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APPLICATION OF COUNTERPROPAGATING NONLINEAR WAVES TO INHOMOGENEOUS PRESTRESS CHARACTERIZATION

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Abstract

Counterpropagation of two longitudinal waves in a physically nonlinear inhomogeneously prestressed elastic material (structural element) is studied theoretically. The main attention is paid to the wave induced oscillations on two parallel boundaries of the material. It cleared up that the nonlinear effects of boundary oscillations are sensitive to the values of the parameters of multi-parametric prestressed state of the material. Comparisons of nonlinear boundary oscillation profiles on opposite boundaries enables qualitatively and quantitatively characterize inhomogeneous prestress. This is illustrated by the numerical experiments for the case of two-parametric prestressed state.

INTRODUCTION

Due to promising perspectives in nondestructive testing (NDT) of materials wave-wave and wave-material interaction is under intensive investigation [2]. Variation of material properties leads to the necessity to look for the extra sources of information in ultrasonic NDT. One possibility is to complicate the NDT set-up by involving besides ultrasound different generators of deformation waves. The second choice is to increase the precision of equipment and to take advantage of higher order small effects of wave propagation and interaction [4].

In this paper the second approach is used. The problem of nondestructive evaluation of inhomogeneous prestress in the physically nonlinear elastic material is solved theoretically on the basis of nonlinear effects of interaction of counterpropagating ultrasonic waves. Two longitudinal waves with arbitrary smooth initial profiles are excited simultaneously on the parallel boundaries of the specimen.

The wave motion is described on the basis of the nonlinear theory of elasticity [1]. The special attention is paid to the oscillations caused by waves on the boundaries of the specimen. Influence of the prestress on the wave profile distortion and on the evolution of nonlinear effects of wave propagation and interaction is clarified.

The model problem of counter-propagation of two harmonic waves in the material undergoing pure bending with compression or tension is considered. Analyses of the results of numerical simulations leads to the conclusion that the nonlinear effects of boundary oscillations caused by counterpropagating waves are informative and may be used for nondestructive characterization of inhomogeneous prestress field. The methods for qualitative and quantitative nondestructive characterization of two-parametric inhomogeneous prestressed state of the material are proposed.

MATHEMATICAL FORMULATION

A specimen with two parallel boundaries is considered. In the civil engineering such objects are supported plates, beams, thin-walled structures, etc. The material of the specimen is isotropic and homogeneous. The small but finite deformations of the material are described by the five constant nonlinear theory of elasticity [1]. The geometrical and physical nonlinearity of the problem is taken into account.

The quasi one-dimensional problem of one-dimensional wave motion in the material undergoing two-dimensional inhomogeneous prestress is studied in Lagrangian rectangular coordinates. The wave motion is described by the nonlinear second order hyperbolic differential equation with variable in space coefficients. The coefficients are functions of the physical properties of the material and the parameters of the prestressed state. The problem is solved by assumption that the kind of prestressed state and the physical properties of the material are known and the coefficients of the equation of motion are known functions that involve unknown parameters of the prestressed state.

Consequently, some preliminary information about the prestressed state is available. Here, it is assumed that the material (structural element) is undergoing prestressed state that corresponds to the plane strain. In this case the coefficients of the equation of motion are determined as solutions to a set of two nonlinear second order elliptic differential equations that describe equilibrium of the material in the static prestressed state. The solution to the equations of equilibrium is sought in the form of polynomials making use of the perturbation technique. The intention is to solve the prestress nondestructive characterization problem by evaluation of the coefficients of polynomials as the parameters of the prestressed state.

The analytical solution for the special case of prestress that corresponds to the pure bending with compression or tension is derived. The main domain of the prestress is determined by the component $T_{22}=a+b\ X_1$ of the Kirchhoff pseudostress tensor where a and b are constants and direction of the coordinate X_1 is perpendicular to the surface of the plate. The constant a characterizes the constant part of prestress and the constant b the linearly variable part of it.

