ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

MODELING CONSIDERATIONS FOR THE DYNAMIC BEHAVIOR OF A RIGID ROTOR SUPPORTED BY HYDRODYNAMIC JOURNAL BEARINGS

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Abstract

Rotating machines are very common in industry. To understand their behaviour is therefore, very important. When considering numerical analysis of such systems, their modelling needs to consider several important effects which are normally disregarded, such as cross-coupled damping and stiffness coefficients associated with the hydrodynamic interactions between the shaft and the supporting bearings. This paper deals with an analysis of the dynamic behaviour of rotating shafts supported on hydrodynamic journal bearings. The model employed to represent the rotor-bearing system is based on the Stodola-Green rotating shaft model, in which the inertia effects associated with the gyroscopic moment and rotary inertia are taken into account, and on the classical Reynolds equation for the hydrodynamic journal bearings. A finite element procedure specially devised to solve the zeroth- and first-order lubrication equations generated from a perturbation technique applied on the Reynolds equation is used to predict the dynamic force coefficients of the hydrodynamic journal bearings. Lagrange equations are used for obtaining the four equations of motion for the rotor lateral vibration. By using the classical method of Runge-Kutta is possible to obtain the frequency response of the system due to the unbalance distribution along the shaft. The influence of the bearing damping and stiffness cross-coupling coefficients on the rotor unbalance response is analysed. Numerical results show that the inclusion of the bearing force coefficients on the dynamic analysis of rotating machines supported on hydrodynamic bearings plays an important role on the determination of the unbalance response of rotors.

INTRODUCTION

Turbo machines are widely employed in several industrial processes. The most common cause of vibration in turbo machines is the rotor mass unbalance. The unbalance centrifugal forces are transmitted to the machine support system and foundation. Such forces may damage the system and, in some cases, even affect others equipments in the vicinity. The use of computational procedures to analyze the dynamic behavior of turbo machines has provided significant data for the preliminary stages of the machine design. The rotating system modeling usually is based on simplified models for the support system, which commonly do not account for the hydrodynamic bearing dynamic force coefficients. Even though the bearing coefficients play an important role on the rotor response, the technical literature lacks analyses of rotating machines that include the cross-coupled stiffness and damping coefficients associated with the hydrodynamic journal bearings.

Regarding hydrodynamic bearings, Reynolds equation describes the hydrodynamic lubrication and defines the bearing pressure field as a function of motion (displacement and velocity) in the bearing [1, 2]. Glienicke et al [3] determined the dynamic coefficients considering four different bearing types under controlled conditions. Hashimoto et al [4] calculated the oil film forces for short bearings using analytical formulation. Stiffness and damping hydrodynamic bearings coefficients can be determined using numerical formulation via finite differences and finite elements from the zero and first order Reynolds equation through perturbation analysis of the system [5, 6, 7].

This paper deals with an analysis of a rigid rotor supported by hydrodynamic journal bearings. The Stodola-Green shaft model is employed to represent the rigid rotor, taking into account the gyroscopic moments and the rotary inertia. The hydrodynamic bearing model uses eight linearized damping and stiffness coefficients computed using a finite element procedure specially devised to solve the zeroth- and first-order lubrication equations obtained from the classical Reynolds equation. The equations of motion are obtained using Lagrange formulation considering four degrees of freedom. The responses of the system are obtained via numerical integration from a random unbalance distribution along the shaft. Some simulated examples are presented in order to clarify the importance of considering a full model in mathematical modeling process of rotating systems, besides showing the influence of the cross-coupled bearing coefficients. The paper is organized as follows: section 2 shows the mathematics behind the model used. Then, the determination of the rotor response in the frequency domain using the Runge-Kutta integration method is presented in section 3. The results are shown in section 4, together with the necessary comments. Finally, section 5 shows the conclusions drawn.

MODEL

The equations of motion for the lateral vibration of a rigid rotor can be obtained from the Lagrangian calculated in terms of the Euler angles [1], as presented in Figure 1. Initially the body-fixed plane *xyz* and the space-fixed plane *XYZ* are coincident. The order of the rotations for the Lagrangian is the following: a) α rotation about y; b) β rotation about x; c) ψ rotation about z.

