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OUTDOOR PROPAGATION USING A HIGHER-ORDER PARABOLIC EQUATION

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Abstract

This paper shows that the Higher-Order Parabolic Equation (HOPE) ensures the computation of outdoor propagation at very large angles. The derivation of the HOPE based on the approximation of a square operator using a Padé expansion with a high order is presented. Its numerical implementation using a finite difference technique and the alternative directions technique is investigated. Comparisons with the wide-angle PE show that the HOPE improves the results at wide angles and at very long range. In contrast, the CPU time rises in proportion to the Padé order.

INTRODUCTION

Theoretical and experimental studies on long range sound propagation in the low atmosphere are investigated from few decades. Several complementary methods have been developed for predicting outdoor propagation. The ray-acoustics allows us to model realistic scenes. The gaussian beam approach is an alternative to the ray-approximation, including the diffraction effects. The Fast Field Program (FFP) is efficient when the meteorological conditions are independent of the range. More recently, the Euler's equations have been applied on short ranges to handle rotational flows. The Parabolic Equation (PE) remains well suited to predict long range sound propagation. Since the introduction of the PE to the outdoor propagation, many developments and comparisons with the experiments have been investigated. The PE has been applied with an atmospheric model to take into account the kinematic turbulence [3] and its capability to take into account the diffraction by a screen has been examined [12], [7]. Three-dimensional models have been implemented to include the azimuthal diffraction effects [6], [13]. The fast Green's function method reduces the CPU time for propagation above the ground [8]. The Wide-Angle Parabolic Equation (WAPE) has extensively been used for outdoor propagation. It provides accurate results, including a 40 degrees propagation aperture angle. However, in some cases a wider angle of propagation is required. For instance, to predict the sound pressure level on the ground, almost straight to an aircraft, during take-off or landing. This paper aims to show that the Higher-Order Parabolic Equation (HOPE) provides the outdoor propagation at very wide angles.

The code PARABOLE based on the WAPE has been developed at ONERA for the prediction of sound propagation above the ground [9], [11]. Recently, the HOPE has been implemented in PARABOLE to predict in-duct propagation in the high frequency range [10].

The derivation of the HOPE is briefly presented. Its numerical implementation using a finite difference scheme and the alternative directions technique is investigated. Comparisons between the WAPE and the HOPE are illustrated.

THE HIGHER ORDER PARABOLIC EQUATION

Derivation of the Higher-Order Parabolic Equation

Like the other PEs, the HOPE, based on the wave equation, neglects the back-scattered field and then approximates a square root operator *Q*. Let us consider the two-dimensional Helmholtz equation in heterogeneous atmosphere, with cylindrical co-ordinates (*r, z*):

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}+k_0^2N^2\right]p=0\,, \tag{1}$$

where r represents the range of propagation and z the altitude. $N = c_0/c(r,z)$ is the index of refraction, c_0 is a reference sound celerity (c_0 = 344 m/s) and $c(r, z)$ is the local sound speed depending on the wind speed and the temperature. k_0 is a reference wave number. The acoustic pressure is written in the form $p(r,z) = u(r,z)/r^{1/2}$. Applying the far-field approximation ($k_0 r >> 1$), equation (1) becomes

$$\left[\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial z^2}+k_0^2N^2\right]u=0 \tag{2}$$

The linear partial differential [*P*] and [*Q*] operators are introduced:

$$[P]=\frac{\partial}{\partial r} \text{ and } [Q]=[1+X]^{1/2} \text{ with } [X]=\left[N^2-1+\frac{1}{k_0^2}\frac{\partial^2}{\partial z^2}\right] \tag{3}$$