COUNTERPROPAGATING WAVES

Two longitudinal waves with arbitrary smooth initial profiles are excited on opposite parallel boundaries of the material (sample, structural element). The kind of inhomogeneous prestressed state is known and it is described analytically by polynomials with unknown values of coefficients. The governing equation for wave propagation is solved making use of the perturbation technique and the Laplace integral transform. The resulting solution describes the initial stage of nonlinear wave propagation, reflection and interaction and is valid in some time interval. It is assumed that in this initial stage the distortion of wave profile is weak and the shock wave is not generated. The small parameter introduced by the perturbation procedure ensures the smallness of strain, i. e., the absence of plastic deformations.

Counter-propagation of harmonic waves in the nonlinear elastic material undergoing two-parametric prestressed state is studied in detail. Harmonic waves with the same amplitude and frequency are excited on the opposite boundaries in terms of particle velocity. The distorted wave profiles are recorded on the same boundaries in term of stress. The recorded data are analysed on the basis of the perturbative solution that makes it possible to separate the linear and nonlinear effects of counter-propagation of harmonic waves. The first term in perturbative series describes linear propagation of waves in the physically linear prestress free material. The second and the subsequent terms correct the solution and take the influence of nonlinearity and inhomogeneity into account. The second term describes the main domain of nonlinear effects including the evolution of the second harmonic, influence of the prestress to the evolution of the first harmonic, nonlinear interaction between two first harmonics and influence of the nonlinear physical properties of the material to the wave propagation. This term in the dimensionless form $\varepsilon U_{1,1}^{(2)}$ is studied in detail. Here ε denotes a small parameter and $U_{1,1}$ is the derivative of the particle displacement U_1 with respect to the spatial coordinate X_1. Function $\varepsilon U_{1,1}^{(2)}$ characterizes the ratio of the magnitude of nonlinear effects to the magnitude of linear oscillation. The subsequent terms are neglected in discussions as the higher order small terms.

The main attention is paid to the wave induced oscillations on two parallel boundaries of the material. Due to the complexity of the analytical solution these oscillations are studied on the basis of this solution numerically in duralumin Al 7475 and Steel Hecla 17. The behaviour of these oscillations in both materials under different kinds of prestress is qualitatively similar. The amplitude of oscillations caused by the excitation of the same amplitude is on the boundaries of the sample of Al 7475 about 20-30 per cent large than on the boundaries of the sample of Steel Hecla 17. Wave motion in prestressed aluminium is studied in several papers [3, 4]. In this paper the main attention is paid to counter-propagation of waves in Steel Hecla 17 with density $\rho_0 = 7850$ kg/m^3, constants of elasticity $\lambda = 111$ Gpa, $\mu = 82$ GPa, $\nu_1 = -222.7$ GPa, $\nu_2 = -261$ GPa, $\nu_3 = -22.3$ GPa [3] and thickness of the sample h = 0.1 m.

Two harmonic waves with the frequency $\omega = 1.1868 * 10^6$ rad/s and with the same amplitudes that are characterized by the strain amplitude $\varepsilon = 5 * 10^{-4}$ are excited simultaneously on the parallel boundaries of the sample. Waves propagate into the depth of the material, reach the opposite boundary, reflect and come back to the boundary of excitation. This time interval is taken under consideration. The corresponding nonlinear boundary oscillations in the prestress free Steel Hecla 17 are illustrated in Fig. 1 where τ denotes the dimensionless time $\tau = t\, h/c$. Here t is the time and c is the linear wave velocity in the prestress free material. Oscillations on both boundaries $X_1 = 0$ and $X_1 = h$ coincide in the prestress free material. It is possible to distinguish two intervals on time axis – the interval of propagation $0 \leqq \tau < 1$ and the interval of interaction $1 \leqq \tau < 2$.

