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AN OPTIMIZATION METHOD FOR REDUCING RESIDUAL VIBRATIONS OF COMPLIANT MANIPULATORS

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Abstract

The paper describes a motion planning procedure that allows to eliminate or reduce the vibratory effects arising on the end-effector of an industrial robot after the execution of high-speed movements. The adopted technique employs a mathematical model of the manipulator, a particular class of parametric motion profiles and an optimization procedure that acts like a motion planner and selects automatically, for each degree of freedom of the machine, the motion laws that allow vibration reduction.

INTRODUCTION

Vibration of the end-effector is a problem that frequently takes place during the operation of an industrial robot, in particular during the execution of high-speed movements. In many cases the gripper oscillations persist even after the end of the motion time (residual vibrations) and therefore they need overtime to be damped.

It is clear that these vibratory effects are due, for the most part, to the compliance of the mechanical transmissions, since the links of the robot can be considered substantially rigid.

A great number of industrial processes can not be carried out if the manipulators exhibit a vibratory behaviour: so it is very important to develop motion strategies, that allow these machines to execute fast movements with good dynamic performances, and high positioning precision.

The motion planning procedure described in this paper has been developed in order to eliminate or to reduce the residual vibrations of a manipulator, without augmenting the motion time, nor introducing overtime for vibration damping (clearly both these solutions reduce the rhythm of production of the industrial plant where the robot operates, with obvious economical consequences).

The proposed technique is based on a suitable choice of the motion profiles assigned to the robot arms. As it will be clarified in the following paragraphs, the motion of each link is defined through some coefficients that determine the shape of the displacement, velocity and acceleration functions; these coefficients can be modified without changing the motion time (usually imposed by production requirements), nor altering the continuity conditions imposed on the displacement and on its time derivatives at the initial an final time instants.

This calculation procedure can be used if the end-effector doesn't have to follow a pre-defined trajectory inside the workspace and therefore it is well-suited for "pick and place" operations, where only the initial and final position of the robot gripper are assigned.

It is important to underline that this technique can be implemented on an actual machine with very low costs; namely it is not necessary to modify the mechanical structure of the machine, neither to use complex control algorithms for the servomotors or additional feedback sensors to measure the vibration amplitude: it is only sufficient to modify the reference motion profiles memorized in the robot controller to improve the dynamic performances of the manipulator.

MODEL OF A ROBOT WITH COMPLIANT JOINTS

As it has been already outlined, the elasticity of a robot manipulator is concentrated for the most part, into the mechanical transmission. For example the SCARA robots (see Fig. 1) have a slightly compliance in the horizontal plane (the acronym SCARA means: **S**elective **C**ompliance **A**ssembly **R**obot **A**rm); in fact the end-effector of the robot, if forced, can move slightly in the horizontal plane (the one containing the two links), but non in the vertical one. This depends on the structural characteristics of the robot and on the torsional elasticity of the Harmonic-Drive (HD) speed reducers [2].

It must be considered that a certain level of compliance is deliberately introduced in the mechanical transmission between the motors and the robot arms, in order to compensate little positioning errors: that happens, for example, when a manipulator must insert a cylindrical piece into the corresponding hole: if there is a misalignment between the axes of the two parts, the insertion operation can be made easier by the robot compliance.

Fig. 2 shows the transmission of a SCARA robot and its schematization through a simple model with lumped parameters: in this model the elastic properties are defined by an equivalent torsional stiffness k_i and the energy dissipations, due to friction effects, are taken into account by introducing a viscous damper, with damping constant c_i. We will use these parameters to calculate the driving torques of the robot actuators.

Owing to the elasticity of the transmissions, the motion of each link of the manipulator is slightly different the motion imposed by the corresponding actuator; these effects can be studied through a dynamic analysis of the machine. For this purpose it is necessary to deduce the motion equations of the mechanical system and to carry out a numerical integration, in order to calculate the motion of each degree of freedom of the manipulator. The calculations will be here presented for a SCARA robot, but they can be extended to other types of manipulators. The motion equations will be written using the method described in [3]. For the meanings of the symbols, see Fig. 1a and Table 1.