Assuming a weak dependence of N with the r coordinate, the operators [P] and [Q] commute, and equation (3) is transformed into:

$$[P + ik_0 Q][P - ik_0 Q]u = 0 \tag{4}$$

This wave equation exhibits two terms related to the forward and the backward fields. The parabolic approximation neglects the back-scattering field with respect to the forward-field, and the wave equation reduces into:

$$[P]u = ik_0 [Q]u \tag{5}$$

Finally, considering an envelop function, $\Psi(r, z)$, with smooth variations with respect to the phase of the wave, $u(r,z) = \psi(r,z)\exp(ik_0 r)$, the parabolic equation is:

$$[P]\psi = ik_0 [Q - 1]\psi \tag{6}$$

The principle of the Higher-Order Parabolic Equation is to approximate Q with a Padé expansion of order n. The theoretical background was analysed by Bamberger *et al.* [2]. The HOPE has been used by Collins [5] for underwater acoustics in presence of elastic bottom to take into account scattering effects at very large angles:

$$[Q] = \left[1 + \sum_{j=1}^{n} \frac{a_{j,n} X}{1 + b_{j,n} X}\right] + O(X^{2n+1}) \tag{7}$$

The coefficients $a_{j,n}$ and $b_{j,n}$ are determined in order that the n first derivatives of Q and equation (7) agree when $X = 0$:

$$a_{j,n} = [2/(2n+1)]\sin^2[j\pi/(2n+1)] \tag{8a}$$

$$b_{j,n} = \cos^2[j\pi/(2n+1)] \tag{8b}$$

The HOPE reads:

$$[P]\psi = ik_0 \left[\sum_{j=1}^{n} \frac{a_{j,n} X}{1 + b_{j,n} X}\right]\psi \tag{9}$$

From (9), the WAPE proposed by Claerbout [12] is obtained, when n = 1, $a_{j,n}$ = 1/2 and $b_{j,n}$ = 1/4:

$$[1 + X/4][P]\psi = ik_0 [X/2]\psi \tag{10}$$

Error between Q and its approximation using the Padé expansion

Let us consider the difference e_p^n between the square root operator Q and its approximation with a Padé expansion:

$$e_p^n(X) = (1+X)^{1/2} - \left[1 + \sum_{j=1}^{n} \frac{a_{j,n} X}{1 + b_{j,n} X}\right] \quad (16)$$

Table 1 below summarizes e_p^n for several values of X and n. An excellent agreement between the square root operator and the Padé expansion is found, when increasing the order n. When $X > 1$ (the radius of convergence of the Padé expansion is greater than one) the Padé expansion also provides a good approximation. Figure 1 plots e_p^n as function of n, when X is fixed to 5. It exhibits an exponential decrease of the error e_p^n.

Table 1 - Differences between the square root operator and the Padé expansion

X	$(X+1)^{1/2}$	order n	Padé	e_p^n x 10^7
0.1	1.0488088	1	1.0487804	284
0.5	1.2247449	1	1.2222222	25227
		2	1.2247190	259
		3	1.2247446	4
0.8	1.3416407	1	1.3333334	83073
		2	1.3414634	1773
		3	1.3416370	37
1.	1.4142135	1	1.4000001	142134
		2	1.4137931	4205
		3	1.4142013	123
		4	1.4142132	4
5.	2.4494898	1	2.1111112	3383787
		2	2.3861384	633514
		3	2.4381833	113065
		4	2.4474897	20001
		5	2.4474897	3531
		6	2.4491367	625
		7	2.4494739	110
		8	2.4494879	19
		9	2.4494896	2

