



## **PIEZOELECTRIC VIBRATOR FOR MEDICAL VIBROACOUSTIC DIAGNOSTICS**

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### **Abstract**

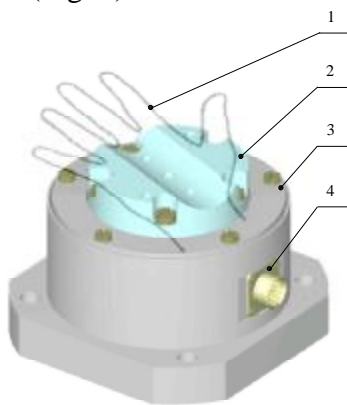
Problems of calculation and design of piezoelectric vibrator for new methods of medical diagnostics based on excitation of vibrations of human body, joints, muscles and ligaments are considered in this paper. A mathematical model of a vibrator is presented. The model allows one to design a vibrator piezotransducer both of resonance and wide-band types and to calculate vibrator characteristics. Results of calculations of the electroacoustic coefficient and optimal design parameters of a vibrator for the quasistatic load up to 2000 N in the wide frequency band (from few Hz to tens kHz) are presented. Our calculations confirm the principal possibility of creation of medical vibrators for the new methods of vibroacoustic diagnostics.

### **INTRODUCTION**

The acoustic method of medical diagnostics, viz. percussion, and related resonance vibration method are widely known. They imply listening of sounds from different body regions generated by percussion or other sound vibration generators. The tone of a percussion sound, i.e. resonance response from internal parts being listened, provides information on conditions and tonography of such organs, as liver, kidneys etc., which do not emit sound alone. In late nineties, a new percussion method has been proposed, the locaphony [1]. It enabled to obtain 2-D acoustic images of human internal parts' pathology with accuracy of 2 to 3 mm. The latter is determined by the size of the acoustic receiving sensor. The frequency range of the initiating sound signal is 300 Hz to 3000 Hz. A number of other methods of vibration medical

diagnostics are known, such as impedance mechano-myography, which uses the forced (nonresonant) mode excitation of surface tissue layers and measuring mechanical impedance, reflecting the elastic and viscous properties of those tissues. These methods use various vibrating-reed converters, which excite vibration signals of a required level and frequency in the minor area of the human body (up to 2-10 cm<sup>2</sup>). Similar vibrating-reed converters have been widely used in Russia since 1994 to treat various deceases /Ref. Vytaphon vibro-acoustic device, Spec. 9444-003-33159359-95/.

It is a question of principle, that the parameters of the signal, excited in the human body, shall not depend on the loaded mass up to the whole body mass (up to 120-150 kg). One of design versions of such a medical vibrator, operating in acoustic range, is proposed in this report (Fig. 1).



*Figure 1 – Medical vibrator: physical configuration*

1 – symbolic pattern of hand; 2 – moving armature (plate); 3 – fixed armature (casing);  
4 – electric connector

## **DESIGN DESCRIPTION**

A piezoelectric transducer, is secured in a casing, and comprises an elastic member with a piezoelectric cell (PE) glued to its lower surface. The elastic member is made of 36NChTYu alloy as a flat diaphragm with a central cylindrical stem, and the piezoelectric cell is made as a disk of PZT-19 piezoceramic material with electrodes on its ends.

For vibrator's excitation constant or oscillatory voltage of some hundreds volts or a specified pulse are applied to the piezoelectric cell. The patient puts the measured limb, such as hand, to the vibrator bracket (plate) with the receiving electroacoustic transducer attached, to listen the response to applied mechanical agitation.

## **MATHEMATICAL MODEL**

To develop a vibrator operating in a wide frequency range from several Hz to several tens of kHz it should be taken into account the transducer's resonant mode and how

the load of the patient's body part affects the amplitude-frequency response. The problem has been solved using mathematical simulation and piezoelectric transducer design optimization, based on methods described in [2, 3]. These methods imply PE electric elasticity calculations at three-dimensional stress of the transducer with allowance for patient's limb acoustic properties and electric signal generator parameters.