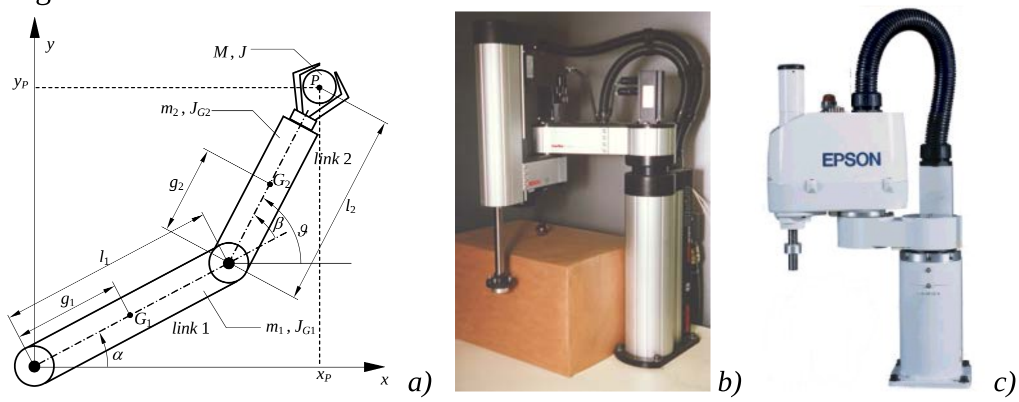


Figure 1 – a) Scheme of a SCARA robot. b) Bosch Turbo SCARA SR60 robot. c) Epson E2C25 SCARA robot.

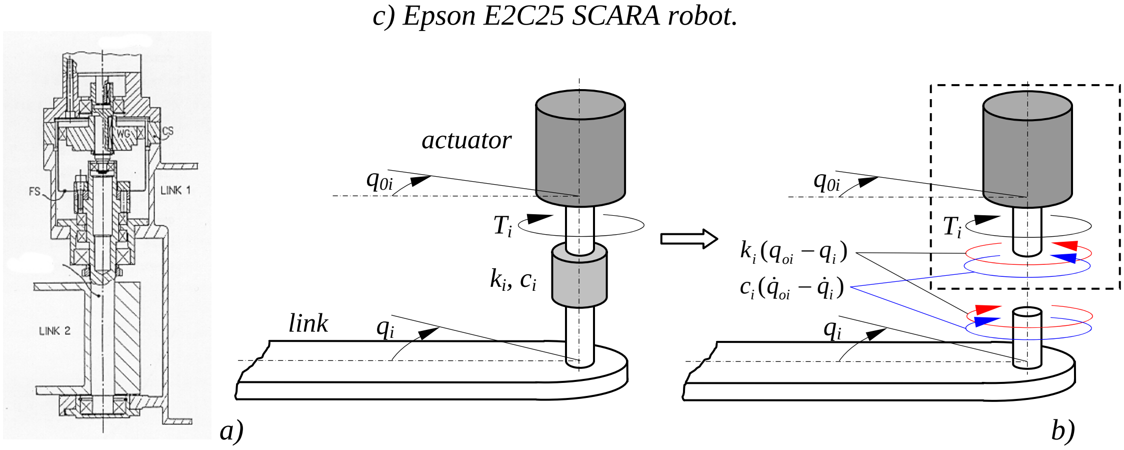


Figure 2 – The transmission between the motor and the link (a) and its equivalent model (b).

Symbol		**Description**	**Value**		**SI Unit**
m_1	m_2	*Links masses*	15	12	*kg*
J_{G1}	J_{G2}	*Links moments of inertia about their centres of mass*	0.24	0.11	*kg m*^{2}
M		*End effector mass (gripper + payload)*	5		*kg*
J		*End effector moment of inertia about its c.m.*	9×10^{-3}		*kg m*^{2}
l_1,	l_2	*Links lengths*	0.45	0.35	*m*
g_1	g_2	*Centre of mass positions of the links*	0.25	0.20	*m*
k_1	k_2	*Joints torsional stiffness*	12500	10500	*Nm/rad*
c_1	c_2	*Joints damping constants*	8	6	*Nms/rad*
x_{Pi}	y_{Pi}	*End effector initial position*	0.7	0.1	*m*
x_{Pf}	y_{Pf}	*End effector final position*	−0.3	0.2	*m*

Table 1 – Parameters of the SCARA robot used for the numerical simulations.

