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DESIGN AND EXPERIMENTAL VALIDATION OF OPTIMAL PLACEMENT OF STRAIN SENSORS IN FLEXIBLE STRUCTURES

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Abstract

The purpose of this work is to suggest a criterion for the optimal placement of strain sensors for monitoring vibrations in flexible structures. More precisely, it is proposed here to find the optimal location of strain sensors by maximising the observability of the von Mises equivalent strain of the fundamental vibration modes. The sensors should be placed at points where this equivalent strain presents significant values for the system dynamic behaviour. Experimental validation is provided on a cantilever thin plate instrumented with triaxial rosettes of conventional strain gauges and fiber Bragg grating sensors subjected to impact and harmonic loading.

INTRODUCTION

The research and development of smart structures technology in recent years has provided new ways of measuring and cancelling vibrations. A good choice of the sensor location on smart structures is essential to measure and cancel efficiently these vibrations. The optimal placement of sensors has been addressed by a great number of researchers. For instance, Crawley and de Luis [2] attempt to find the optimal placement by determining the location of high average strain on structures. Hwang *et al.* [6] find the placement of piezoelectric sensors by determining the position sensitivity function of each controlled mode, which is proportional to the sum of the deformation in two directions (0º, 90º). Gawronski [3] addresses the problem of sensor positioning using their notion of modal controllability and observability. Halim and Moheimani [4] propose a method based on the spatial controllability measure.

In this work, we deal with the strain sensor placement problem as applied to three-dimensional flexible structures and particularized to plate structures. To this end, we analyse a three-dimensional strain state by using the von Mises equivalent strain (ES) [1] corresponding to the vibration modes to be observed and/or controlled. It is a unique representative value of a three-dimensional strain state and gives information on optimal locations of sensors. The developed method is used to locate strain sensors and monitor vibrations in an experimental platform based on a cantilever thin plate. In most of the applications, conventional strain gauges and piezoelectric sensors are used as strain sensors. However, other types of sensors, like Fiber Bragg Grating (FBG) sensors, are increasing their importance in several applications, for instance, in structural monitoring for civil, aerospace, marine, and other structures [8]. FBG sensors exhibit several advantages with respect to conventional sensors, such as electrical and magnetic isolation or non-existence of noise measuring. For these reasons, we have also used these sensors in our experimental setup.

This paper continues as follows. A sensor placement criterion based on the von Mises ES and its application to locate strain sensors in a cantilever thin plate is presented. In the next Section the main features of FBG sensors used as strain sensors are expounded. The experimental validation using conventional strain gauges and FBG sensors is provided and some conclusions are finally given.

SENSOR POSITIONING CRITERION

Our objective is to suggest a criterion for the optimal placement of strain sensors on flexible structures. This criterion is based on considering that the ES distributions should be large and nearly equal for applications in monitoring and cancelling vibrations. As a consequence, this method takes into account the mean and the standard deviation of the ES distributions of the considered modes. The optimal placement is proposed to be found by the following optimization problem:

$$\max_{(\underline{x})\in\mathbb{R}} \quad \Psi(\underline{x}) = \frac{1}{\beta(\underline{x})}\frac{\bar{\varepsilon}(\underline{x})}{S(\underline{x})} \tag{1}$$

with $\underline{x}=(x,y,z)$, $\bar{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^{vM}$, $S=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\varepsilon_i^{vM}-\bar{\varepsilon}\right)^2}$, $\beta=\max\left(\bar{\varepsilon}/S\right)$, i indicates the vibration mode, N is the number of considered modes, $\mathbb{R}=\left\{(x,y,z)\middle|0\le x\le L_x, 0\le y\le L_y, 0\le z\le L_z\right\}$ and $\left(L_x,L_y,L_z\right)$ are the structure dimensions. The von Mises ES ε_i^{vM} (in analogy with stress state theory) is derived as follows [1]:

$$\varepsilon_i^{vM}=\frac{2}{3}\left\{\frac{1}{2}\left[\left(\varepsilon_{xi}-\varepsilon_{yi}\right)^2+\left(\varepsilon_{yi}-\varepsilon_{zi}\right)^2+\left(\varepsilon_{zi}-\varepsilon_{xi}\right)^2\right]+12\left[\varepsilon_{xyi}^2+\varepsilon_{yzi}^2+\varepsilon_{xzyi}^2\right]\right\}^{\frac{1}{2}} \tag{2}$$

The procedure is as follows. A modal analysis must be carried out and the modal shapes and the corresponding distributions of each component of the strain tensor must be derived from this. From the strain distributions, we can calculate the ES according to (2) and then the optimization criterion (1) can be carried out.

