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COMPONENT MODE SYNTHESIS FOR HIGH FREQUENCY ACOUSTICS

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Abstract

In this paper a Component Mode Synthesis (CMS) method using characterstic constraint(CC)modes is developed for finding the natural frequencies of large acoustic cavities with high modal density. The cavity under consideration is a periodic structure. Since conventional FE techniques require large number of elements due to the high modal density, two new formulations using Chebyshev polynomial and trigonometric shape functions are proposed. These result in significantly smaller number of elements. The accuracy and computational efficiency is clearly demonstrated here. the cavity is divided into subcavities; each subcavity has been analyzed using finite element method and they are coupled using normal modes and constrained modes of subcavities. Characteristic constrained modes (CC modes) are used for reducing the large number of constraint modes.

INTRODUCTION

Acoustic cavities are currently modelled using three-dimensional brick elements. At higher frequencies these type of geometries require numerous elements, as the requirement is atleast 10 elements per wavelength, in order to provide good aspect ratios and to adequately capture required results at critical sections. This paper presents a method which can reduce the number of degrees-of-freedom(DOF) required for such problems while capturing behaviour with equal or better resolution.This method has the potential for exceptional computational efficiency, for structures which are prismatic or quasiprismatic. Shape functions based on orthogonal functions is employed in the direction of prismatic direction[1,2]. Shape functions of Quasi Prismatic(FQP) element and Trigonometric finite element(TFE) are derived from the orthogonal functions like Chebyshev polynomial and trigonometric functions respectively. These new elements are also capable of being assembled with conventional three dimensional finite elements.

Component mode synthesis(CMS) was developed as a practical and efficient approach to modeling and analyzing the dynamics of the global structure. In CMS, the dynamics of a structure are described by selected sets of normal modes of the individual component structures, plus a set of static vectors that account for the coupling at each interface where component structures are connected. One of the most accurate and widely used CMS methods is the Craig-Bampton method[3]. In this paper, a new technique is presented for reducing the size of a CMS model by performing an eigenanalysis on the constraint-mode partitions of the mass and stiffness matrices. This is a computationally superior variant of the technique proposed by Tan et. al[4]. The resultant eigenvectors are called the characteristic constraint(CC) modes. These modes may then be truncated to yield a highly reduced- order model(ROM).

NEW FINITE ELEMENT FORMULATIONS

The below formulations are mainly used for the cavities which are prismatic or quasi-prismatic.

Finite quasiprismatic element(FQP)

The finite quasi-prismatic element is a three dimensional finite element which uses conventional interpolating functions in two directions and functions based on Chebyshev polynomials in the third direction[1]. This Chebyshev polynomial expansion is better behaved than other family members of ultra spherical expansions. It is similar to conventional finite elements, the difference being in the shape functions used. The coordinates are approximated by

$$\begin{aligned} x(\xi,\eta,\zeta) &= \sum_{i=1}^{nc}\sum_{j=0}^{no(i)} x_{ij}N_i(\xi,\eta)T_j(\zeta) \\ y(\xi,\eta,\zeta) &= \sum_{i=1}^{nc}\sum_{j=0}^{no(i)} y_{ij}N_i(\xi,\eta)T_j(\zeta) \\ z(\xi,\eta,\zeta) &= \sum_{i=1}^{nc}\sum_{j=0}^{no(i)} z_{ij}N_i(\xi,\eta)T_j(\zeta) \end{aligned} \tag{1}$$

where nc is the number of axode curves per element (see Figure (1)) and $no(i)$ is the order of the Chebyshev expansion at node i. The unknown pressure is interpolated in a very similar fashion. The $N_i(\xi,\eta)$ are the conventional two-dimensional shape functions for isoparametric elements with the number of nodes capable of varying between three to nine. The $T_j(\zeta)$ are shape functions derived from Chebyshev polynomials:

$$T_0(\zeta) = (1-\zeta)/2, \quad T_1(\zeta) = (1+\zeta)/2,$$

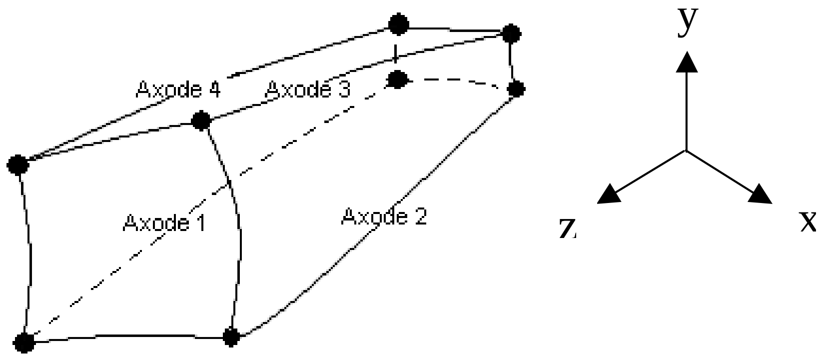


Figure 1: FQP element with axodes

$$T_n(\zeta) = \begin{cases} \tau_n(\zeta) - 1 & \text{for } n > 1 \text{ and even} \\ \tau_n(\zeta) - (\zeta) & \text{for } n > 1 \text{ and odd} \end{cases} \tag{2}$$

Here $\tau_n(\zeta)$ are standard Chebyshev polynomials [5].