To write the motion equations of the robot it is first necessary to define a vector **S,** containing the geometrical coordinates of the points where the external and inertial actions are applied; in our case this vector contains the Cartesian coordinates of the points P (end-effector), G_1 (centre of mass of the 1st link) and G_2 (centre of mass of the 2nd link) and the angular coordinates α (absolute position of the 1st link) and $\vartheta = \alpha + \beta$ (absolute position of the 2nd link); these angular coordinates are needed for taking into consideration the effects of the inertial torques acting on the links. Consequently we obtain for the vector **S** the following expression:

$$\mathbf{S} = [x_P \quad y_P \quad x_{G2} \quad y_{G2} \quad \vartheta \quad x_{G1} \quad y_{G1} \quad \alpha]^T \tag{1}$$

Now let's introduce the vector of the joint coordinates **Q**, that contains the angles used to define the relative position of each link, that is the position of a link with respect the preceding one (see Fig. 1a):

$$\mathbf{Q} = [\alpha \quad \beta]^T \tag{2}$$

If there are external actions applied to the robot, they have to be included in a vector $\mathbf{F_{se}}$; for the case under consideration this vector is null ($\mathbf{F_{se}} = \{0\}$).

As concern the motor torques applied to joints of the manipulator, their analytical expression can be determined using the equivalent scheme illustrated in Fig. 2b. Let us denote with the symbols q_{0i} and q_i respectively the angular position of an actuator and the position of the corresponding link (each actuator is composed by a servomotor coupled with a speed reducer); by imposing an equilibrium condition on the part inside the dashed rectangle, the driving torque T_i can be calculated as

$$T_i = k_i (q_{0i} - q_i) + c_i (\dot{q}_{0i} - \dot{q}_i) \qquad i = 1, 2 \tag{3}$$

where k_i is the equivalent torsional stiffness of the transmission and c_i the equivalent viscous damping coefficient. In our case Eqs. (3) can be rewritten as

$$\mathbf{F_q} = \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{Bmatrix} k_1(\alpha_0 - \alpha) + c_1(\dot{\alpha}_0 - \dot{\alpha}) \\ k_2(\beta_0 - \beta) + c_2(\dot{\beta}_0 - \dot{\beta}) \end{Bmatrix} \tag{4}$$

where $\mathbf{F_q}$ is the vector of the driving actions.

The inertial properties of the robot can be expressed through the mass matrix **M**, that, for the system under consideration, is defined as

$$\mathbf{M} = diag[M \quad M \quad m_2 \quad m_2 \quad (J + J_{G2}) \quad m_1 \quad m_1 \quad J_{G1}] \tag{5}$$

This is a $n \times n$ diagonal matrix, whose dimension n is equal to the number elements of contained in the vector **S**.

The position coordinates in the vectors **S** and **Q** are related by geometrical relationships, that can be rewritten in compact form as:

$$\mathbf{S} = \mathbf{S}(\mathbf{Q}) \tag{6}$$

Consequently the jacobian matrix **J** is:

$$\mathbf{J} = \frac{\partial \mathbf{S}}{\partial \mathbf{Q}} \tag{7}$$

Now, if we define the following matrices:

$$\overline{\mathbf{M}}(\mathbf{Q}) = \mathbf{J}^{\mathbf{T}}\mathbf{M}\,\mathbf{J} \qquad \mathbf{V}(\mathbf{Q},\dot{\mathbf{Q}}) = \mathbf{J}^{\mathbf{T}}\mathbf{M}\,\dot{\mathbf{J}}\,\dot{\mathbf{Q}} \qquad \mathbf{G}(\mathbf{Q},\mathbf{F}) = -(\mathbf{J}^{\mathbf{T}}\mathbf{F}_{\mathbf{se}} + \mathbf{F}_{\mathbf{q}}) = -\mathbf{F}_{\mathbf{q}} \tag{8}$$

the motion equations of the robot can be expressed as:

$$\overline{\mathbf{M}}(\mathbf{Q})\ddot{\mathbf{Q}} + \mathbf{V}(\mathbf{Q},\dot{\mathbf{Q}}) + \mathbf{G}(\mathbf{Q},\mathbf{F}) = \{0\} \qquad \Rightarrow \qquad \overline{\mathbf{M}}(\mathbf{Q})\ddot{\mathbf{Q}} + \mathbf{V}(\mathbf{Q},\dot{\mathbf{Q}}) = \mathbf{F}_{\mathbf{q}} \tag{9}$$