The Lagrangian is composed only by the translational and rotational kinetic energy for the rotor bearing system as given by equation (1):

$$L = T = \frac{1}{2}M(\dot{X}^2 + \dot{Y}^2) + \frac{1}{2}I_t(\dot{\alpha}^2 + \dot{\beta}^2) + \frac{1}{2}I_p(\omega^2 - 2\omega\dot{\alpha}\beta) \tag{1}$$

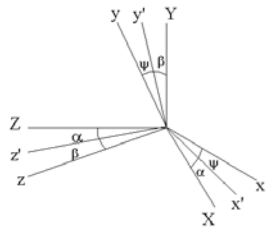

Figure 1 – The Euler angles

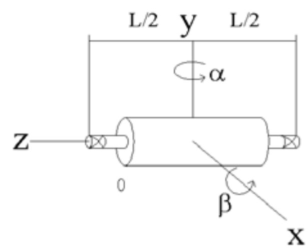

Figure 2 – Rigid rotor

In this work, the rotor bearing system model uses the Stodola-Green shaft model in which a rigid rotor is supported by hydrodynamic journal bearings localized on Z = ± L/2, as shown in Figure 2.

The equations of motion (eq. 2) for the rotor bearing system are obtained using Lagrange formulation [2], with X, Y, α e β as the generalized coordinates of the system and F and M as the generalized forces and moments, respectively. The terms involving the polar moment of inertia (I_p) are known as gyroscopic moments.

$$\begin{aligned} M\ddot{X} &= \sum F_X \\ M\ddot{Y} &= \sum F_Y \\ I_t\ddot{\beta} + I_p\omega\dot{\alpha} &= \sum M_X \\ I_t\ddot{\alpha} - I_p\omega\dot{\beta} &= \sum M_Y \end{aligned} \tag{2}$$

The generalized forces and moments are obtained by the dynamic coefficients of damping and stiffness of the journal bearings and by the rotor unbalancing.

Equations (3) and (4) represent the zeroth and first order Reynolds equations for an incompressible, isothermal and isoviscous lubricant. From these equations, damping and stiffness dynamic formulation can be obtained considering a perturbation analysis [5]. In this work the bearings dynamic coefficients are calculated using numerical formulation via finite element procedure [7].

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\frac{\rho h_0^{\ 3}}{12\mu}\frac{\partial p_0}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h_0^{\ 3}}{12\mu}\frac{\partial p_0}{\partial z}\right) = \frac{1}{2}\frac{u}{R}\frac{\partial(\rho h_0)}{\partial\theta} \tag{3}$$

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\frac{3\rho {h_0}^2 h_\sigma}{12\mu}\frac{\partial p_0}{\partial\theta}+\frac{\rho {h_0}^3}{12\mu}\frac{\partial p_\sigma}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{3\rho {h_0}^2 h_\sigma}{12\mu}\frac{\partial p_0}{\partial z}+\frac{\rho {h_0}^3}{12\mu}\frac{\partial p_\sigma}{\partial z}\right)= \\ \frac{1}{2}\frac{u}{R}\frac{\partial(\rho h_\sigma)}{\partial\theta}+i\omega\rho h_\sigma \quad (4)$$

The damping and stiffness forces are calculated considering, respectively, the velocity and displacement in the X and Y direction. The cross-coupling stiffness (damping) coefficients are obtained assuming that a displacement (velocity) in the X direction produces a force in the Y direction, and vice-versa. It is known [2] that the cross-coupling stiffness coefficients have opposite sign, whereas the cross-coupling damping coefficients have the same sign. However, it is important to point out here that the cross-coupled coefficients are normally ignored in most models available for rotor-bearing systems, although they have a strong influence on the rotor response. So, equation (5) represents the generalized forces and moments in terms of the damping and stiffness coefficients, including the cross-coupling terms.

