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OPERATIONAL MODAL ANALYSIS OF THE SAKER RACE CAR WITH ENGINE AS EXCITATION

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Abstract

This paper presents the application of a new technique for operational modal analysis (i.e. blind identification) to a high performance race-car. The technique combines two very powerful signal processing tools: cyclostationarity and the complex cepstrum. Cyclostationarity is employed to isolate the contribution to the responses of the engine combustion alone, effectively reducing a multiple-input-multiple-output system to single-input-multiple-output. The cepstrum is then employed to separate the source and transmission path, and curve fitted. The frequency response functions are thereby recovered up to an overall scaling constant, which can be readily obtained from a finite element model or a previous similar measurement, allowing the mode shapes to be correctly scaled. Unlike most other operational modal analysis techniques, the input need not be white, only frequentially smooth.
The resonance frequencies, damping and mode shapes are compared with those obtained from the industry standard Operational PolyMAX software from LMS.

INTRODUCTION

Blind system identification, often referred to as Operational Modal Analysis (OMA) when applied to mechanical systems, has become increasingly popular in recent years. New techniques have become available which allow the properties of a system to be identified while the system is in service, thereby ensuring the properties identified correspond with the working environment of the system. However, when the system is in service, it is not possible to measure the excitations, only the responses, so traditional input-output modal analysis techniques cannot be applied and a blind identification technique is required. Some popular techniques include Frequency Domain Decomposition [1], Stochastic Subspace Iteration and Maximum Likelihood [2], and the new PolyMAX algorithm [3]. This paper presents a new technique that exploits two powerful tools; cyclostationarity and the cepstrum.

Cyclostationary signals are a special class of non-stationary signals which exhibit periodicity in their statistics. They occur quite commonly in both the natural and mechanical worlds, examples including ECG signals in biomedicine, seasonal fluctuations in meteorology, and of particular relevance here, the combustion signal from an internal combustion engine.

Researchers have previously shown how the spectral redundancy offered by cyclostationarity can be exploited to blindly identify system transfer functions. The power spectral density (PSD) may be evaluated at a particular frequency shift, corresponding to the unique statistical periodicity in the cyclostationary signal. At this frequency shift, the only contribution to the spectrum, known as the cyclic spectrum, will come from that cyclostationary signal. The system is thereby reduced from a Multiple-Input-Multiple-Output (MIMO) scenario to a Single-Input-Multiple-Output (SIMO) scenario, and the transfer function can then be identified. This may be applied to many systems where at least one cyclostationary input exhibits unique statistical periodicity, such as a vehicle with only one engine, or multiple engines that all run at different speeds. However, even if the system can be reduced to a SIMO form, the responses will still be a convolution of the input and transfer function. For the system properties to be identified, these must be separated.

The cepstrum is defined as the inverse Fourier transform of a log spectrum. Like all measures in the cepstrum domain, it derives its name from reversing the first syllable of the frequency domain analogue, i.e. spectrum becomes cepstrum, and frequency becomes quefrency. It was previously shown [4] that, by curve fitting a SIMO response autospectrum in the cepstrum domain, the input and transmission path effects can be separated because they occupy different quefrency regions. It was further shown that if one input in a MIMO system was at least four times larger than any other, then their cepstrum technique could be applied in conjunction with the singular value decomposition to blindly identify the system properties [5]. The technique employed in this paper relaxes this condition for systems excited by at least one cyclostationary input with a unique cyclic frequency. It was demonstrated on a simple laboratory test rig using idealised input excitation in [6] and is extended in this paper to a more realistic application: operational modal analysis of a race car using the engine as the excitation.

THEORETICAL BACKGROUND.

Cyclic Spectral Density

The cyclic spectral density is analogous to the ordinary power spectral density, but evaluated at a frequency shift of $\pm\beta\alpha$, where β is any positive integer. Indeed, when α=0, the cyclic spectral density reduces to the ordinary PSD. The cyclic spectral density is represented in Figure 1, where it can be seen that non-zero spectral densities exist only for $\pm\beta\alpha$. Furthermore, it can be shown that if only one cyclostationary source exhibits a cyclic frequency of α, then the cyclic spectral density evaluated at $\pm\beta\alpha$ contains contributions from that source alone. In theory, even noise should be absent. This has the effect of reducing a MIMO system at α=0 to a SIMO system at $\pm\beta\alpha$.

