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MODAL ANALYSIS AND CONTROL OF ADAPTIVE STRUCTURES WITH SHEAR PIEZOELECTRIC ACTUATORS

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Abstract
The shear actuated control of structures has occupied a considerable portion of the literature in the previous years. Shear controlled mechanisms has proven effective in active vibration damping with advantages that has been reported repeatedly. In this paper, we present a model for the control of the sandwich beam vibration using the assumed modes method. A velocity feedback control algorithm is used. The results have proven the accuracy of the model as well as the effectiveness of the control mechanism. Also, it was shown that the control was more effective when based on the second mode vibration.

INTRODUCTION

The use of piezoelectric actuators in the field of vibration damping and shape control has been extensively studied during the past decades. The main setup of the controlled structures was through the bonding of a piezoelectric patch on the surface of the structure which in turn gets actuated by a control signal or passive techniques. The choice of bonding the piezoelectric material to the surface was taken to obtain maximum strain at the surface of the structure; consequently, the piezoelectric patches were subjected to high stresses as well as being subject to damage by foreign objects. The use of the embedded shear piezoelectric controllers overcame those drawbacks [1-4]. Finite element formulations that describe the behaviour of this new family of controllers were also introduced [5-7] to facilitate the analysis of realistic structures. Enhancements on the analytical models were developed in parallel to the numerical techniques [8-9] together with studies on different control techniques [10-12]. The assumed modes analysis was used to study the control of thin beams by Abreu et al. [13]. In this paper, we will present a numerical study of the control of a shear actuated beam using the assumed modes method. A PD controller is used to demonstrate the method. The results are verified using published results.

THEORETICAL FORMULATION

A sandwich beam with thin, elastic face beams and a core that behaves as a Timoshenko beam is considered for the modal vibration analysis (Figure 1). Linear theories are used to derive the governing equations. The transverse displacement w of all points in any cross section of the sandwich beam is considered to be the same. For simplicity, the face beams are assumed to have the same thickness t_b, density ρ, and modulus of elasticity E. The sandwich beam has length L and width b.

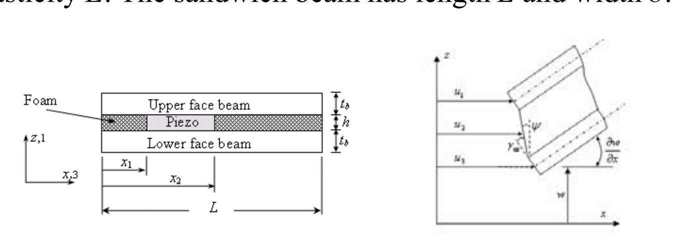

Figure 1. Geometry and deflections of a sandwich beam with shear piezoelectric core

Kinematics

Considering the geometry of motion shown in Figure 1, the normal strain ε_x and the shear strain γ_{xz} of the piezoelectric core layer are given by $\varepsilon_x = \partial u_2/\partial x - z\,\partial\psi/\partial x$, $\gamma_{xz} = \partial w/\partial x - \psi$; where ψ denotes the shear angle of the core (i.e. rotation of the cross section) and u_2 is the longitudinal displacement of the core. Note that $\psi = \partial w/\partial x$ for the Euler-Bernoulli face beams. The shear angle, described by the longitudinal displacements of the face beams u_1, u_3 and the transverse displacement, w, is given by $\psi = (u_3 - u_1 - t_b\,\partial w/\partial x)/h$.

Thus, using the above strain-displacement relations, the shear strain is described by $\gamma_{xz} = (u_1 - u_3 + d\,\partial w/\partial x)/h$; where $d = h + t_b$, with subscripts 1, 2, and 3, denoting upper face beam, core, and lower face beam respectively. Because of the symmetry of the sandwich beam, the longitudinal displacement of the core u_2 is found to be $u_2 = (u_1 + u_3)/2$. From the above, the displacement field could be fully described using u_1 , u_3 , and w.