$$\begin{aligned}
\sum F_X &= -2C_{XX}\dot{X}-2C_{XY}\dot{Y}-2K_{XX}X-2K_{XY}Y \\
\sum F_Y &= -2C_{YY}\dot{Y}-2C_{YX}\dot{X}-2K_{YY}Y+2K_{YX}X \\
\sum M_X &= -C_{YY}\frac{L^2}{2}\dot{\beta}-C_{YX}\frac{L^2}{2}\dot{\alpha}-K_{YY}\frac{L^2}{2}\beta+K_{YX}\frac{L^2}{2}\alpha \\
\sum M_Y &= -C_{XX}\frac{L^2}{2}\dot{\alpha}-C_{XY}\frac{L^2}{2}\dot{\beta}-K_{XX}\frac{L^2}{2}\alpha-K_{XY}\frac{L^2}{2}\beta
\end{aligned} \quad (5)$$

Therefore, the equations of motion for the system (eq. 6) can be obtained substituting equation (5) into (2) and including the generalized forces and moments caused by the rotor unbalance. The latter is represented by the RHS of equation (6).

The cross-coupled stiffness terms usually reduce the effective system damping, what can lead to large rotor vibration amplitudes [1, 2]. That stresses the importance of including such terms during the modeling stage, since their exclusion will underestimate the unbalance responses of the system, as will be shown in the Results.

$$\begin{aligned}
&M\ddot{X}+2C_{XX}\dot{X}+2C_{XY}\dot{Y}+2K_{XX}X+2K_{XY}Y=\sum_{i=1}^{m} m_i\omega^2 u_i\cos(\omega t+\psi_i) \\
&M\ddot{Y}+2C_{YY}\dot{Y}+2C_{YX}\dot{X}+2K_{YY}X-2K_{YX}X=\sum_{i=1}^{m} m_i\omega^2 u_i\mathrm{sen}(\omega t+\psi_i) \\
&I_t\ddot{\beta}+I_p\omega\dot{\alpha}+C_{YY}\frac{L^2}{2}\dot{\beta}+C_{YX}\frac{L^2}{2}\dot{\alpha}+K_{YY}\frac{L^2}{2}\beta-K_{YX}\frac{L^2}{2}\alpha=\sum_{i=1}^{m}(m_i\omega^2 u_i\cos(\omega t+\psi_i))l_i \\
&I_t\ddot{\alpha}-I_p\omega\dot{\beta}+C_{XX}\frac{L^2}{2}\dot{\alpha}+C_{XY}\frac{L^2}{2}\dot{\beta}+K_{XX}\frac{L^2}{2}\alpha+K_{XY}\frac{L^2}{2}\beta=\sum_{i=1}^{m}(m_i\omega^2 u_i\mathrm{sen}(\omega t+\psi_i))l_i
\end{aligned} \quad (6)$$

INTEGRATION OF THE EQUATIONS OF MOTION

The fourth order *Runge-Kutta* integration method, as presented by equation (7), can be used to obtain the time responses of the rotor bearing system from equation (6), in terms of coordinates X, Y, α e β.

$$y_{n+1} = y_n + \frac{1}{6}\left(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}\right) \tag{7}$$

where:

$$\begin{aligned} k_{n1} &= f(t_n, y_n)\Delta t \\ k_{n2} &= f(t_n + \frac{1}{2}\Delta t, y_n + \frac{1}{2}k_{n1})\Delta t \\ k_{n3} &= f(t_n + \frac{1}{2}\Delta t, y_n + \frac{1}{2}k_{n2})\Delta t \\ k_{n4} &= f(t_n + \Delta t, y_n + k_{n3})\Delta t \end{aligned} \tag{8}$$

The smaller the time increment (Δt) considered for integration, the better the resolution obtained for the time responses. From that, a Fast Fourier Transform (*FFT*) is used to give the responses of the system in the frequency domain.

For the present study, the responses of the system are obtained for the center of the rotor and for the two journal bearings of the system.

RESULTS

In order to illustrate the formulations presented, the rotor bearing system was simulated using theoretical data. The rotor parameters employed in the modeling process are described in table 1. The length and diameter are such that the rotor can be considered a rigid structure and the material employed simulates a common one found in the industry.

