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NONPLANAR NONLINEAR VIBRATION ANALYSIS OF AN EXTERNALLY EXCITED CIRCULAR CANTILEVER BEAM

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Abstract
A symmetrical circular cantilever beam with periodic external excitation shows a lot of nonlinear response. A flexible circular cantilever beam has non-linear terms of inertia, spring, and viscosity. Each term represents different characteristic of the nonlinear response. In lower order modes, the non-linearity due to spring is dominant while the non-linear effect of inertia is dominant in higher order modes. In the analysis of the response characteristics of a cantilever beam, the jump phenomena in planar and nonplanar direction was investigated by examining d in the analysis of the frequency response characteristics. During the non-linear phenomena, the superharmonic and subharmonic motions of the beam were analyzed by using autospectrum and cepstrum. The phase portrait method is used for the analysis of phase change due to change in excitation frequency. In the analysis, it can be seen that the subharmonic motion plays an important role in causing non-linearity in the beam.

INTRODUCTION

A circular cantilever beam excited to a nonlinear vibration shows very interesting dynamic characteristics of nonlinear response. A flexible circular cantilever beam has nonlinear terms of inertia, spring, damping, gravity, warping, and so on. When the external excitation force is weak, the beam shows a linear motion, but as the force increases, the linear motion transforms to a nonlinear motion due to the nonlinear terms. It is needed to study the characteristics of the nonlinear terms of a system by analyzing the nonlinear phenomena and to find causes of the phenomena in the view of dynamics. Also, it is necessary to study on the physical characteristics of those phenomena. To study them, an experimental method is adopted and a long slender circular cantilever beam is used. The transition process of the motion from linear state to nonlinear state is analyzed, applying an external base harmonic excitation of the beam. The subharmonic and the superharmonic motions are analyzed using autospectrum of FFT and cepstrum. The phase change in the nonlinear motion is traced by using an oscilloscope. Quasi periodic phenomena are analyzed by investigating the torus structure [1],[3].

ANALYSIS OF NONLINEAR VIBRATION

Analysis of Subharmonic and Superharmonic motions

When nonlinear phenomena occur in a system under an external excitation force, the system shows subharmonic or superharmonic motions. And the relation of the excitation frequency Ω to the response frequency of the system can be written as the following [7], [10].

$\omega = m\Omega$ superharmonic motion,

$\omega = \frac{1}{n}\Omega$ subharmonic motion,

$\omega = \frac{m}{n}\Omega$ super-subharmonic motion, (1)

where m and n are integers. $(m \neq 1, n \neq 1)$

The analysis of the time series is necessary to study the harmonic motions, but when the response is complicated, the analysis is hard to be performed. In that case, the study can be done easily if the time signal is converted to a frequency signal by performing FFT analysis for the response signal. When the peak value of autospetrum is analyzed, it is possible to analyze the superharmonic, subharmonic, and super-subharmonic motions.

When the response signal with the nonlinearities is investigated using the autospectrum, many components of the harmonic motions are detected. To analyze the periodicity of those components, the cepstrum is used. The cepstrum is defined as the power spectrum of the logarithm power spectrum. The power cepstrum is defined as the following.

$$C_{AA}(\tau) = F^{-1}[\log S_{AA}(f)]\,. \tag{2}$$

$C_{AA}(\tau)$: Power cepstrum

S_{AA} : Auto spectrum

F^{-1} : Inverse Fourier Transform

The cepstrum is obtained when the autospectrum is applied to the inverse Fourier transform. In the autospectrum, when the spectrums, which are harmonic components of a high frequency or components of a side band frequency, are arranged at a fixed interval, the interval can be presented in a period. Since the harmonic components of a high frequency and the components of a side band frequencies yield only one peak value in the time axis, the phenomena of frequency modulation can be analyzed when the system has nonlinearities.