In this work, we focus on a cantilever thin plate, which often appears in spacecraft applications and it is a typical light damped structure. The low-frequency vibrations of this structure decrease very slowly with time. Hence, most control applications concentrate on cancelling these low frequencies. Since our problem deals with a cantilever plate, it is necessary to measure the strain in three different directions at each point of interest [1]. This is the reason why a triaxial rosette is needed. There are two possible configuration of a triaxial rosette: a star configuration and a triangle configuration. Both configurations are typical for traditional strain gauges and the triangle configuration is more common for FBG sensors. Figure 1 shows a diagram of these two possible configurations.

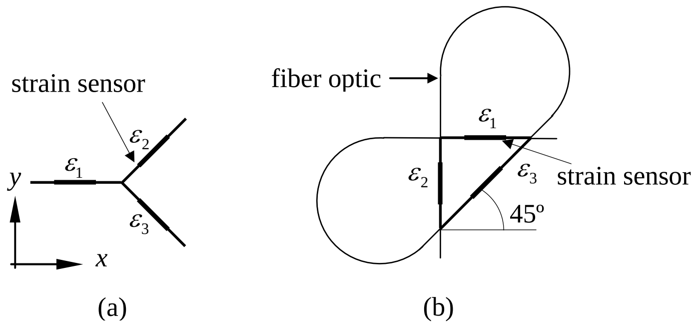


Figure 1 – Strain rosette sensors. a) Star configuration. b) Triangle configuration

The measured longitudinal strains at each point, ε_1, ε_2 and ε_3 can be expressed in terms of the components of the strain tensor ε_x, ε_y and ε_{xy} as follows:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} n_{1x}^2 & n_{1y}^2 & n_{1x}n_{1y} \\ n_{2x}^2 & n_{2y}^2 & n_{2x}n_{2y} \\ n_{3x}^2 & n_{3y}^2 & n_{3x}n_{3y} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \tag{3}$$

where n_{lx}, n_{ly} with $l = 1,2,3$ are the direction cosines of the axis of each sensor with respect to the X-axis. In the case of the triangle configuration shown in Figure 1b), the angles with respect to the X-axis are 0º, 90º and 45º. Substituting the corresponding direction cosines, (3) can be rearranged and the strain state ($\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$) can be derived.

From the Kirchhoff's assumptions of thin plate theory [5], ε_z, ε_{xz} and ε_{yz} are negligible. Then, (2) can be simplified as follows:

$$\varepsilon_i^{vM} = \frac{2}{3}\left\{\varepsilon_{xi}^2 + \varepsilon_{yi}^2 - \varepsilon_{xi}\varepsilon_{yi} + 12\varepsilon_{xyi}^2\right\}^{\frac{1}{2}} \tag{4}$$

where $\varepsilon_{xi} = z\,\partial^2\phi_i\left(x, y\right)/\partial x^2$, $\varepsilon_{yi} = z\,\partial^2\phi_i\left(x, y\right)/\partial y^2$, $\varepsilon_{xyi} = z\,\partial^2\phi_i\left(x, y\right)/\partial x\partial y$ and $\phi_i\left(x, y\right)$ is the orthonormalized modal shape with respect to mass matrix [9], as it is typically considered.

FIBER BRAGG GRATING STRAIN SENSORS

Fiber optic sensors are displacing the traditional sensors for acoustic, strain, vibration, etc, due to their inherent advantages, like their ability to be lightweight, resistant to electromagnetic interference, high sensitivity and environmental ruggedness. The sensor is fabricated by “writing” a fiber grating onto the core of an optical fiber [7]. Bragg writing technology allows us to couple numerous sensors on a single fiber, with no physical splicing. The installation is easy and has a single point connection to the optical source as opposed to multiple connections required in traditional sensors.