Trigonometric Finite Element(TFE)

This is similar to FQP formulation.In this element shape functions based on trigonometric functions are used in the third direction. Here also the unknown pressure can be written as

$$p(\xi, \eta, \zeta) = \sum_{i=1}^{nc} \sum_{j=0}^{no(i)} p_{ij} N_i(\xi, \eta) T_j(\zeta) \tag{3}$$

Here $N_i(\xi, \eta)$ are two dimensional shape functions and $T_j(\zeta)$ are shape functions based on trigonometric function, those are given below

$$\begin{gathered} T_0(\zeta) = (1 - \zeta)/2, \quad T_1(\zeta) = (1 + \zeta)/2, \\ T_{n+1}(\zeta) = \begin{cases} \cos(n\pi\zeta)/2 & \text{for } n \text{ is odd} \\ \sin(n\pi\zeta)/2 & \text{for } n \text{ is even} \end{cases} \end{gathered} \tag{4}$$

COMPONENT MODE SYNTHESIS WITH CC MODES

Consider an acoustic cavity that is partitioned into two sub cavities, termed component cavities. The boundary where the component cavities are connected will be referred to as the interface. The set P of physical coordinates of the component cavities may be divided into a set J of junction coordinates,u_j, and a set I of interior coordinates, u_i. The junction coordinates are those coordinates where components are joined together. The equation of motion (undamped free) for a component cavity may be written in partitioned form as:

$$\begin{bmatrix} m_{ii} & m_{ij} \\ m_{ji} & m_{jj} \end{bmatrix} \begin{bmatrix} \ddot{u}_i \\ \ddot{u}_j \end{bmatrix} + \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

The physical coordinates, u, may be represented in terms of component cavity generalized coordinates, p, by the coordinate transformation

$$u = \Phi p \tag{6}$$

where Φ is a matrix of preselected component modes (an assumed-modes, or Ritz approximation, of the physical displacement vector u) of the following types: rigid-body modes, normal modes of free vibrations and constraint modes.

The Craig-Bampton method of component mode synthesis employs fixed-interface component normal modes, as well a a set of vectors called constraint modes. A constraint mode is the static deflection induced in the structure by applying a unit displacement to one interface DOF while all other interface DOF are held fixed. Therefore, the interface partition of the constraint modes is an identity matrix of dimensions N_j, and the set of constraint modes Ψ^c is of the form

$$\Psi = \begin{bmatrix} \Psi^c \\ I \end{bmatrix} \tag{7}$$

where Ψ^c is the static shape induced in the sub cavity by the unit displacement at the interface. This shape is determined by posing the following statics problem:

$$\begin{bmatrix} k_{ii} & k_{ij} \\ k_{ji} & k_{jj} \end{bmatrix} \begin{bmatrix} \Psi^c \\ I \end{bmatrix} = \begin{bmatrix} 0 \\ R \end{bmatrix} \tag{8}$$

where R is a vector of forces that would be needed to impose the successive unit displacement at the interface DOF. The first row of Eq.(8)yields

$$\Psi^c = -k_{ii}^{-1} k_{ij} \tag{9}$$

Using these solutions, the constraint modes of Eq.(7) are known. Next, the component normal modes are obtained by performing a modal analysis for each component cavity. Because the Craig-Bampton method uses fixed-interface modes, they are determined from the following eigenvalue problem:

$$k_{ii}\Phi^N = \lambda m_{ii}\Phi^N \tag{10}$$

In general, these modes are truncated to reduce the size of the model. Then,by the use of the constraint modes and selected sets of component normal modes, the tranformation from finite element coordintes u to CMS generalized coordinates p is

$$u = \begin{bmatrix} \psi^N & \Psi^c \\ 0 & I \end{bmatrix} p = \Phi p \tag{11}$$

Applying the tranformation of Eq.(11), the global mass and stiffness matrices for the CMS model have the form

$$\begin{aligned} M_{CMS} &= \Phi^T M \Phi \\ K_{CMS} &= \Phi^T K \Phi \end{aligned} \tag{12}$$