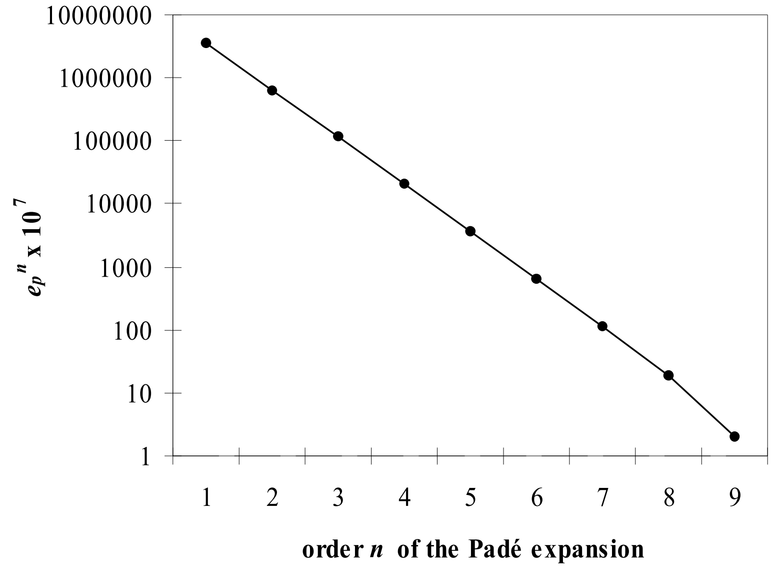


Figure 1 – Error between the square root operator and the Padé expansion as function of the order n; X = 5.

Numerical implementation

The HOPE is solved using a finite difference method. For compactness, we set for the HOPE:

$$[P]\psi = ik_0\left[\sum_{j=1}^{n} L_{j,n}\right]\psi$$

Discretizing the P operator and using the alternating directions technique, the HOPE is transformed into a set of n successive PEs. The j^{th} PE reads:

$$\left[1 - ik_0\Delta r\theta\sum_{j=1}^{n} L_{j,n}\right]\psi^{l+j/n} = \left[1 + ik_0\Delta r(1-\theta)\sum_{j=1}^{n} L_{j,n}\right]\psi^{l+(j-1)/n}$$

The first PE, when $j = 1$, makes use of the known field ψ^l at range r, and the last one, when $j = n$, provides the expected field ψ^{l+1}at range $r+\Delta r$. The CPU time for the HOPE is then n-times the CPU time of the WAPE. θ is a weighting coefficient of the L operator at two successive ranges. θ is fixed to 1, providing an implicit finite difference scheme.

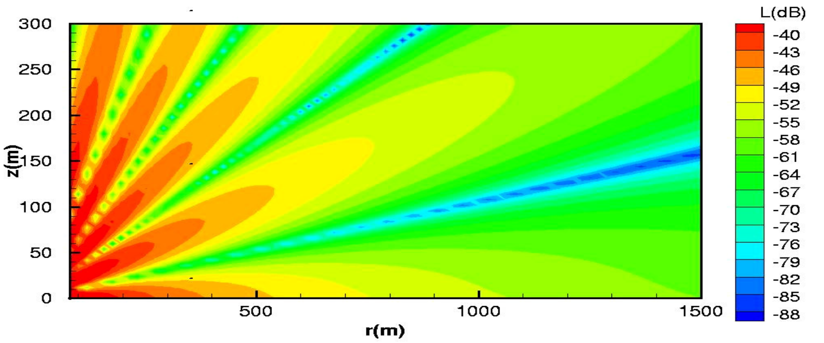
Boundary conditions

The gaussian function is generally used as the starting field, at $r = r_0$, to reduce the aperture angle of the field, in agreement with the PE approximation. Using the HOPE, including a large aperture angle of propagation, a spherical radiation from a point source above a plane can be introduced. On the ground an impedance condition is applied. The locally reacting impedance is derived from the model proposed by Attenborough [1]. In the upper part of the mesh, an artificial attenuation is generally considered to avoid numerical reflections. A radiation boundary condition derived from the wave impedance is here implemented. The wave impedance, at $z = z_{Max}$, is derived from the pressure and acoustical velocity, obtained from the solution of a point source above the ground in homogeneous medium. In comparison with the absorbing layers, this reduces the mesh size and the CPU time.