The basic principle of operation of FBG sensors is based on the shift wavelength of the return signal with the changes in the strain measure. Then, if the strain is given by $\varepsilon = \Delta L/L$, the relationship from the Bragg wavelength $\left(\lambda_B\right)$ to its shift $\left(\Delta\lambda_B\right)$ can be expressed as [7]:

$$\Delta\lambda_B/\lambda_B = \left(1 - p\right)\varepsilon \tag{5}$$

where p is the effective photoelastic constant and depends on the strain optic sensor.

EXAMPLE OF APPLICATION

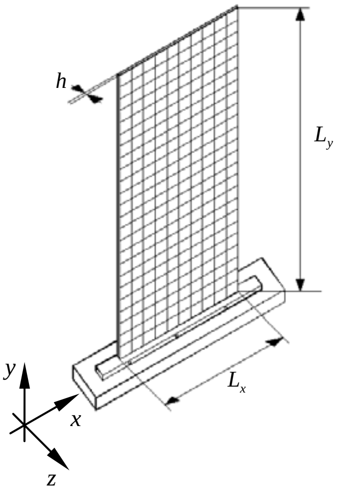
The example consists of a cantilever thin plate made of aluminium. Figure 2 shows a schematic diagram, dimensions and physical characteristics of the plate.

The differential equation of undamped motion of plates has the form [9]:

$$D\nabla^4 w\left(x, y, t\right) + \rho h \frac{\partial^2 w\left(x, y, t\right)}{\partial t^2} = 0 \tag{6}$$

where $w(x, y, t)$ is the transversal deflection of each point of the plate, ∇^4 is the Biharmonic operator and $D = Eh^3/12\left(1 - \upsilon^2\right)$ is the plate flexural rigidity.

Equation (6) has not analytical solution for the cantilever case [9]. Because of this, the vibration modes of the plate are obtained using the finite element method. The plate has been modeled by 250 shell finite elements [5] and the computation has been performed with the finite element code ANSYS [10].

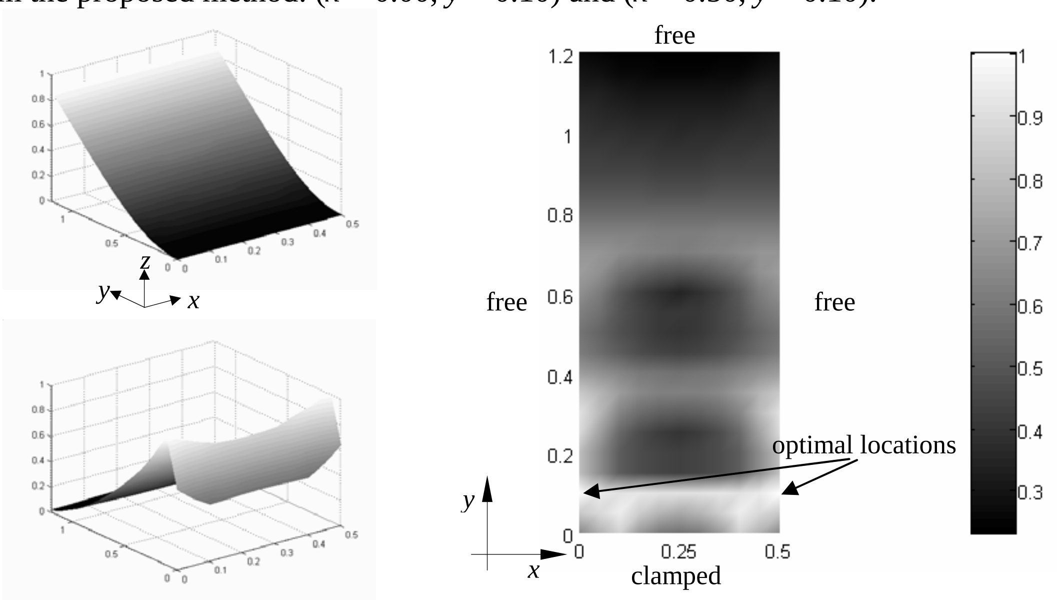


Elastic Modulus	E	$0.69e11$	N/m^2
Mass density	ρ	2950	Kg/m^3
Poisson's ratio	υ	0.30	
Width	L_x	0.500	m
Thickness	h	0.003	m
Length	L_y	1.2	m