Note that this transformation yields only inertial coupling between the normal modes and the constraint modes. The number of normal-mode DOF depends,of course, on how many normal modes are selected for each component structure.However, the number of constraint-mode DOF is, the number of FEM DOF that are in the interface. Thus, the size of the constraint-mode partitions is determined by the finite element mesh. If there are many finite element nodes in the interface region,then the constraint-mode partitions of the CMS matrices may be relatively larger.
It is now suggested that the number of necessary constraint modes may be reduced by seeking a new set of modes that correspond to more natural physical motion at the interface.This is posed as an eigenvalue problem for the constraint-mode partitions of the CMS matrices[4]:

$$k_{jj}\Phi^{cc} = \lambda m_{jj}\Phi^{cc} \tag{13}$$

These eigenvectors are then truncated as in traditional modal analysis. This selected set of eigenvectors may be used to tranform the mass and stiffness matrices to yield a reduced-orede model(ROM). The tranformation from CMS generalized coordinates p to ROM generalized coordinates q may be defined as

$$p = \begin{bmatrix} I & 0 \\ 0 & \Phi^{cc} \end{bmatrix} q = \Gamma q \tag{14}$$

The mass and stiffness matrices in ROM generalized coordinates are, thus

$$\begin{aligned} M_{ROM} &= \Gamma^T M_{CMS}\Gamma \\ K_{ROM} &= \Gamma^T K_{CMS}\Gamma \end{aligned} \tag{15}$$

Now, compared to the mass and stiffness matrices of Eq.(5), the size of every matrix partition has been reduced. The two sub cavities are assembled[3] using the constraint equation such as ${u_j}^1 = {u_j}^2$ can be written in terms of the generalized coordinates q and combined to form as ${\Phi_{cc}}^1 {q_j}^1 = {\Phi_{cc}}^2 {q_j}^2$

$$q = \begin{bmatrix} q^1 & q^2 \end{bmatrix}^T = \begin{bmatrix} q_i^1 & q_j^1 & q_i^2 & q_j^2 \end{bmatrix}^T = [S]\begin{bmatrix} q_i^1 & q_j^1 & q_i^2 \end{bmatrix}^T \tag{16}$$

Where the suffix i denotes interior modal coordinates, and j the junction physical coordinates. The transformation matrix S to be used in Eq.(16) is

$$[S] = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & {\Phi_{cc}}^{2^T} m_{jj}^2 {\Phi_{cc}}^1 & 0 \end{bmatrix} \tag{17}$$

Reference[4] used the CC modes transformation after assembling the two sub cavities. But here in this paper the CC mode transformation is included for finding the ROM of individual sub cavities before those sub cavities are assembled. This leads to smaller models and better computational efficiency.

RESULTS

The rectangular cavity shown in Fig.(2) is now considered as an example structure.The dimensions of cavity are 3 X 4 X 10m and properties are,density of air is $\rho = 1.2kg/m^3$, and velocity of sound is $c = 340m/s$. The cavity is considered to consist of two subcavities as shown.A FEM of this cavity was constructed using a 9 X 12 X 30 mesh of ANSYS three dimensional fluid elements. The first 100 natural frequencies of the cavity are calculated analytically and these were used as a benchmark. The finite element mesh is sufficiently fine that the first 100 FEM natural frequencies are all within 2.5% of the analytical results. A Craig-Bampton model[3] of the cavity was then generated. The first 100 normal modes were retained for each component cavity. In addition there were 130 constraint modes, yielding a CMS model with 230 DOF.The first 100 natural frequencies of this CMS model are within 1.5% of the corresponding FEM natural frequencies.Next,CC modes were calculated,and three different ROMs were constructed by keeping the lowest 10, 15 and 20 CC modes. Because of 100 component normal modes were selected for the CMS model,the smallest ROM considered contains 110 DOF and largest contains 120 DOF. The error percentages for the first 100 natural frequencies of the rectangular cavity in shown in Fig(3(a)). This error is calculated relative to the full model.

Next the same rectangular cavity is meshed using 432 FQP/TFE elements i.e 9 X 12 X 4. The first 100 natural frequencies of the cavity are calculated using the same above procedure of CMS with CC modes, are within $\pm$1.5 % of the corresponding FEM natural frequencies. The error percentages for the first 100 natural frequencies of the rectangular cavity in shown in Fig.(3(b)). This error is calculated relative to the full model.