Constitutive Relation

Using the plane stress assumption, the constitutive equations of a piezoelectric material in shear mode can be expressed as:

$$\sigma_x = \overline{C}_{11}\varepsilon_x \qquad \& \qquad \tau_{xz} = \overline{C}_{55}\left(\gamma_{xz} - \gamma_p\right) \tag{1}$$

With $\overline{C}_{11} = C_{11} - C_{13}^2/C_{33}, \overline{C}_{55} = C_{55}, \& \gamma_p = d_{15}\overline{E}_1$; where C_{ij} (i,j=1,2,…,6), d_{15} and $\overline{E}_1$ denote the stiffness matrix components, the piezoelectric shear coefficient, and the electric field applied across the thickness of the shear mode actuator, respectively. Also, γ_p stands for the piezoelectric induced shear strain. For the isotropic, Euler-Bernoulli face beams, no transverse shear strain exists and the elastic coefficients are totally defined using Young's modulus E and Poisson's ratio ν. The constitutive relation for the face beams in this case is given by $\sigma_x = E_{11}\varepsilon_x$ with $E_{11} = E/(1-\nu^2)$.

System Energy

The potential energy of both face beams U_b can be expressed as

$$U_b = \frac{1}{2}\int_0^L \left[2E_{11}I\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + E_{11}bt_b\left(\frac{\partial u_1}{\partial x}\right)^2 + E_{11}bt_b\left(\frac{\partial u_3}{\partial x}\right)^2 \right] dx \tag{2}$$

where I denotes the moment of inertia of the face beams. The potential energy U_p of the piezoelectric core is given by:

$$U_p = \frac{1}{2}\int_0^L \left[\overline{C}_{11}I_p\left(\frac{\partial \psi}{\partial x}\right)^2 + \overline{C}_{11}bh\left(\frac{\partial u_2}{\partial x}\right)^2 + \overline{C}_{55}bh(\gamma_{xz} - d_{15}\overline{E}_1)^2 \right] R(x)\, dx \tag{3}$$

where $R(x) = H(x - x_1) - H(x - x_2)$. With x_1 and x_2 denoting the position of the piezo actuator along the beam axis which is defined using the Heaviside function H, and I_p denotes the second moment of area about the piezo layer neutral axis. Upon substitution of strain-displacement relations in the last equation and performing some manipulations, the potential energy term of the piezo core becomes

$$\begin{aligned} U_p = \frac{1}{2}\int_0^L \Bigg[& (\alpha + \overline{\beta})\left(\frac{\partial u_1}{\partial x}\right)^2 + (\alpha + \overline{\beta})\left(\frac{\partial u_3}{\partial x}\right)^2 - 2(\alpha - \overline{\beta})\frac{\partial u_1}{\partial x}\frac{\partial u_3}{\partial x} + \alpha t_b^2\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2\alpha t_b \frac{\partial u_3}{\partial x}\frac{\partial^2 w}{\partial x^2} \\ & + 2\alpha t_b \frac{\partial u_1}{\partial x}\frac{\partial^2 w}{\partial x^2} + \beta(u_1^2 + u_3^2 - 2u_1u_3) + \beta d^2\left(\frac{\partial w}{\partial x}\right)^2 + 2\beta d u_1 \frac{\partial w}{\partial x} - 2\beta d u_3 \frac{\partial w}{\partial x} \\ & + \beta\left(\frac{d_{15}\overline{E}_1}{h}\right)^2 + 2\overline{C}_{55}bd_{15}\overline{E}_1 u_3 - 2\overline{C}_{55}bd_{15}\overline{E}_1 u_1 - 2\overline{C}_{55}b\,d\,d_{15}\overline{E}_1\frac{\partial w}{\partial x} \Bigg] R(x)\, dx \end{aligned} \tag{4}$$

where $\beta = \overline{C}_{55}b/h, \overline{\beta} = \overline{C}_{11}bh/4, \& \alpha = \overline{C}_{11}I_2/h^2$. In the same manner, the kinetic energy, neglecting rotary inertia, of both face beams and the piezoelectric core respectively are:

$$T_b = \frac{1}{2}\int_0^L \left[2m_b\left(\frac{\partial w}{\partial t}\right)^2 + m_b\left(\frac{\partial u_1}{\partial t}\right)^2 + m_b\left(\frac{\partial u_3}{\partial t}\right)^2 \right] dx \tag{5}$$

