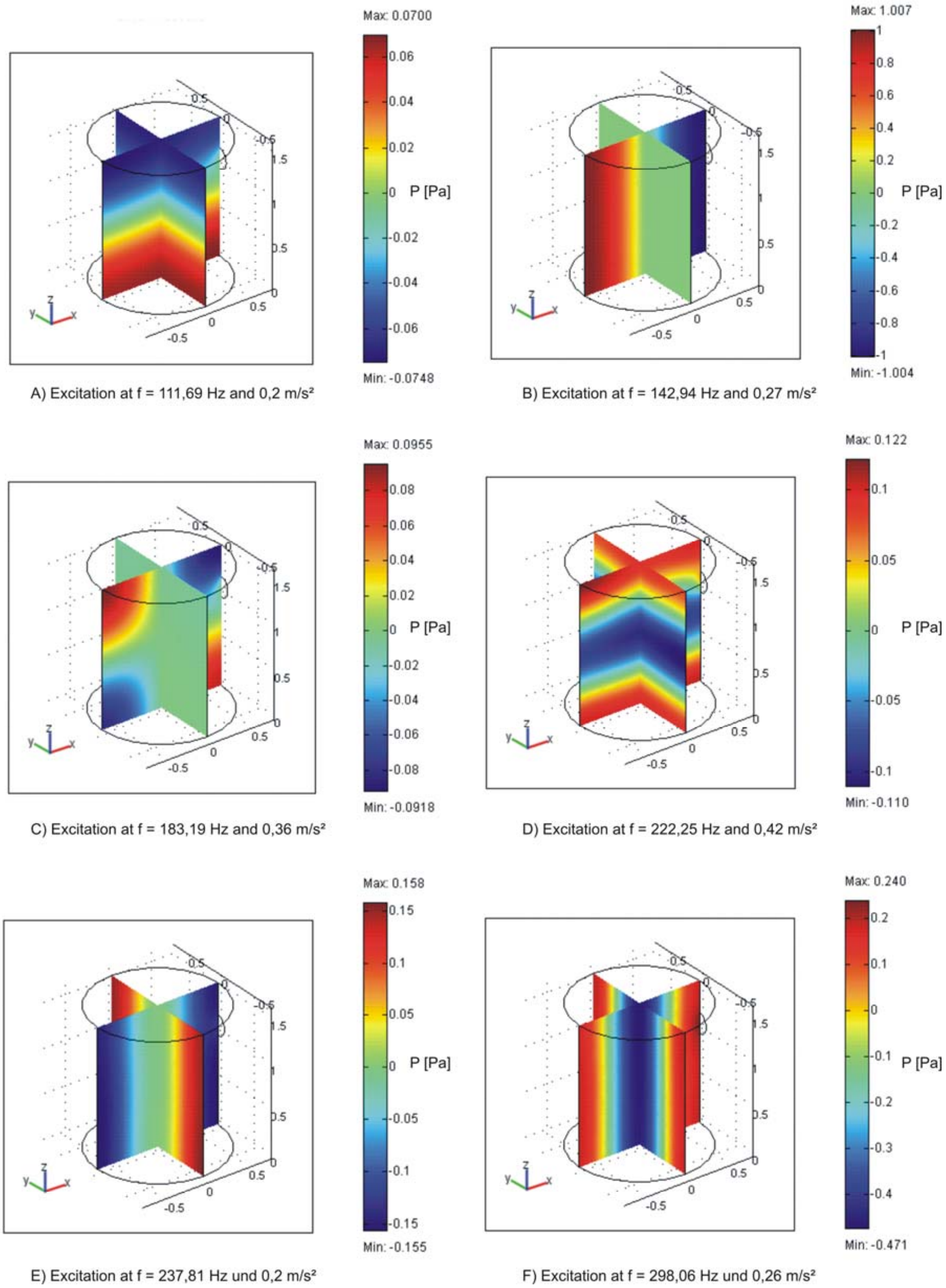
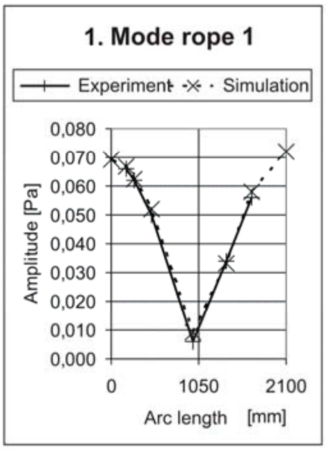
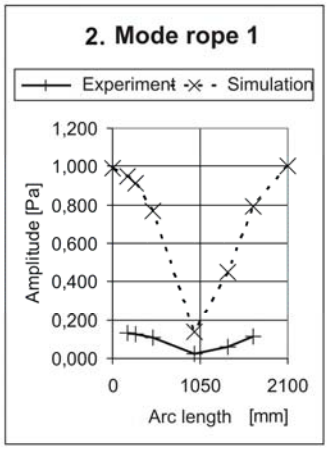