Figure 2 – Schematic diagram, dimensions and physical characteristics of the plate structure

Application of the optimization criterion

We have considered the first three modes which adequately approximate the dynamic behaviour of the plate. A modal analysis has been carried out and the modal shapes have been derived. Then, the ES distributions of each vibration mode can be obtained. As an example, Figure 3a) depicts the first vibration mode and its corresponding ES distribution. The clamped edge is placed at $y = 0$. Note that the first mode presents the maximum values of the ES close to the clamped edge as it is expected. Figure 3b) shows the cost function ($\Psi(x, y)$) and the placement of the optimal points obtained from the proposed method: (x = 0.00, y = 0.10) and (x = 0.50, y = 0.10).



a) (First bending mode,1.66 *Hz*)

b)

Figure 3 – a) First vibration mode and the corresponding ES distribution. b) Sensitivity map

Experimental validation

To verify the above results, we have performed a harmonic analysis to obtain the frequency response of the system in terms of the ES.

The applied force at the point (x = 0.40, y = 0.20) is the following:

$$F(t) = F_0 \cos(2\pi ft) \tag{7}$$

and the response of the structure in terms of the ES is as follows:

$$\varepsilon^{vM}(t) = \varepsilon_0^{vM} \cos(2\pi ft + \varphi) \tag{8}$$

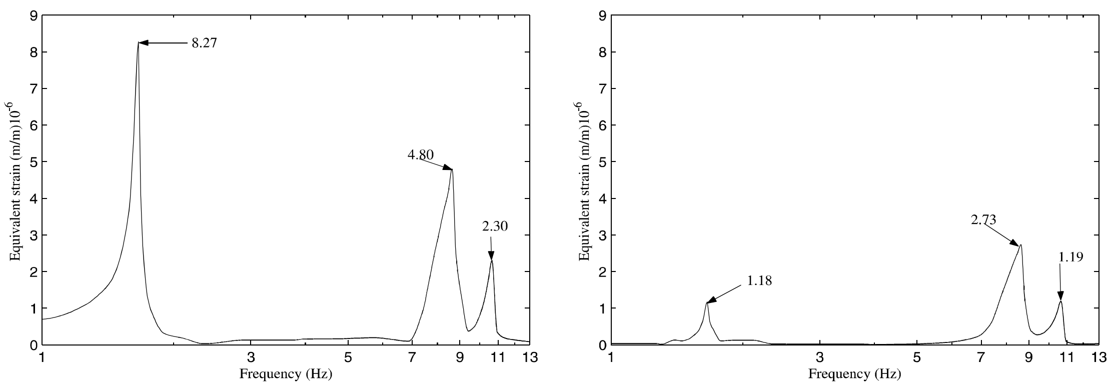
F_0 being the amplitude of the excitation , $f \in (0,\ 13)\,Hz$ the range of frequency variation , t the time, ε_0^{vM} the amplitude and φ the phase of the response.

On the other hand, we have carried out several impact analyses by means of an instrumented hammer to complete the validation of the obtained location.

We shall now present some experimental results by using conventional strain gauges and FBG sensors.

Conventional strain gauges

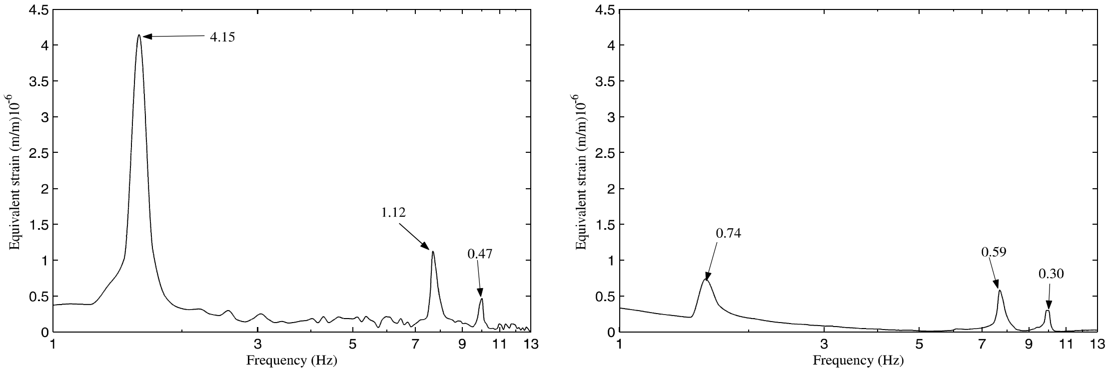
Several conventional strain rosettes are cemented over the structure and one example of them is given here. Figure 4a) shows the amplitude of the frequency response of the harmonic analysis for a sensor placed at the optimal location (x = 0.0, y = 0.10) and Figure 4b) for a sensor placed at a non-optimal location (x = 0.25, y = 0.80). It can be observed that the response registered by the optimal placed sensor reflects the first three modes more clearly than the response registered by a non-optimal placed sensor.