$$\begin{aligned} T_p = \frac{1}{2}\int_0^L \Bigg[& m\left(\frac{\partial w}{\partial t}\right)^2 + \mu t_b^2\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 + (\mu + \frac{m}{4})\left(\frac{\partial u_1}{\partial t}\right)^2 + (\mu + \frac{m}{4})\left(\frac{\partial u_3}{\partial t}\right)^2 \\ & - 2(\mu - \frac{m}{4})\frac{\partial u_1}{\partial t}\frac{\partial u_3}{\partial t} + 2\mu t_b \frac{\partial u_1}{\partial t}\frac{\partial^2 w}{\partial x \partial t} - 2\mu t_b \frac{\partial u_3}{\partial t}\frac{\partial^2 w}{\partial x \partial t} \Bigg] R(x)\, dx \end{aligned} \tag{6}$$

where $m_b = \rho b t_b, \mu = \dfrac{\rho_2 I_2}{h^2}, \& m = \rho_2 b h$

Assumed Modes Analysis

The assumed modes method, in terms of generalized coordinates, is used to discretize the energy equations and expand the transverse displacement w in the form $w(x,t) = W(x)^T \theta(t)$, and the longitudinal displacements of the face beams u_1 and u_3 as $u_1(x,t) = U(x)^T \phi(t)$ and $u_3(x,t) = V(x)^T \xi(t)$ respectively. The displacement functions W, U, and V are sets of admissible functions that are chosen to satisfy the geometrical boundary conditions [14]. The assumed modes method has been successfully used to analyze the dynamics of three-layered damped sandwich beams [15]. Substituting the assumed modes in the strain and the kinetic energy terms and applying Lagrange's equation, the differential equation of motion, in matrix form becomes

$$[M]\{\ddot{q}(t)\} + [K]\{q(t)\} = \{\delta_p\}\overline{E}_1 + \{F\} \tag{7}$$

where $[M]$ and $[K]$ denote the mass and stiffness matrices of the sandwich beam while $\{q(t)\} = \{\theta(t), \phi(t), \xi(t)\}^T$ and $\{F\}$ are vectors of the generalized coordinates and generalized forces respectively. The vector $\{\delta_p\}\overline{E}_1$ denotes the non-conservative work done by shear actuation forces and moments with

$$\delta_p = d_{15}\overline{C}_{55} b \left[d\int_0^L \frac{\partial W}{\partial x} R(x)\, dx, \quad \int_0^L U\, R(x)\, dx, \quad -\int_0^L V\, R(x)\, dx \right]^T \tag{8}$$

Shear Actuation Forces and Moments

To further demonstrate the shear actuation mechanism in terms of longitudinal forces and bending moments, consider an element of the sandwich beam with a piezoelectric core. The induced shear stresses by the piezoelectric actuator form equal and opposite shear stresses on the face beams. On the other hand, the induced shear stresses form a moment about the core's neutral axis as well as a moment about each face beam's neutral axis. The induced shear actuation force per unit length on any of the face beams is defined as $F_p = \tau_p b = \gamma_p \overline{C}_{55} b = d_{15} \overline{E}_1 \overline{C}_{55} b$. This force, upon multiplication by the rod displacement modes and integration over the length of the piezo, will represent the distributed induced shear actuation work. Similarly, the summation of moments about each layer's neutral axis caused by the induced shear forces is found to be $m_p = F_p \cdot \frac{t_b}{2} + F_p \cdot \frac{t_b}{2} + F_p \cdot h = F_p \cdot d$. The resulting moment, upon multiplication and integration with the bending slope, represents the actuation work done on the sides of the piezo. Thus, it can be seen that the induced shear forces and moments agrees with the non-conservative work obtained in the last section.

Active Control

A proportional plus derivative feedback controller is considered for this study. Accordingly, the electric field, in Laplace domain, may be expressed as

$$\overline{E}_1 = -(K_p + K_d s)\{C\}^T \{q(s)\} \quad \textbf{(9)}$$

where K_p and K_d are the proportional and derivative control gains, respectively, and {C} is a vector denoting the modes upon which the control effort is applied. Also, s denotes the Laplace complex number. The equation of motion for the closed-loop system takes the form

$$s^2[M]\{q(s)\} + [K]\{q(s)\} = -k_p\{\delta_p\}\{C\}^T\{q(s)\} - sk_d\{\delta_p\}\{C\}^T\{q(s)\} + \{F\} \quad \textbf{(10)}$$

Rearranging the last equation and grouping similar terms results in

$$\left(s^2[M] + s[K_d] + [K + K_p]\right)\{q(s)\} = \{F\} \quad \textbf{(11)}$$

where $K_p = k_p\{\delta_p\}\{C\}^T$ and $K_d = k_d\{\delta_p\}\{C\}^T$ denote the control gain stiffness and damping matrices respectively.