Table 1 – Rotor Parameters

Rotor Parameters	
length (m)	0,6
diameter (m)	0,05
specific mass (kg/m^3)	7800

The rotor is supported by hydrodynamic journal bearings. The damping and stiffness dynamic coefficients of the bearings employed in the system simulation are presented in table 2. They were found solving the zeroth and first order Reynolds equations via finite element analysis, as described by equations (2) and (3).

Table 2 – Dynamic Coefficients

Journal Bearings Dynamic Coefficients			
Stiffness Coefficients (N/m)			
K_{XX}	K_{XY}	K_{YY}	K_{YX}
0,2310E+06	0,2449E+05	0,2020E+06	-0,2182E+05
Damping Coefficients (N.s/m)			
C_{XX}	C_{XY}	C_{YY}	C_{YX}
1,718E+03	1,672E+03	1,713E+03	1,672E+03

From a random unbalancing distribution along the shaft, the response of the system was obtained using the parameters given in table 1 and the dynamic coefficients given in table 2, considering a balancing speed (ω) of 400 rpm. The unbalancing responses are obtained at the rotor center and at the bearings. Figure 3 shows the synchronous whirl and frequency spectra for the center of the rotor. The same procedure is done for the bearings position and similar results are obtained. The unbalancing response represented in Figure 3 was obtained considered an eccentricity in both directions equal to 0.7 (i.e., $e_x = e_y = 0.7$) making the system operate in a pre-defined synchronous whirl.