The boundary equations may be divided into two groups of conjugate field conditions on the PE boundary, viz. mechanical and electrical ones. The boundary conditions for the mechanical component are reduced to usual relations of the elasticity theory. Thus, if on surface of the body the vector of mechanical stresses is specified, it is necessary to take advantage of dependencies of the generalized Hooke's law and to equate stresses on the surface of body to their specified values. The obtained three equations relate components of displacements vector and electrostatic potential:

$$S_r^p = \frac{s_{11}^E}{(s_{11}^E)^2 - (s_{12}^E)^2} \cdot \left( \frac{\partial^2 u_z}{\partial r^2} - \frac{s_{12}^E}{s_{11}^E \cdot r} \cdot \frac{\partial u_z}{\partial r} \right) - \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot E_z \quad , \quad (1)$$

$$S_q^p = \frac{s_{11}^E}{(s_{11}^E)^2 - (s_{12}^E)^2} \cdot \left( \frac{1}{r} \cdot \frac{\partial u_z}{\partial r} - \frac{s_{12}^E}{s_{11}^E} \cdot \frac{\partial^2 u_z}{\partial r^2} \right) - \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot E_z \quad , \quad (2)$$

$$D_z = \frac{d_{31}}{s_{11}^E + s_{12}^E} \cdot \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u_z}{\partial r} \right) + \left( e_{33}^S - \frac{2 \cdot d_{31}^2}{s_{11}^E + s_{12}^E} \right) \cdot E_z \quad , \quad (3)$$

where  $S_r^p, S_q^p$  - radial and tangential mechanical stresses in PIC respectively;  $s_{11}^E, s_{12}^E$  - elastic flexibilities of piezoceramics at constant electric field;  $d_{31}$  - piezoelectric module of piezoceramics;  $E_z$  - electric field strength over thickness;  $D_z$  - electrostatic flux density,  $e_{33}^S$  - permittivity constant of PE material at constant strain.

Taking into account the design, a one-dimensional electric field  $E_z$  is assumed in the proposed mathematical model, while the field strength can be determined as an average over the PE surface with electrodes  $S_p$ :

$$\bar{E} = \frac{1}{S_p} \iint_{S_p} E_z dS \quad . \quad (4)$$

Then, in respect the relation of potential drop  $(j_1 - j_2)$ , electric field strength  $\bar{E}$ , current  $I$  and generator's full resistance  $Z_e$ ,  $(j_1 - j_2 = \int \bar{E} dz = \frac{1}{S_p} \iiint_V E_z dV = -I \cdot Z_e)$ , we obtain the relation that is a basic one

for the further analysis:

$$I = \frac{-1}{S_p \cdot Z_e} \iiint_V E_z dV, \quad (5)$$

where  $V$  is the PE volume.

The electrical boundary condition is formulated for the electrical component of conjugate field, and it depends on how electrical energy is picked off PE. With allowance for the adopted electric field one-dimensional hypothesis flux density  $D$  is proportional to current  $I$  through the PE:

$$D = \frac{I}{j \cdot w \cdot S_p}, \quad (6)$$

where  $w$  is the circular frequency.

The PE to the generator output connection condition can be expressed as:

$$j_1 - j_2 = U_e - I \cdot Z_e, \quad (7)$$

where  $U_e$  is the idle generated voltage of the electric signal generator.

Developing relation (5), substituting into equation (7), and integrating over  $z$  we obtain the electric boundary condition:

$$U_e = -\frac{I \cdot Z_p}{1 - k_p^2} + I \cdot Z_e + \frac{(h_p^2 - 2h_p \cdot c) \cdot k_p^2}{2d_{31} \cdot (1 - k_p^2) \cdot R_p^2} \cdot \int_{r_0}^{R_p} \left( r \frac{\partial^2 u_z}{\partial r^2} + \frac{\partial u_z}{\partial r} \right) dr, \quad (8)$$

$$Z_p = \frac{1}{j \cdot w \cdot C_0}, \quad (9)$$

$$k_p = \sqrt{\frac{2d_{31}^2}{e_{33}^T \cdot (s_{11} + s_{12})}}, \quad (10)$$

where  $Z_p$  - electric impedance of PIC,  $k_p$  - radial electromechanical relation ratio;  $c$  - distance from the neutral surface to the Y-axis origin (to the PIC/elastic member gluing surface),  $C_0$  - compressed component's capacity,  $e_{33}^T$  - permittivity at constant mechanical stress.