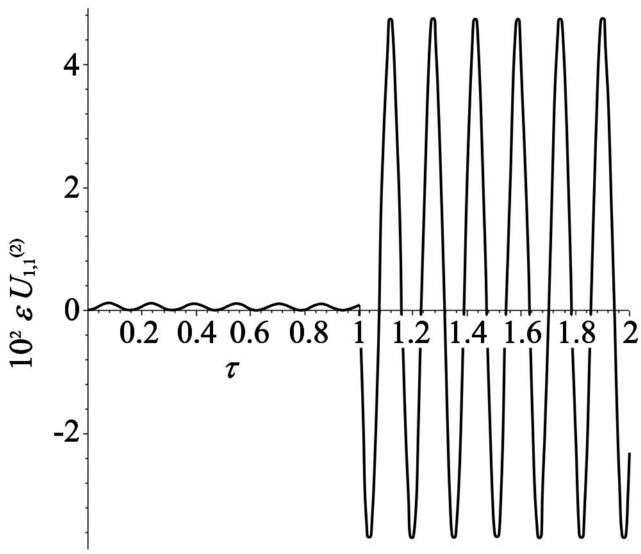

Figure 1 - Oscillations on the boundaries of the prestress free Steel Hecla 17

Boundary oscillations in the interval of propagation are plotted in Fig. 2. Essential is, that the nonlinear wave interaction amplifies the boundary oscillation amplitude in the interval of interaction about a hundred times. This facilitates utilization of nonlinear effects of wave interaction data in the algorithms of nondestructive prestress characterization.

QUALITATIVE PRESTRESS CHARACTERIZATION

Analysis of the results on numerical simulations leads to the conclusion that the amplitude of boundary oscillations is dependent on the physical properties of the material. In the prestress free material boundary oscillations have a constant amplitude in the interval of propagation (Fig. 2) and in the interval of interaction (see Fig. 1). The homogeneous prestress modulates these oscillations (Figs. 3 and 4). The oscillations on both boundaries coincide. The amplitude and the depth of modulation

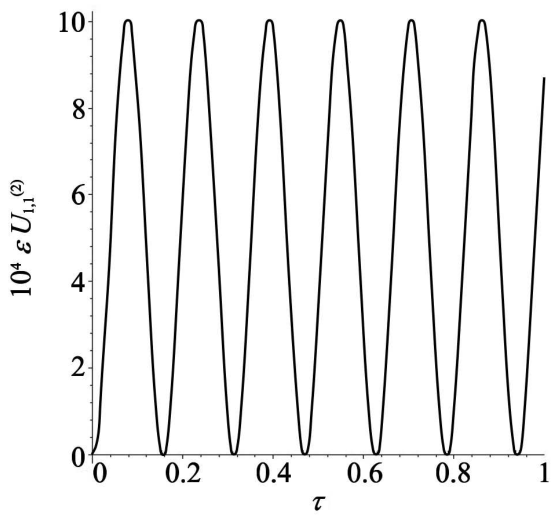

Figure 2 - Oscillations on the boundaries of the prestress free Steel Hecla 17 in the interval $0 \leqq \tau < 1$

are dependent on the value and the sign (compression or tension) of prestress. The inhomogeneous prestress (bending of the sample, for example) involves disparity in oscillation profiles on different boundaries (Fig. 5). In the special case of pure bending the boundary oscillation profiles coincide with phase shift.

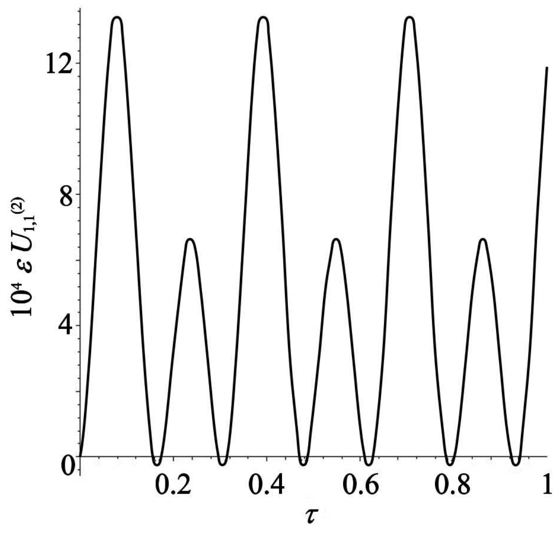

Figure 3 – Modulation of oscillations on the boundaries of Steel Hecla 17 in the interval $0 \leqq \tau < 1$ by the homogeneous prestress (a = -50 MPa)