Analysis of Phase Analysis

The phase portrate of a cantilever beam can be analyzed two dimensionally in the planar direction (x-axis) and in nonplanar direction (y-axis). Since the planar and nonplanar motions have the form of a periodic vibration in one-to-one resonance, they can be written as the following [11].

$$x = a_1 \cos(\omega_1 t - \alpha_1)\,,$$
$$y = a_2 \cos(\omega_2 t - \alpha_2)\,. \tag{3}$$

Since $\omega_1 \cong \omega_2$ in one-to-one resonance,

$$x = a_1 \cos(\omega t)\,,$$
$$y = a_2 \cos(\omega t + \varphi)\,. \quad \varphi \text{ : phase difference} \tag{4}$$

Thus, various types of phase portrate can be shown depending on φ and the change of phase between the planar and the non-planar motions in one-to-one resonance are possible to be analyzed.

Analysis of Phase Portrate

In a nonlinear system, the response signals are possible to be converted from signals in time to signals in phase space. When the acceleration signal is converted to the signal in phase space by applying the integral-differentiation, it has the form of non-autonomous periodic orbit. In figure 2, each black dot represents a portrate that shows particular phases for the frequency of external excitation. The path from one point to the next represents a periodic component with the excitation frequency $T = 2\pi/\Omega$. It is useful to present the motions of the system in the expanded phase space with the time variable $x_3 = t$. The motions of the system in this space are autonomous and each portrate doesn't intersect each other. In the planes of the phase space $(x, \dot{x})$, which are normal to each other, considering the intersection of these portrates, Poincare`Map can be obtained. Poincare`Map can be obtained when the transverse planes are separated with the excitation period T along the axis of time. Its usefulness derives from the uniqueness property, but it is the weak point that it has a lot of planes to present many motions in the wide space of phase. Connecting two planes in the phase space, Solid Torus Region (DXS`) is formed and the center of torus becomes the axis of time. The portrate of torus has a singular cross section that can slice it continuously. Thus the toroidal region is very useful in analyzing the dynamic phenomena of a nonlinear system even though it is not a real phase space [5], [8], [11].

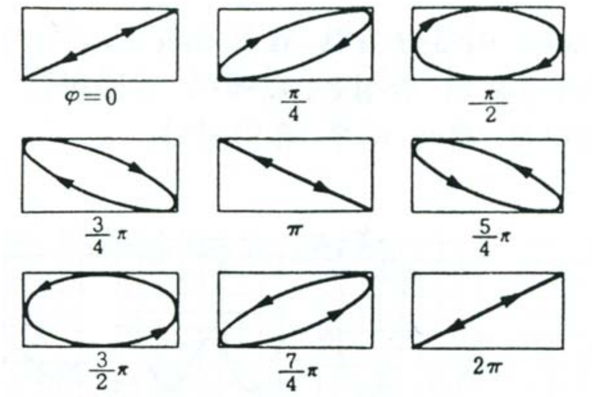

Figure 1 – Phase space on the same frequency

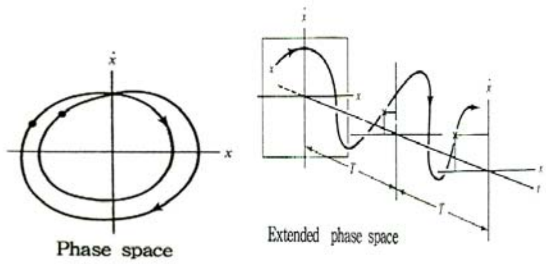

Figure 2 – Phase space and Extended phase space of the subharmonic

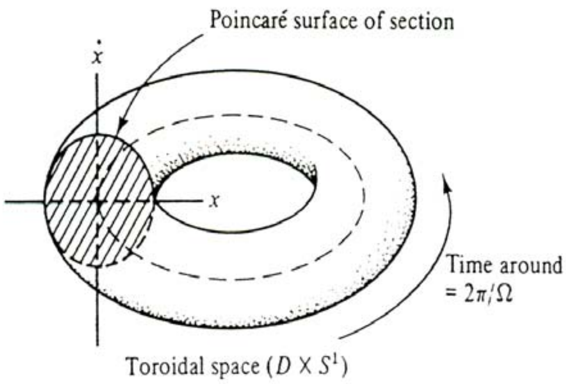

Figure 3 – Poincare' surface of section on the toroidal space

EXPERIMENT ON NONLINEARITIES

To investigate the nonlinearities, a circular beam of aluminum alloy was used as a uniform elastic material. The dimensions of the beam were: the modulus of elasticity E=72GPa, the coefficient of stiffness G=27GPa, Poisson's ratio $\nu = 0.3333$, mass per unit length the beam $m = 0.0336 Kg/m$, diameter $\phi = 5mm$, length L=675mm. For the excitation of the beam, base harmonic excitation was applied to the fixed part of the base in the form of a sine wave with constant amplitude.