In case of calculation of a stepped piezoelectric transducer formed by multilayer plates plate areas with constant thickness shall be considered separately (Fig. 2). The balance of displacement, rotational displacement, moments, radial moments, and cutting forces on correspondent plate edges are the mechanical conditions. Therefore, for the considered piezoelectric transducer in form of a double-layered plate, rigidly mounted on the outer and inner edges and loaded with force  $Q$ , the

boundary conditions are equated for two areas:

- I* – double-layered area (bimorph transducer) with thickness  $h$ , bounded by cylinder sections with radii  $r_0$  and  $R_p$ ;
- II* – single-layered area with thickness  $h_m$ , bounded with radii  $R_p$  and  $R_0$ .

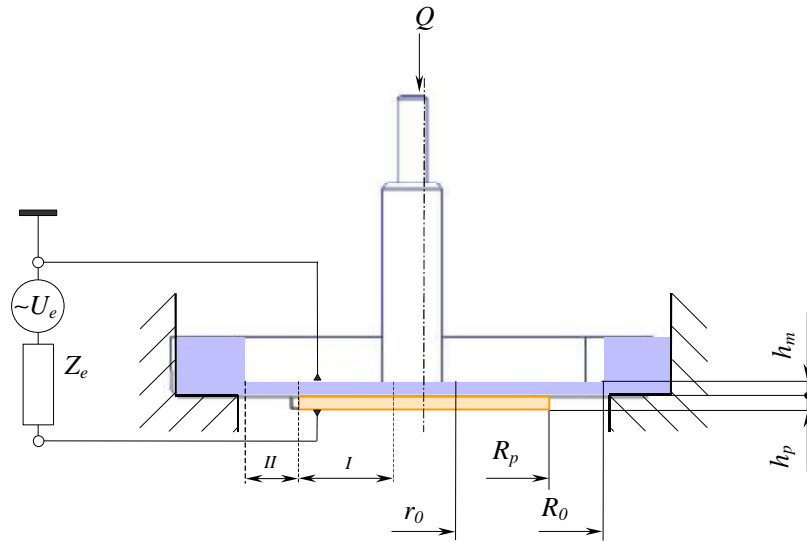


Figure 2 - Piezoelectric transducer of the vibrator: design model

$U_e$  - idle generated voltage of electric signal generator;  $Z_e$  - internal impedance of the electric signal generator;  $Q$  - load force,  $r_0$  - inner transducer circuit radius (elastic element stem radius);  $R_p$  - piezoelectric cell radius;  $R_0$  - outer transducer circuit radius (elastic diaphragm radius);  $h_m$  - elastic diaphragm thickness;  $h_p$  - piezoelectric cell thickness

In this case the boundary equation system comprises the following conditions:

- zero  $u_z''$  displacement and angle displacement  $y'' = \frac{\partial u_z''}{\partial r}$  at the outer edge  $R_0$ ;
- equal displacement  $u_z^I = u_z''$ , angle displacement  $y^I = y''$ , radial bending moment  $M_r^I = M_r''$  and transverse cutting force  $Q_r^I = Q_r''$  on edge  $R_p$ ;
- load force  $Q$ , applied to edge  $r_0$ ;
- gravity forces;
- and electric boundary condition for transducer connection to generator output.

Thus, the following boundary equation system, relating the deformation tensor components with the electric current, may be written for the transducer as per Fig.2:

$$\left\{ \begin{aligned}
 &u_z^{II} \Big|_{r=R_0} = 0 \\
 &\frac{\partial u_z^{II}}{\partial r} \Big|_{r=R_0} = 0 \\
 &u_z^I \Big|_{r=R_p} = u_z^{II} \Big|_{r=R_p} \\
 &\frac{\partial u_z^I}{\partial r} \Big|_{r=R_p} = \frac{\partial u_z^{II}}{\partial r} \Big|_{r=R_p} \\
 &C_I \cdot \left( \frac{\partial^2 u_z^I}{\partial r^2} + \frac{m}{r} \frac{\partial u_z^I}{\partial r} \right) \Big|_{r=R_p} = C_{II} \cdot \left( \frac{\partial^2 u_z^{II}}{\partial r^2} + \frac{m_m}{r} \frac{\partial u_z^{II}}{\partial r} \right) \Big|_{r=R_p} \\
 &C_I \cdot \left( \frac{\partial}{\partial r} \left( \frac{\partial^2 u_z^I}{\partial r^2} + \frac{1}{r} \frac{\partial u_z^I}{\partial r} \right) \right) \Big|_{r=R_p} = C_{II} \cdot \left( \frac{\partial}{\partial r} \left( \frac{\partial^2 u_z^{II}}{\partial r^2} + \frac{1}{r} \frac{\partial u_z^{II}}{\partial r} \right) \right) \Big|_{r=R_p} \\
 &C_I \cdot \left( \frac{\partial}{\partial r} \left( \frac{\partial^2 u_z^I}{\partial r^2} + \frac{1}{r} \frac{\partial u_z^I}{\partial r} \right) \right) \Big|_{r=r_0} = \frac{Q}{2 \cdot p \cdot r_0} \\
 &C_{II} \cdot \left( \frac{\partial}{\partial r} \left( \frac{\partial^2 u_z^{II}}{\partial r^2} + \frac{1}{r} \frac{\partial u_z^{II}}{\partial r} \right) \right) \Big|_{r=R_0} = [Q + p \cdot g \cdot (r_m \cdot h_m + r_p \cdot h_p) \cdot (R_p^2 - r_0^2) + \\
 &\quad + p \cdot g \cdot r_m \cdot h_m \cdot (R_0^2 - R_p^2)] / 2 \cdot p \cdot R_0 \\
 &U_e = -\frac{I \cdot Z_p}{1 - k_p^2} + I \cdot Z_e + \frac{(h_p^2 - 2h_p \cdot c) \cdot k_p^2}{2d_{31} \cdot (1 - k_p^2) \cdot R_p^2} \cdot \int_{r_0}^{R_p} \left( r \frac{\partial^2 u_z^I}{\partial r^2} + \frac{\partial u_z^I}{\partial r} \right) dr
 \end{aligned} \right. \quad (11)$$

where  $C_I, C_{II}$  is cylinder rigidity of transducer in areas  $I$  and  $II$ ;  $g$  - intensity of gravity;  $r_m$  - elastic member density;  $r_p$  - piezoceramics density.

Taking into account a relative low mass of parts, secured to the elastic members stem, load force  $Q$  can be expressed via mechanical impedance of the patient's limb  $Z$ :

$$Q \approx j \cdot \omega \cdot Z \cdot u_z \Big|_{r=r_0}, \quad (12)$$

and at low frequencies through the mass of measured patient's part  $M$ :

$$Q \approx -\omega^2 \cdot M \cdot u_z \Big|_{r=r_0}. \quad (13)$$

The lower configuration of axially symmetric oscillations of a round plate with provision of gravity is given by:

$$\begin{aligned}
 u_z(r, \omega) = & B_1 \cdot J_0(I(\omega) \cdot r) + B_2 \cdot Y_0(I(\omega) \cdot r) + B_3 \cdot I_0(I(\omega) \cdot r) + B_4 \cdot K_0(I(\omega) \cdot r) - \\
 & - \frac{g \cdot r \cdot h \cdot r^4}{64 \cdot C}, \quad (14)
 \end{aligned}$$

$$I(w) = \sqrt[4]{\frac{r \cdot h \cdot w^2}{C}} \quad , \quad (15)$$

where  $B_{1,2,3,4}$  are constants, depending on boundary conditions;  $J_0(I(w) \cdot r)$ ,  $Y_0(I(w) \cdot r)$ ,  $I_0(I(w) \cdot r)$ ,  $K_0(I(w) \cdot r)$  - Bessel function;  $r$  - plate material density;  $h$  - plate thickness;  $C$  - plate cylinder rigidity.