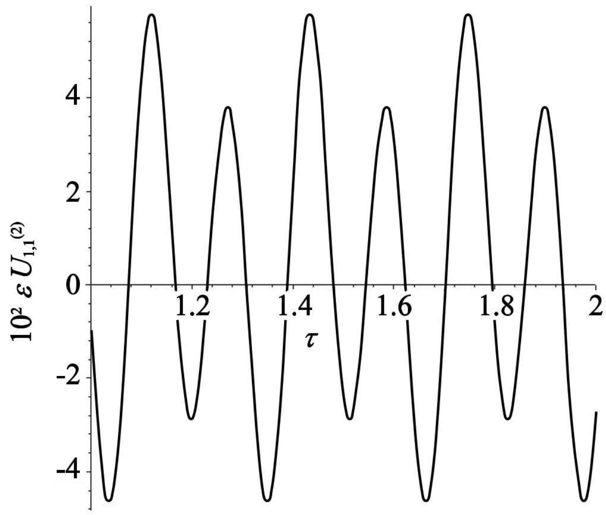

Figure 4 – Modulated oscillations on the boundaries of the homogeneously prestressed (a = -50 MPa) Steel Hecla 17 in the interval $1 \leqq \tau < 2$

Consequently, the analysis of boundary oscillations evoked by nonlinear effects of counterpropagating harmonic waves makes it possible to solve the problem of qualitative nondestructive characterization of two-parametric prestressed state in the material. It is easy to determine qualitatively the presence and the nature of prestress and to distinguish (i) prestress free material, (ii) homogeneously prestressed material, (iii) material undergoing pure bending and (iv) material undergoing pure bending with tension or compression.

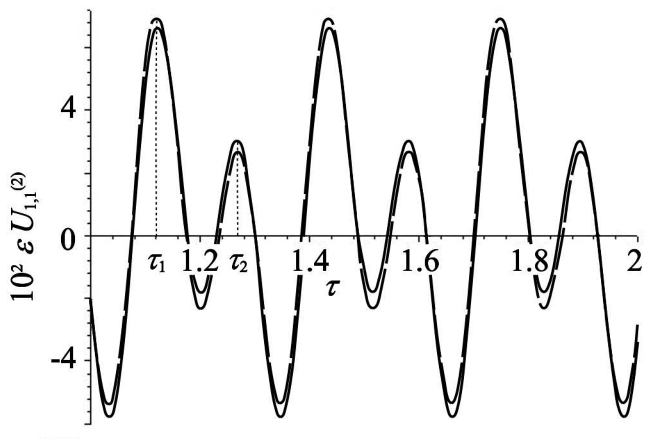

Figure 5 – Oscillations on the boundaries of the inhomogeneously prestressed Steel Hecla 17 in the interval $1 \leqq \tau < 2$ (line – X_1 = 0, dashed line – X_1 = h)

QUANTITATIVE PRESTRESS CHARACTERIZATION

The problem of quantitative nondestructive characterization of the two-parametric prestress field is solved by the assumption that the geometry and the physical properties of the material (structural element) are known. The derived analytical solution is used and the plot nonlinear oscillations on the boundary of the prestress

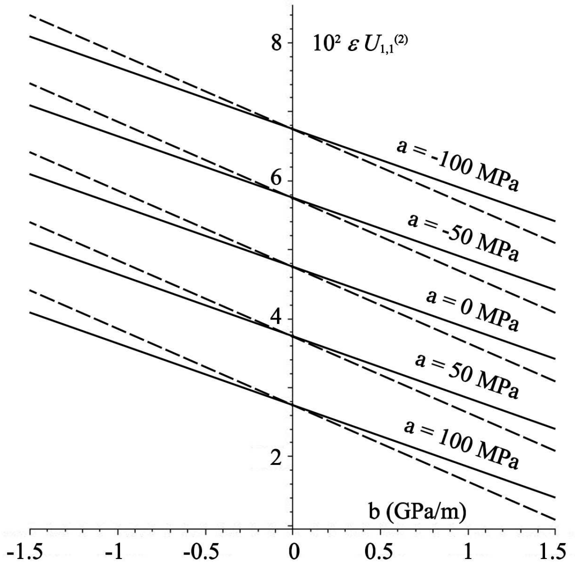

Figure 6 – Boundary oscillation amplitude versus prestress parameters a and b at the instant $\tau_1 = 1.118$ (line – $X_1 = 0$, dashed line – $X_1 = h$)