Solving Eq. (9) with respect to $\ddot{\mathbf{Q}}$ we obtain:

$$\ddot{\mathbf{Q}} = \overline{\mathbf{M}}(\mathbf{Q})^{-1}[\mathbf{F}_{\mathbf{q}}(\mathbf{Q}) - \mathbf{V}(\mathbf{Q},\dot{\mathbf{Q}})] \tag{10}$$

If we know the motion of the actuators, that is the functions $\alpha_0(t)$, $\beta_0(t)$ and their time derivatives (see the following paragraph for a detailed description of these functions), the motion equations (10) can be integrated without particular difficulties, using a numerical procedure (for example a Runge-Kutta algorithm with fixed or adaptive step size). Thus it is possible to analyse the dynamic behaviour of the system and to study the effects of the adopted motion strategy on the residual vibration of the end-effector.

DEFINITION OF THE MOTION PROFILES

After deducing the motion equations of the manipulator, it is needed to define the motion laws for each DOF of the machine.

We have already stated that such profiles have to be defined in parametric form, so that it is possible to modify their shape by changing the numerical values of some coefficients, without altering the boundary conditions (i.e. the conditions at the limits of the integration interval).

In the technical literature we can find many classes of motion profiles which depend on a set of coefficients: the functions that define these profiles may have a single analytic expression, valid for the total motion interval (like the polynomial motion laws), or different expressions inside this interval (like the motion laws with a seven segments acceleration profile). In this work we will use a modified version of the polynomial motion laws, where the exponents of the various terms may assume real (rather than integer) positive values.

These exponents represent the design parameters, that is the variables of the optimization procedure and they must be determined in such a way as to minimize a performance index, that will be defined in the next paragraph.

To define a generic displacement profile $y(t)$, it is firstly necessary to fix some boundary conditions, which must be fulfilled by the function itself and by its time derivatives up to order m. If we denote with the symbols h and T respectively the required displacement and the corresponding motion time, the conditions that are usually imposed are:

$$\begin{array}{ll} y(0)=0 & y(T)=h \\ \dot{y}(0)=0 & \dot{y}(T)=0 \\ \vdots & \vdots \\ y^{(m)}(0)=0 & y^{(m)}(T)=0 \end{array} \qquad (11)$$

If we use the following expression for $y(t)$:

$$y(t)=h\left[1-\sum_{j=0}^{m} C_j\left(1-\frac{t}{T}\right)^{\lambda_j}\right] \qquad (12)$$

where λ_j is a real positive number, the k^{th} order time derivative ($1 \le k \le m$) is:

$$y^{(k)}(t)=(-1)^{k+1}\frac{h}{T^k}\left\{\sum_{j=0}^{m}\left[\prod_{r=0}^{k-1}(\lambda_j-r)\right]C_j\left(1-\frac{t}{T}\right)^{\lambda_j-k}\right\} \qquad (13)$$

From Eqs. 12 and 13 we deduce immediately that the conditions (11), pertinent to instant $t = T$, are always fulfilled; the remaining $m + 1$ conditions (pertinent to instant $t = 0$) can be used to determine the coefficients C_j ($j = 0, 1,\dots,m$), that will result dependent on parameters λ_j. These coefficients are the solution of the following linear system of equations:

$$\begin{cases} \displaystyle\sum_{j=0}^{m} C_j = 1 \\ \displaystyle\sum_{j=0}^{m}\left[\prod_{r=0}^{k-1}(\lambda_j-r)\right]C_j = 0 \qquad k=1,\dots,m \end{cases} \qquad (14)$$

For the SCARA manipulator described in the previous paragraph, the arm rotations $\alpha_0(t)$ and $\beta_0(t)$ can be defined by using Eq. 12; the motion time T is fixed according to the technological and manufacturing requirements of the plant, where the robot is installed, and the execution of the movement is performed by moving the two axes of the machine simultaneously.