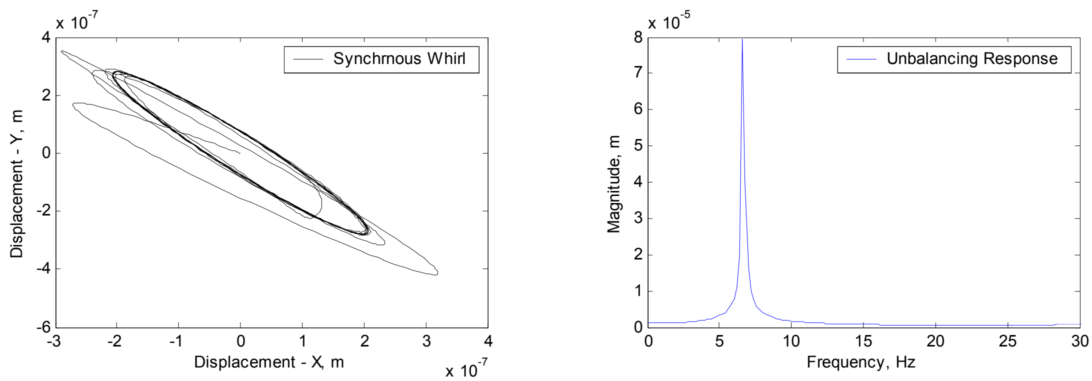

Figure 3 – The unbalancing response

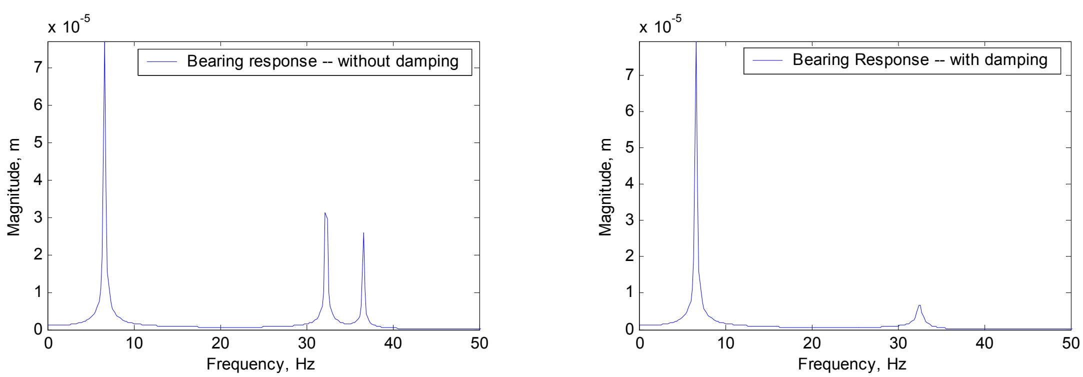

Figure 4 – Minimization of the backward whirl

Analyzing table 2, it can be seem that the bearing stiffness coefficients present some elastic asymmetry. So, the backward whirl excited by the rotor unbalancing can occur when the rotor speed is between two critical speeds related to the lower and higher stiffness, respectively. However, the bearing damping coefficients, considered in this study, were able to make the backward whirl effect disappear, as represented in Figure 4. Similar results are obtained for the center of rotor and the bearing positions.

Other important effect observed during the study is the influence of the cross-coupled stiffness coefficients in reducing the system effective damping, therefore, increasing the magnitude of the unbalancing response. Figure 5 represents 10 different random unbalancing distributions along the shaft that emphasize the improvement of rotor-bearing system unbalancing response when the cross-coupling stiffness coefficients effects are considered.

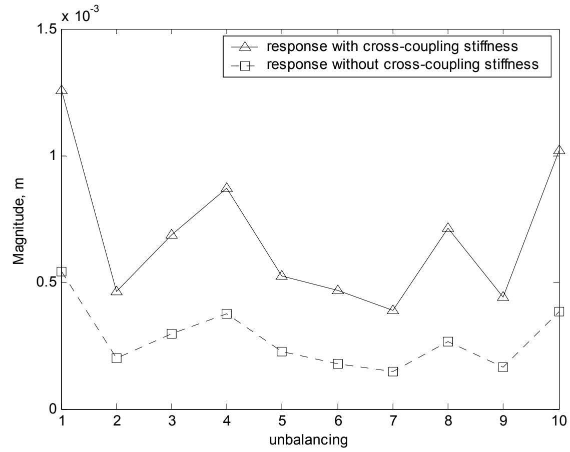

Figure 5 – Cross-coupling effect

The results showed in Figure 5 highlight the importance of the cross-coupled stiffness coefficients in the modeling process. The unbalancing responses of the system increased about 150% when these coefficients were employed in the simulation of the rotor-bearing system. Although it is not shown in this work [8], it should be mentioned that the rotor-bearing system presents a great instability when the cross-coupling stiffness coefficients have order of magnitude equal or higher than the direct stiffness bearing coefficients.

CONCLUSIONS

Modeling process of the rotating system must include the main effects present in this kind of system such as gyroscopic effect, rotating inertia and bearings contributions represented by the damping and stiffness dynamic coefficients. Most of the dynamic analyses developed for rotor-bearing systems exclude the cross-coupled terms that are very important for the levels of the responses obtained.

The cross-coupled stiffness coefficients changed the stability and the magnitude of the system response. It was observed for the coefficients considered in this study that the cross-coupled stiffness increased the magnitude of the unbalancing responses acting reducing the system effective damping. Besides the system presents a great instability when the cross-coupling stiffness coefficients have order of magnitude equal or higher than the direct stiffness bearing coefficients, which influences the numerical convergence of the procedure implemented.

It is well known that unbalance can excite the backward whirl when the rotor operation speed is between the two critical speeds associated to the two lateral whirl modes generated by the bearing stiffness asymmetry. The damping coefficients considered in this work were able to eliminate the backward critical speeds associated with the bearing stiffness asymmetry.

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NOMENCLATURE

X, Y, α, β = generalized coordinates

ω = rotor angular velocity

M = rotor mass

I_p, I_t = polar and transversal moment of inertia

$K_{XX}, K_{XY}, K_{YY}, K_{YX}$ = stiffness coefficients

$C_{XX}, C_{XY}, C_{YY}, C_{YX}$ = damping coefficients

m_i = mass of rotor section

l_i = length of rotor section

Δt = time increment

u_i = eccentricity of unbalancing mass

L = rotor length