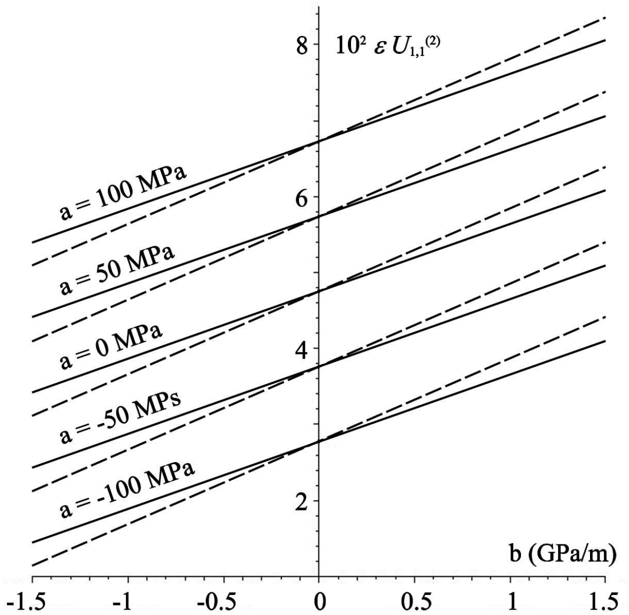

Figure 7 – Boundary oscillation amplitude versus prestress parameters a and b at the instant $\tau_2 = 1.274$ (line – $X_1 = 0$, dashed line – $X_1 = h$)

free material is composed. The values τ_1 and τ_2 of the dimensionless instants that correspond to two first local maxima of the boundary oscillations are determined. The plots boundary oscillation amplitude versus parameters *a* and *b* of the two-parametric prestress state are computed for the instants τ_1 (Fig. 6) and τ_2 (Fig. 7).

The experiment to evaluate the prestress parameters *a* and *b* for the real material is implemented. The counterpropagating harmonic waves are excited in the material and the oscillation profiles on opposite boundaries in the interaction interval are recorded (Fig. 5). The difference between the values of oscillation amplitudes on opposite boundaries is calculated for both instants τ_1 and τ_2 . Resorting to Figs. 6 or 7 two possible values of the parameter *b* are determined making use of the value of determined difference. The value of the parameter *a* and the final value of the parameter *b* are determined making use of the value of the recorded oscillation amplitude on one of the boundaries (Fig. 5) at the instant τ_1 or τ_2.

CONCLUSIONS

Counter-propagation of longitudinal harmonic waves in the physically nonlinear elastic material (structural element) with two parallel boundaries is studied theoretically. The nonlinear part of boundary oscillations is investigated in detail. The information that contains in the nonlinear effects of boundary oscillations is sufficient to solve problems of qualitative and quantitative nondestructive characterization of the two-parametric inhomogeneous prestressed state of the material.

ACKNOWLEDGEMENT

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REFERENCES

[1] Eringen A.C., *Nonlinear Theory of Continuous Media*. (McGraw-Hill, New-York, 1962).

[2] Gusev V., Bailliet H., Lotton P., M. Bruneau, “ Interaction of counterpropagating waves in media with nonlinear dissipation and in hysteretic media”, Wave Motion, **29**, 211-221 (1999).

[3] Hauk V., *Structural and Residual Stress Analysis by Nondestructive Methods*. (Elsevier, Amsterdam, 1997).

[4] Ravasoo A., B. Lundberg, “Nonlinear interaction of longitudinal waves in an inhomogeneously predeformed elastic medium”, Wave Motion, **34**, 225-237 (2001).