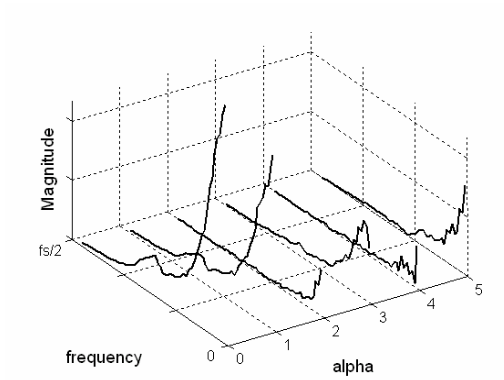

Figure 1 Cyclic spectral density

The cyclic spectral density is estimated using the averaged cyclic periodogram [7], and is expressed as:

$$S_{YX}^{(W)}(f;\alpha;L)=\tfrac{1}{K\Delta}\sum_{k=1}^{K}Y_{N_w}^{(k)}(f+\beta\alpha)X_{N_w}^{(k)}(f-\beta\alpha)^{*} \tag{1}$$

where α is the cyclic frequency, and β is any integer (±1,2,…). $Y_{N_w}^{(k)}$ and $X_{N_w}^{(k)}$ are the short time Fourier transforms of windowed sections of signal, frequency shifted by $\beta\alpha$, *i.e.*

$$Y_{N_w}^{(k)}(f)=\Delta\sum_{n=R}^{R+N_w-1}w_k[n]y[n]e^{-j2\pi fn\Delta} \tag{2}$$

where $y[n]$ is a section of the original signal y, stretching from sample number R to sample number $R+N_w-1$, and $w_k[n]$ is a window function of length N_w. The frequency shifted equivalent, $Y_{N_w}^{(k)}(f+\beta\alpha)$, is obtained by substituting $y[n]e^{-j\beta\alpha n}$ for $y[n]$ in (2) [8].

Estimation of the Cyclic Frequency

The cyclostationary properties of the signal can be used to obtain an accurate estimate of the cyclic frequency. The cyclostationary signal can be modelled as a centred, stationary random process multiplied by a periodic window function, which in the frequency domain may be expressed as [9]:

$$\bar{X}_v(f) = \sigma_r^{\,2} W_k(f) * W_k(f) \tag{3}$$

where * represents convolution, $\sigma_r^{\,2}$ is the variance of the random component x_r, which by definition is a constant, and W_k is the spectrum of the periodic window function. So, $\bar{X}_v(f)$ is a scaled convolution of the spectrum of the window with itself. Since the spectrum of the window only contains harmonics of the window frequency, so does the convolution, i.e. it correlates every time the spectrum is displaced by the harmonic spacing, producing a harmonic series. The sample number of the first harmonic peak corresponds with the cyclic frequency α.

Separation in the Cepstrum Domain

Assuming that only one cyclostationary source excites the system with the cyclic frequency α, then it can be shown that the cyclic cepstrum of the response $C_y^{\pm\alpha}(\tau)$ is an estimate of the cepstrum of the transfer function $C_h(\tau)$ [9]:

$$C_y^{\pm\alpha}(\tau) \approx C_h(\tau), |\tau| > \tau_0 \tag{4}$$

provided that the second order cyclostationary component of the input is frequentially smooth. The system poles and zeros can then be obtained from curve fitting the cyclic spectral density in the positive quefrency region [4].

An improved result may be obtained by including more than one cyclic frequency in the analysis. In this case, the average of the cepstra from a number of harmonics of the fundamental cyclic frequency can be used, i.e.

$$c_h(\tau) \approx \frac{1}{N_\alpha} \sum_{i=1}^{N_\alpha} c_y^{\alpha_i}(\tau) = \frac{1}{N_\alpha}\left(c_y^{1/T}(\tau) + c_y^{-1/T}(\tau) + c_y^{2/T}(\tau) + c_y^{-2/T}(\tau) + \ldots\right) \tag{5}$$

RESULTS

Experiment Set-up

The car was tested in finished condition, supported on its own suspension, as represented in Figure 2.