a) b)

Figure 4 – Frequency response of the harmonic analysis. a) Optimal point. b) Non-optimal point

We have undertaken several impact analyses and an example of them is provided here. Figure 5a) shows the amplitude of the frequency response of the impact analysis for a sensor placed at the optimal location and Figure 5b) for a sensor placed at a non-optimal location. Again, the better observability provided by the first sensor is clearly shown.

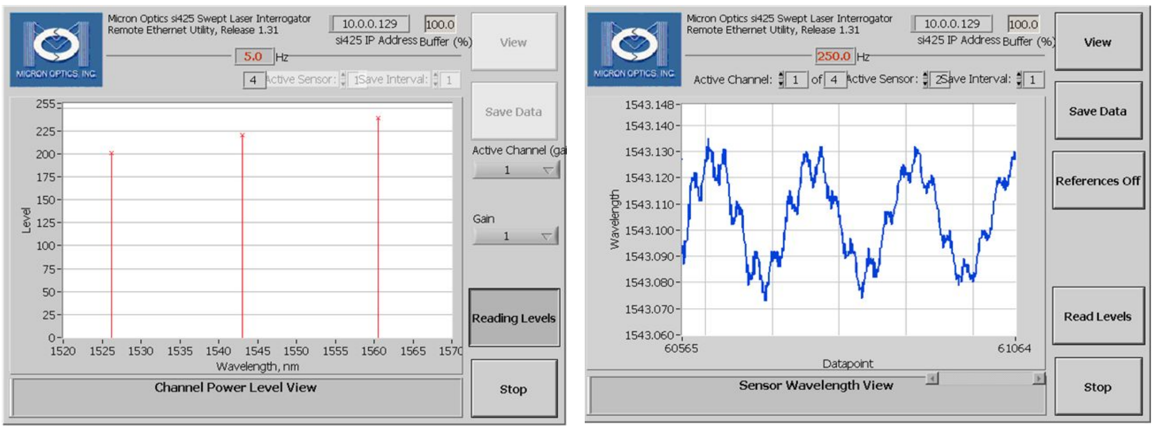


a) b)

Figure 5 – Frequency response of the impact analysis. a) Optimal point. b) Non-optimal point

Fiber Bragg Grating Sensors

The same analyses as those carried out with conventional sensors have been undertaken with FBG sensors. We have cemented a FBG rosette at the optimal location which is demodulated by si425 Optical Sensing Interrogator [11]. Si425 uses a swept laser source, a tuneable narrow-band filter to interrogate the sensor and is able to interrogate up to 512 optical sensors simultaneously at maximum scan rates of 250 *Hz*. Figure 6a) shows the power level of the signal of each sensor and Figure 6b) gives an examples of monitoring vibrations with FBG sensors and si425.



a) b)

Figure 6 – Monitoring vibrations with si425 Optical Sensing Interrogator

The results obtained by means of the FBG sensors are very similar to those represented in Figures 4a) and 5a) and consequently are not shown here.

CONCLUSIONS

A technique for placing strain sensors on flexible structures has been presented. The proposed sensor positioning criterion, based on the measurement of the von Mises equivalent strain, has been experimentally validated by means of a cantilever thin plate subjected to a harmonic excitation and to an impact loading.

Because of the advantages of Fiber Bragg Grating sensors, we have concentrated our interest on monitoring the plate not only with conventional strain gauges but also with FBG sensors.

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