PERFORMANCE INDEX AND OPTIMIZATION

During assembly and manipulation operations the tracking of an assigned trajectory is not required: therefore the motion profiles of the actuators can be selected with a certain degree of freedom, in order to optimize the dynamic performances of the manipulator. For example, to eliminate the residual vibrations of the end-effector, it is sufficient that the total system energy E_{tot}, at the time instant T is null; if this is not possible, we can impose that the value of E_{tot} is reduced to a minimum. In this case the residual oscillations will be in any case strongly reduced, even if they will not be completely eliminated. The total system energy can be calculated by adding the kinetic energy E_{kin} to the potential elastic energy E_{pot} due to the joint deformability, that is:

$$E_{tot} = E_{kin} + E_{pot} = \frac{1}{2}\dot{\mathbf{Q}}^T \overline{\mathbf{M}} \dot{\mathbf{Q}} + \frac{1}{2}[k_1(\alpha - \alpha_0)^2 + k_2(\beta - \beta_0)^2] \tag{15}$$

where the position and velocity variables must be calculated at the time instant T.

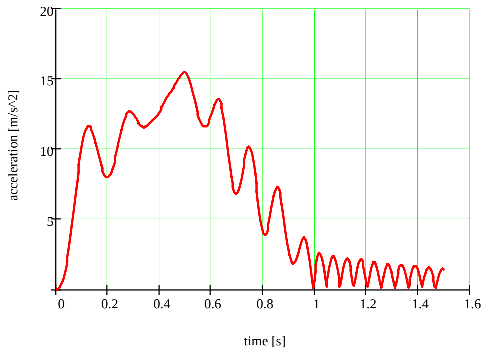
The value of E_{tot}, which is our target function, changes when the motion profiles are modified; having at our disposal two profiles (one for each robot axis), defined in parametric form through the shape coefficients, the this function will depend also on these parameters, which will be opportunely selected, to obtain a minimum.

It may be observed that this is a typical problem of non linear mathematical programming; for the solution of these problems some efficient algorithms are available [5], which allow to determine a local minimum for the selected target function. For the case under consideration an optimization procedure has been implemented using the conjugate gradient method, available inside a software package addressed to the solution of non-linear optimization problems.

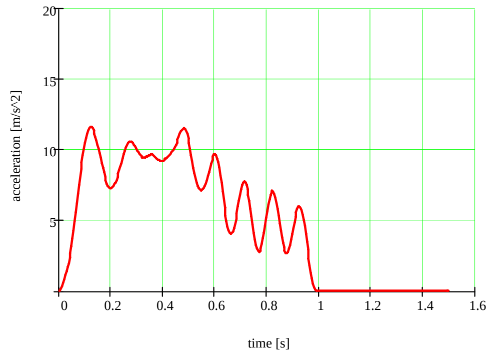
To demonstrate the efficiency of the proposed approach, we show here the results of a dynamic analysis of the SCARA robot, using different motion profiles as input. Fig. 3 represents the modulus of the end-effector acceleration versus time before (Fig. 3a) and after (Fig. 3b) the optimization procedure.

The calculations were performed for a motion time $T = 1$ s, using the parameters indicated in Table 1 and setting $\lambda_1 = \lambda_2 = \{3 \;\; 4 \;\; 5\}^T$ as initial values of the shape coefficients. The values of these parameters at the end of the iterative process for the search of the minimum were $\lambda_1 = \{4.072 \;\; 3.734 \;\; 2.708\}$ and $\lambda_2 = \{4.512 \;\; 5.369 \;\; 6.187\}$.

The effects of the reference motion profiles on the residual vibrations can be clearly observed by the comparison between the diagrams; as it may be noted, the modulus of the acceleration is null for $t > T$, if the optimized motion laws are used.



a)



b)

Figure 3 – Modulus of the end-effector acceleration before (a) and after (b) the optimization procedure. Note the absence of vibrations for t > 1 s.

CONCLUSIONS

The work presented in this paper shows that it is theoretically possible to improve the performances of a manipulator with non rigid transmission by means of an opportune planning of the motion strategy.

The proposed method is based on a mathematical model that reproduces the dynamic behaviour of a robot with elastic transmission. The motion profiles used as reference functions for the actuators are easily definable in parametric form, through few numerical coefficients that define their shape; for this purpose we can utilize the profiles derived by the standard polynomial function. The solution of the optimization problem may be obtained by means of extensively tested algorithms, now available inside commercial software. The optimization procedure is economical and easy to implement on an actual robot, since it requires only a modification of the motion profiles memorized in the motion controller of the machine.

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