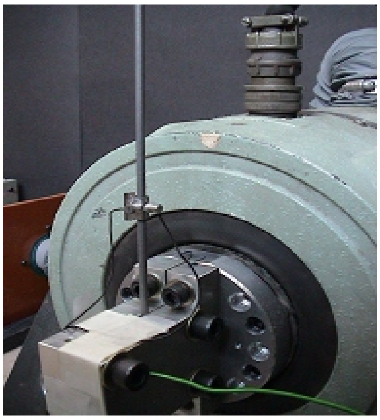

Figure 4 – Accelerometer position on the circular cantilever beam

Figure 5 – One-to-one resonance of the circular cantilever beam on the second mode

Table 1 Measured natural frequency and damping coefficient of the circular cantilever beam

Mode	λ	Theory(Hz)	Meas.(Hz)	Damping
1	1.8751	7.94	7.63	1.510%
2	4.6941	49.78	48.25	0.736%
3	7.8548	139.40	135.13	0.341%

The flexible beam was fixed to the shaker to satisfy the boundary conditions of the cantilever beam. Keeping the voltage applied to the shaker constant, the experiment was performed by increasing or decreasing the excitation frequency. When the voltage to the shaker is kept constant, the speed component in the harmonic vibration has a constant value regardless of the change of excitation frequency. The increase and decrease of excitation frequency were in the form of sine sweeping and the rate of change was 0.030Hz/s.

The experiment was performed by increasing the amplitude of the excitation with the frequency fixed in the second mode (48.25Hz) of the beam. The process of change in the beam from the linear vibration to the nonlinear vibration was investigated as the amplitude of the excitation increases. The nonlinear vibration was investigated using the autospectrum and the cepstrum in the analysis of superharmnomic, subharmonic, and super-subharmonic motions. To investigate the change in the phase of the vibration, the phase portrate was analyzed with the oscilloscope and the analysis of Torus was performed to examine the form of chaos.

To investigate the response of the beam, each B&K 4374 accelerometer was attached to the beam in the planar and nonplane directions. The mass of each accelerometer was about 0.65g, the range of frequency for measurement 1-25 KHz, and the level of measurable acceleration 250,000 m/s^2. The accelerometers were attached to the surface of the beam 100mm above the base with a strong adhesive.

EXPERIMENTAL RESULTS AND DISCUSSION

The nonlinear responses of a flexible cantilever beam were investigated in the second mode of the beam using the autospectrum and the cepstrum. In the autospectrum, it can be seen that the higher orders of harmonic vibration, $2f_0, 3f_0$, and $4f_0$ occurred around the reference frequency $f_0(48.25Hz)$ and the subharmonic vibration of $1/2\,f_o$ and higher subharmonic vibration of $3/2\,f_0$ and $5/2\,f_0$ occurred (Figure 6). When the nonlinear signals of response in the second mode are examined with the cepstrum, the harmonic components in the frequency region can be easily analyzed. In the cepstrum, it can be seen that the large values of $3f_0$ and $4f_0$ appear around $f_0(20.51ms)$ and the components of $1/2\,f_0$ and $1/4\,f_o$ are well shown (Figure 6). In the phase analysis of the planar and the nonplanar motions in the second mode of the beam, it can be seen that the one-to-one resonance makes the planar motion dominant in the planar motion. It is also shown that the difference of phase angle between the planar and the nonplanar is up to 180^o (Figure 7). In the analysis of the system based on torus, it is shown that the element of $1/2\,f_0$ is the main component of the nonlinear vibration in the second mode (Figure 8).