NUMERICAL EXAMPLE

Validation

To validate the present analysis, the natural frequencies of a cantilevered sandwich beam with a piezoelectric shear core are evaluated and compared to results from the literature [5]. The geometric properties of the sandwich beam are shown in Figure 2 and material properties are the same as presented by Lin et al. [16]. For the cantilever structure, rod mode shapes are used to represent the longitudinal displacements u_1 and u_3 while beam mode shapes are used for the transverse displacement w. Good agreement has been found between the two different analyses (Table 1).

Table 1. Natural frequencies of the cantilevered sandwich beam (Hz)

Mode	*Assumed Modes*	*Finite Element [5]*
1	990	985
2	3961	3912
3	8446	8305
4	17176	17273
5	25640	25980

Table 2. Mechanical properties of the face beams and the piezoelectric core

PZT5H					*Foam*			*AL*		
GPa				kg m-3	MPa		Kg m-3	GPa		kg m-3
C11	C13	C33	C55	ρ_2	E	ν	density	E	ν	ρ
126	84.1	110	23	7730	35.3	0.383	32	70.3	0.345	2690

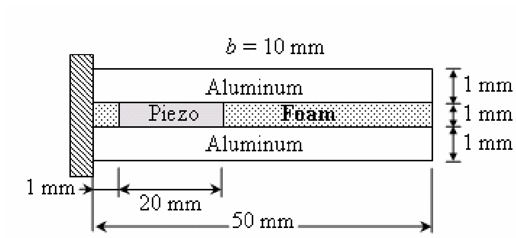

Figure 2. Dimensions of the sandwich beam

Performance of Controlled Structure

In this section, the closed-loop response of the actively controlled structure is compared to the response of the uncontrolled structure for the first two bending modes. The same geometry is used from the free vibration analysis and the material properties are given in Table 2 with the piezoelectric constant $d_{15} = 740 \times 10^{-12}\ m/V$. Using a velocity, derivative, feedback, the structural response to a sinusoidal tip-force is significantly damped as shown in Figure 3 and Figure 4. The first mode feedback regime significantly affects the first mode and moderately affects the second mode as shown in Figure 3. However, feedback of the second mode significantly damp out both modes as shown in Figure 4.

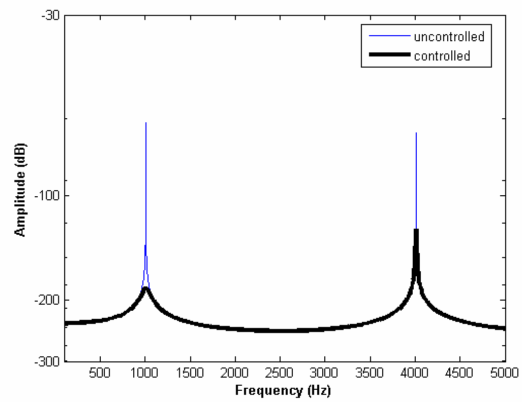

Figure 3. Frequency response of uncontrolled and controlled structures (first mode feedback: K_P=0, K_d=1×10⁴)

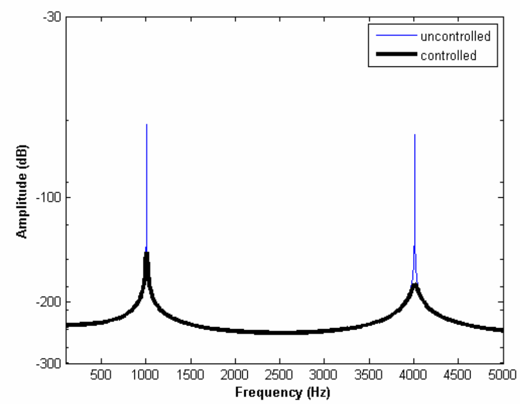

Figure 4: Frequency response of uncontrolled and controlled structures (second mode feedback: K_P=0, K_d=1×10^4)

CONCLUSIONS

The dynamic equations of motion are developed for a sandwich beam with shear piezoelectric core using the assumed modes method. Active damping is generated in the structure using modal feedback control technique. It has been shown that the shear actuation mechanism enhances the active damping characteristics of the structure. For the first two beam bending modes, the second mode feedback results in significant damping in both first and second modes, while the first mode feedback is less significant in damping the second mode.

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