The further calculations of the transducer's technical performance add to:

- the estimation of the position of the neutral surface of the multilayered plate and its cylinder rigidity;
- intermediate determination of oscillation magnitude  $u_z$  and current  $I$  from system (11);
- final calculation of electro-acoustical conversion efficiency and amplitude-frequency response.

The efficiency of conversion of voltage to acceleration and the amplitude-frequency response can be obtained by:

$$x_U = \frac{-w^2 \cdot u_z|_{r=r_0}}{U_e} \quad . \quad (16)$$

Fig. 3 shows the results of calculations together with the experimental values for a vibrator, operating in resonant mode. With  $\times$ -sign the arithmetic mean magnitudes of oscillations of vibrator prototypes are marked.

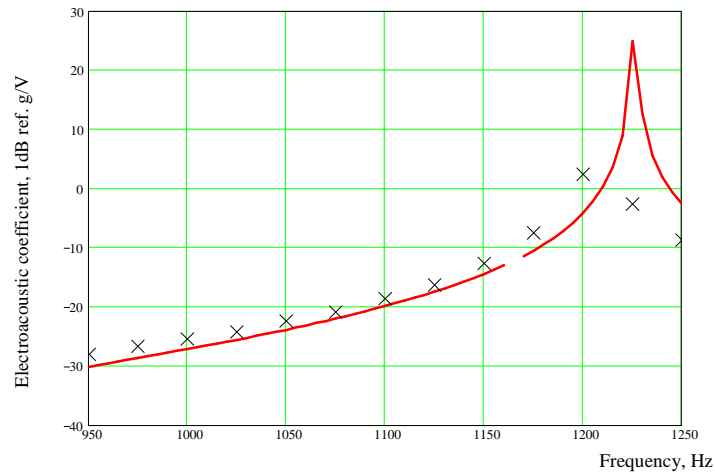


Figure 3 - Amplitude-frequency response characteristic (AFC) of the piezoelectric vibrator  
 3/4 - calculation; ' - experiment

In general the results of calculations meet the experimental data with the disagreement from  $-12,4\%$  up to  $-20,6\%$  with the exception of the area in the neighborhood of resonance (from 1175 Hz up to 1250 Hz), what shall be considered

as a satisfactory result of simulation, taking into account, that oscillation damping in the damping diaphragm pack (not shown in Fig. 2) and the rigidity of the arrangement for fastening of the transducer to the vibrator casing are neglected. This causes a certain overestimation of the calculated values: peak values of resonant oscillations by 22,5 dB and resonant frequency by 2.1%. These errors are predictable enough, so the calculation accuracy may be raised up to some percents, provided the mathematical model is updated with data for damping in the damping diaphragm pack and for fastening.

Considering possible elaborations and updating with any conditions, the presented mathematical model makes it possible to optimize the device's design and layout to predict its operation features at different loads. Using the simulation results the transducer sizes have been chosen, which provide at a quasi-static load under 2000 N, a working range from 300 Hz up to 3 kHz:  $r_0 = 5$  mm,  $R_0 = 28.75$  mm,  $-h_m = 2$  mm,  $R_p = 20$  mm, and  $h_p = 2$  mm. By changing the elastic member thickness only, it is possible to adjust the resonant frequency from several hundreds of Hz to several tens of kHz, providing a rather wide vibration range from several Hz to several tens of kHz using a number of some plug-in piezoelectric transducers in a single vibrator casing.

Such a simulation could be positively carried out using finite element software, such as ANSYS, PiezoFLEX, or ACELAN. However, the accuracy of such calculation is also in the range of some percents. That is why the piezoceramics is distinguished by a significant scatter of electric constants (up to 10%) both for different lots and within a single lot. As for a possibility of updating the mathematical model with any conditions (to the point of economic restrictions), demonstrativeness, and computation simplicity, the presented calculation algorithm seems to be preferable. The presented mathematical model of the piezoelectric transducer enables to optimize not only its design and layout parameters, but also the characteristics of the entire system: "measured object - piezoelectric transducer – signal generator".

## SUMMARY

A mathematical model of a piezoelectric transducer for a medical vibrator formed by multi-layered plates with bending oscillations is presented. The results of calculations satisfy the experimental data. An important advantage is a possibility to account for patient body load effect and to improve the device's acoustic matching.

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