Figure 2 Recording measurements on the Saker race car

The engine was run at approximately constant speed and the structural responses were recorded at eleven locations on the chassis of the car using magnet mounted accelerometers. Each measurement consisted of ten minutes of recording at a sampling frequency of 4096Hz. The engine tachometer signal was also recorded and used to resample the response measurements to overcome any slight variation in engine speed.

Estimation of the Cyclic Spectrum

The cyclic spectrum for one response measurement is plotted for a range of frequency shifts from $\frac{-2.5}{T}$ to $\frac{2.5}{T}$ in Figure 3. It can be seen that non-zero cyclic spectra exist only for the first harmonics of the cyclic frequency, i.e. $\alpha = \frac{\pm n}{T}$, $n \in \mathbb{Z}$. A zero cyclic frequency shift produces the ordinary PSD, as mentioned above.

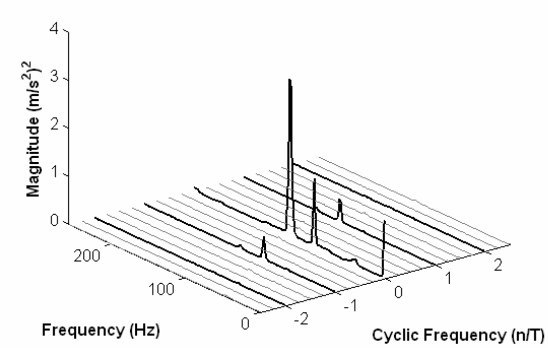

Figure 3 Cyclic spectrum of a typical response for frequency shifts from $\frac{-2.5}{T}$ to $\frac{2.5}{T}$

The cyclic spectrum was initially polluted by a periodic input of unknown origin which made identification of the modes very difficult. This may have come from some of the other engine components which are also locked to engine speed, i.e.

with the same cyclic frequency, but this is yet to be determined. Improved results were obtained using principal component analysis of the cyclic cross spectral density matrix. In this process, only the largest singular value and corresponding vectors (assumed to correspond with the engine combustion) were used to regenerate the cyclic response spectra due to the largest excitation, presumably the engine combustion. These were then curve fitted in the cepstrum domain and the FRFs between each response location and a reference location were regenerated, as represented in Figure 4.

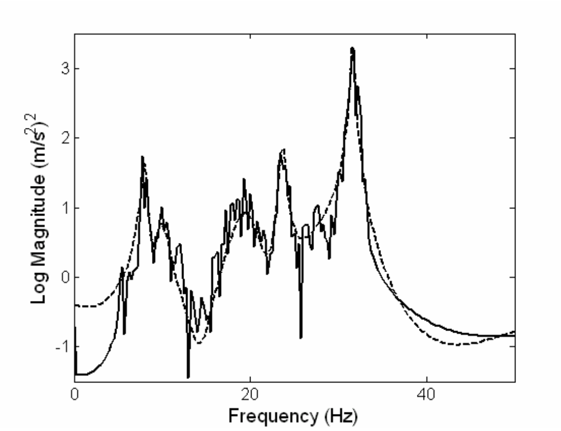

Figure 4 Typical regenerated FRF (dashed line) and cyclic spectrum (solid line)

Estimated Modal Properties

The modal properties of the Saker race car estimated using both the cyclostationary + cepstrum technique described in this paper, and using the industry standard Operational PolyMAX software, are contained in Table 1.

As can be seen, the cyclostationary + cepstrum OMA technique estimates the resonance frequencies quite accurately, but the damping estimates differ. The Operational PolyMAX software did not detect the bounce mode which was of much smaller magnitude than the other modes in this range.

Table 1 Modal properties of the Saker race car estimated using the cyclostationary + cepstrum technique and using the Operational PolyMAX software

Mode	Cyclostationary + Cepstrum		Operational PolyMAX	
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)
Bounce	5.4	2.78		
Pitch	7.74	0.974	7.86	0.47
Roll	9.62	2.06	9.97	3.62
Torsion	19.3	3.21	19.73	1.48

The mode shapes estimated by both techniques are represented in Figure 5 and

Figure 6.