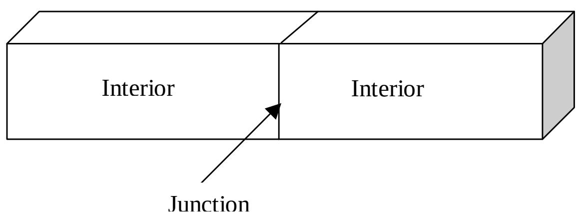
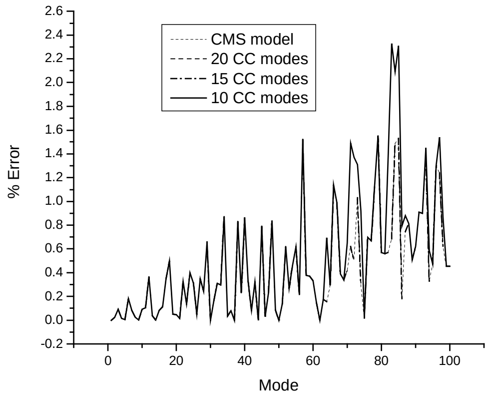
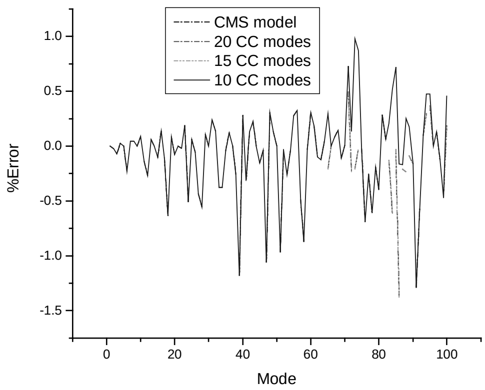


Figure 2: Rectangular box cavity

A segment of a railway coach is made into two sub cavities each of length 2m, height 2.5m and width 3m with two windows on each side, each of 0.5mx0.5m size (Fig.4(a)); these have been modelled and analyzed using FEM and then compared with CMS with CC modes. The first 100 normal modes were retained for each component cavity. In addition there were 33 constraint modes,yielding a CMS model with 133 DOF. Three different ROMs were constructed by keeping the lowest 10,5 and 3 CC modes.Fig.4(b) shows the error of frequencies of segment of railway coach. The error measured for this model using CMS with CC modes are within 0.7% when using 10 and 5 cc modes but for 3 cc mode case the error is about 1.5 %. This error is calculated relative to the full model.

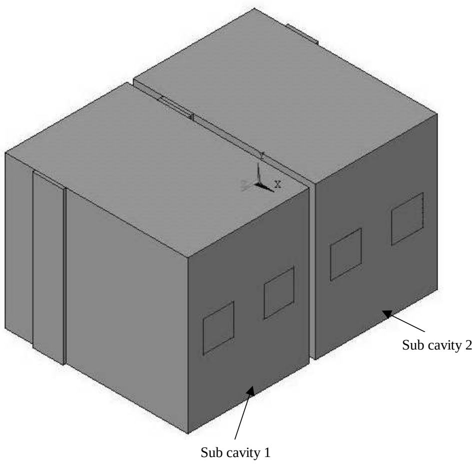


(a)

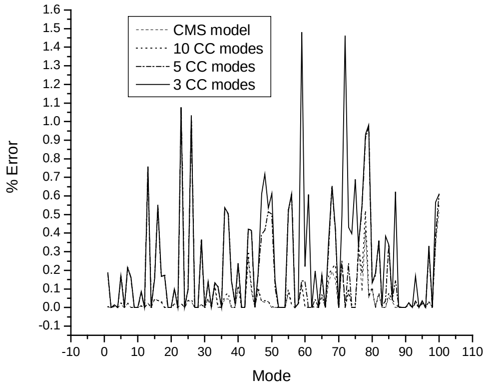


(b)

Figure 3: (a)Error in % of frequencies of rectangular cavity that is meshed by conventional FE (b)Error in % of frequencies of rectangular cavity that is by using FQP/TFE elements



(a)



(b)

Figure 4: (a) Two sub cavities of railway coach(b)Error in % of frequencies of railway coach

CONCLUSION

This paper has shown that the FQP element and TFE can be a very efficient and accurate alternative to conventional finite element and offers a definite advantage over finite elements for high frequency acoustic problems. A technique for reducing the size of a Craig-Bampton CMS model by improving the representation of the interface between component cavities using CC modes has been presented.The CC modes are found by performing an eigenanalysis of the partitions of the CMS mass and stiffness matrices that correspond to the Craig-Bampton constraint modes.This CC modes are truncated as if they were traditional modes of vibration.This truncation leads to highly reduced-order CMS model.This investigation showed that the use of CC modes in CMS allows highly efficient modelling of the dynamics of complex cavities.

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