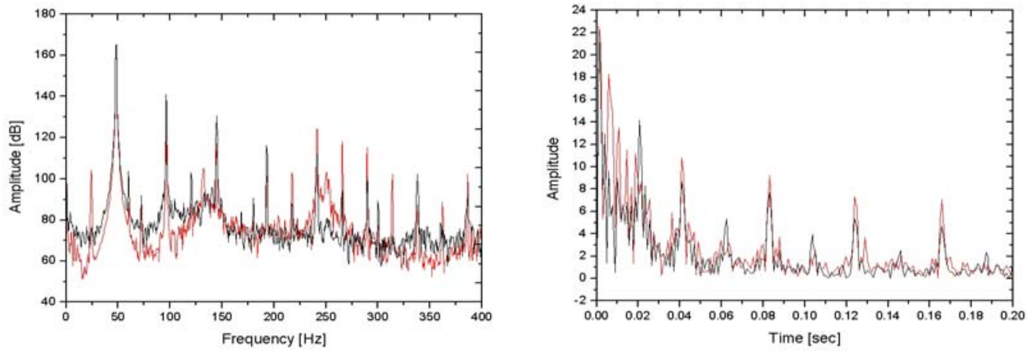

Figure 6 – Autospectrum and Cepstrum of the planar and nonplanar in the second mode

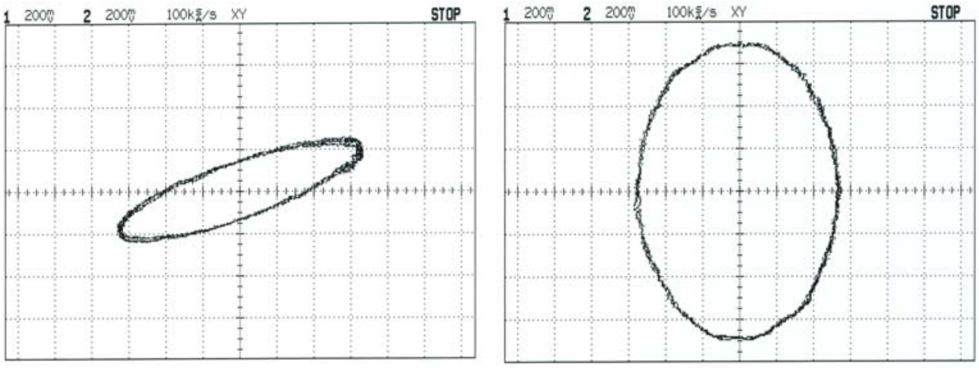

Figure 7 – Phase change in phase space of the second mode (x-planar, y-nonplanar)

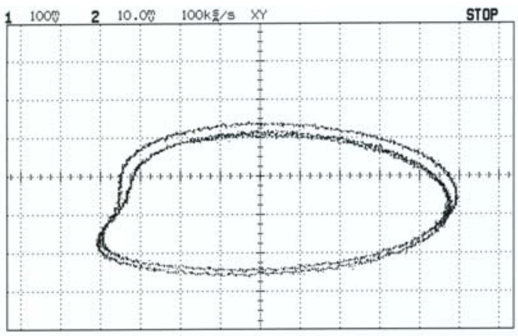

Figure 8 – Torus analysis in the second mode (x-velocity, y-displacement)

CONCLUSION

To investigate the nonlinear vibration of a flexible circular cantilever beam under the base harmonic excitation, the autospectrum, cepstrum, phase portrate, and torus were used. When the nonplanar motion occurred in the second mode of 48.25Hz, due to one-to-one resonance, the nonlinear responses of superharmonic, subharmonic, and super-subharmonic motions were investigated. In the autospectrum, the superhamonic components of $2f_0, 3f_0,$ and $4f_0$ were dominant around $f_0(48.25Hz)$. In the cepstrum, the subharmonic components of $1/2\, f_0$ and $1/4\, f_0$ were dominant around f_0. However, in the phase analysis using torus, the phase component of $1/2\, f_0$ was dominant. Thus, it can be seen that in addition to the component of f_0, $1/2\, f_0$ also plays an important role in causing the nonplanar motion in the one-to-one resonance. In the phase analysis of the planar and the nonplanar, it can be seen that the phase angle varies largely. Finally, it can be concluded that further study is needed to define the role of change of the phase angle in the nonlinear vibration.

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