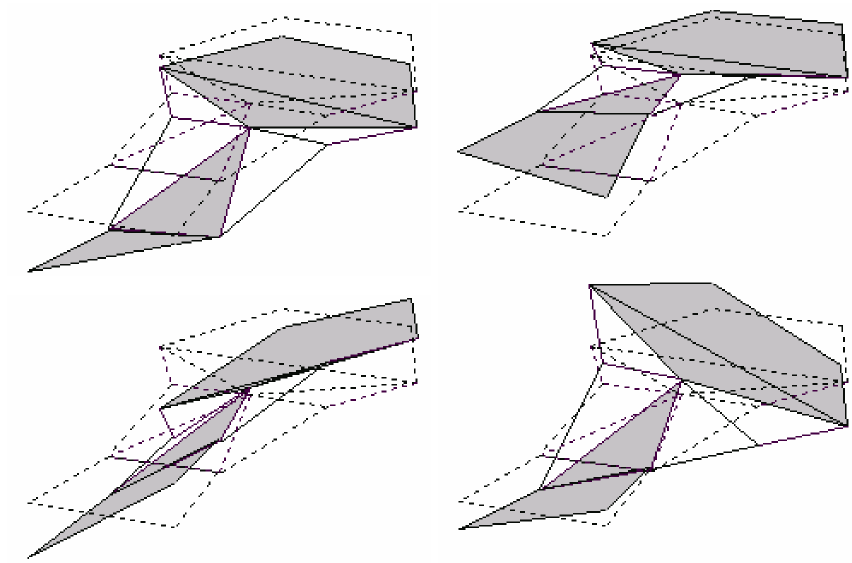

Figure 5 Mode shapes of the car identified using the cyclostationary + cepstrum technique; (clockwise from top left) bounce, pitch, torsion, roll (- - - undeformed structure)

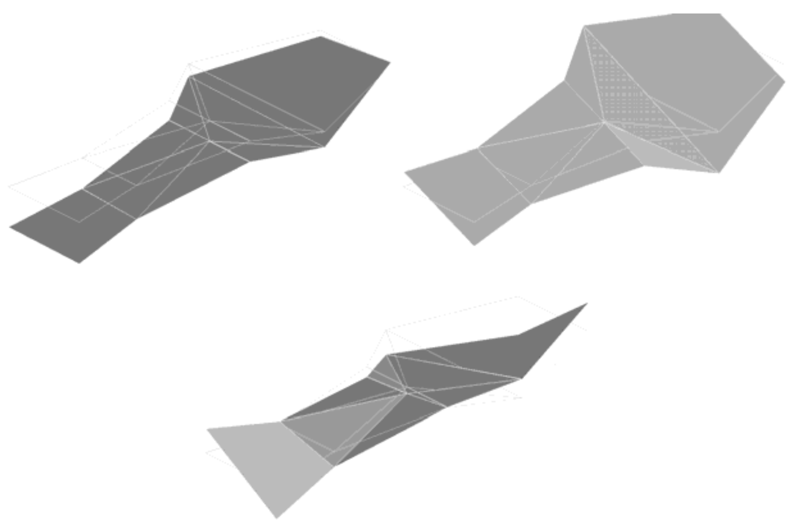

Figure 6 Mode shapes of the car identified using the Operational PolyMAX software from LMS; (clockwise from top left) pitch, roll, torsion

The mode shapes from the cyclostationary + cepstrum technique are relatively well represented with the exception of one or two spurious points, such as the near corner in the bounce mode. It may be possible to improve both the mode shape and damping estimates with an improved cyclic spectrum estimate.

Comparison of these figures reveals good correlation between the mode shapes estimated using Operational PolyMAX and the cepstrum + cyclostationary technique.

SUMMARY

This paper presented an application of a new operational modal analysis technique to measurements taken on the Saker race car. The technique used the engine as the excitation by exploiting the cyclostationary properties of the combustion signal. By estimating the cyclic spectrum of the response measurements at the cyclic frequency of the combustion cycle, the system was effectively reduced from a multiple-input-multiple-output scenario, with excitation from background noise, on board equipment etc, to a single-input-multiple-output scenario with excitation from the engine alone. These SIMO responses were then curve fitted in the cepstrum domain where the transfer function was separated from the relatively broadband excitation.

The mode shapes and resonance frequencies identified using this technique closely matched those obtained using the industry standard Operational PolyMAX software from LMS. The damping estimated could be improved however, possibly with further refinement of the estimation of the cyclic spectrum.

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