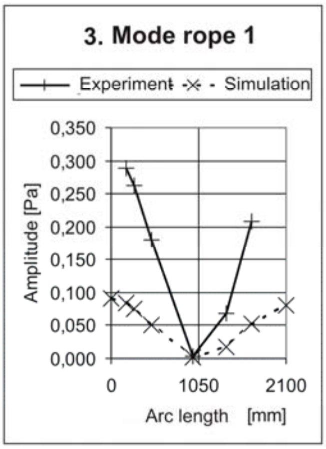
Figure 4 – Spatial distribution of calculated mode shapes



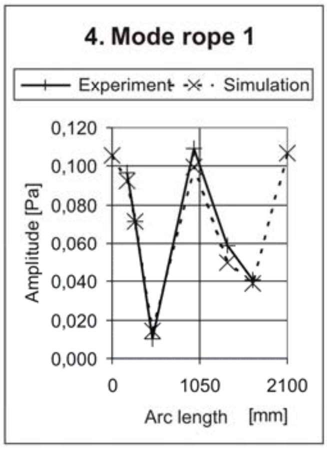
A) Excitation at f = 111,69 Hz and 0,2 m/s²



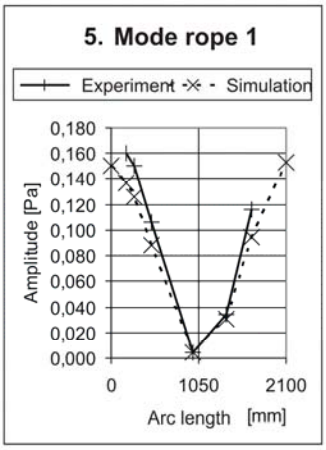
B) Excitation at f = 142,94 Hz and 0,27 m/s²



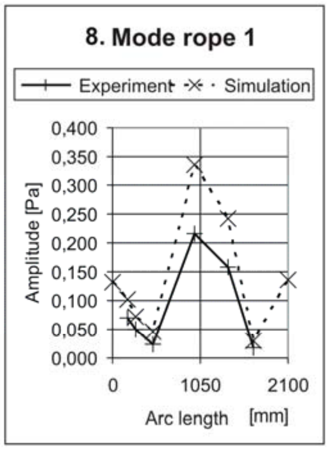
C) Excitation at f = 183,19 Hz and 0,36 m/s²



D) Excitation at f = 222,25 Hz and 0,42 m/s²



E) Excitation at f = 237,81 Hz und 0,2 m/s²



F) Excitation at f = 298,06 Hz und 0,26 m/s²

Figure 5 – Calculated and measured mode shapes

SUMMARY

In order to verify the IFEM for acoustic hot-spot identification, an experimental modal analysis has been performed on a wooden test bed consisting of a barrel with nine loudspeakers mounted at the circumference. It has been found that the numerical results (FC) are in good agreement with the experimental data.

ACKNOWLEDGMENT

The authors thank Mr. Kai Simanowski (project engineer at the professorship for Mechatronics) for his support during the experimental investigations. We also thank Mr. Joachim Drenckhan (former scientific assistant at the professorship for Mechatronics) for his support during the preparation of this paper by providing numerical results of finite element calculations.

Furthermore, the authors gratefully acknowledge the support of the Airbus Germany GmbH and the city of Hamburg, Germany, which funded this work in the framework of the aeronautical research program LUFO HH.

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