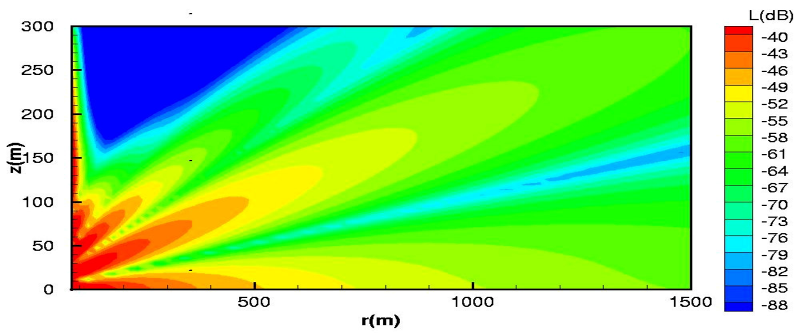
APPLICATION OF THE HOPE TO OUTDOOR PROPAGATION

The radiation of a source at the frequency f = 80 Hz, above a sandy ground at height h_s = 10 m, is considered. Figure 2 compares the exact solution, the WAPE and the HOPE. The plots represent the sound pressure level L(dB) (referenced to 0 decibel at one meter from the acoustical source) in a vertical plane including the source and the receiver. Figure 2 shows that the HOPE improves the solution compared to the WAPE for the wider aperture angles (upper corner on the left-hand side), while the level remains the same between the WAPE and the HOPE on the ground, at smaller aperture angles.

a) Exact solution



b) WAPE



c) HOPE, n = 10

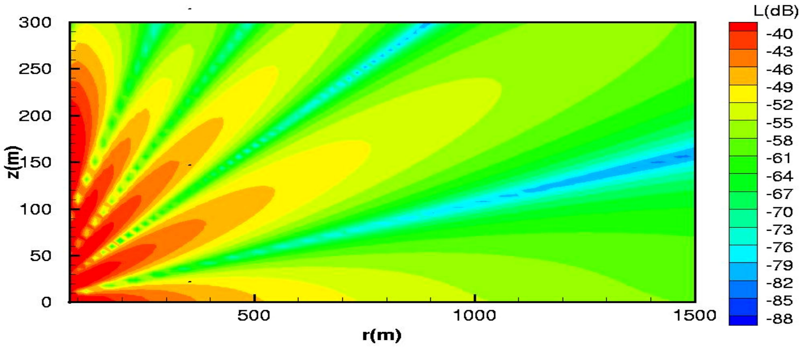
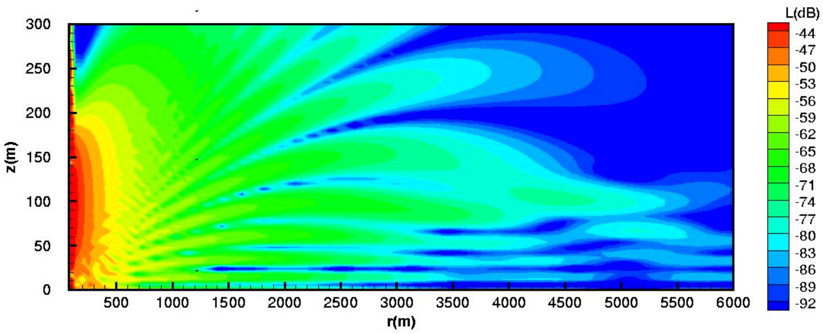


Figure 2 – Sound propagation above a sandy ground

Figure 3 illustrates the sound propagation above a sandy ground in presence of a wind profile. The source is located at 100 m height and its frequency equals 80 Hz. The horizontal component of the wind velocity as function of the altitude is given by an exponential law: $v(z) = v_0\ (z/10)^{0.2}$. v_0, the velocity at the altitude 10 m, equals 5 m/s. The sound speed profile is given by: $c(z) = c_0 + v(z)$. At the range r = 4 000 m from the source, the pressure level is reduced with the WAPE, while it remains almost constant with the HOPE, as expected under downwind configuration. It then appears that the WAPE may lose its accuracy within a long distance from the source, but not the HOPE.

a)WAPE



b) HOPE, n = 5

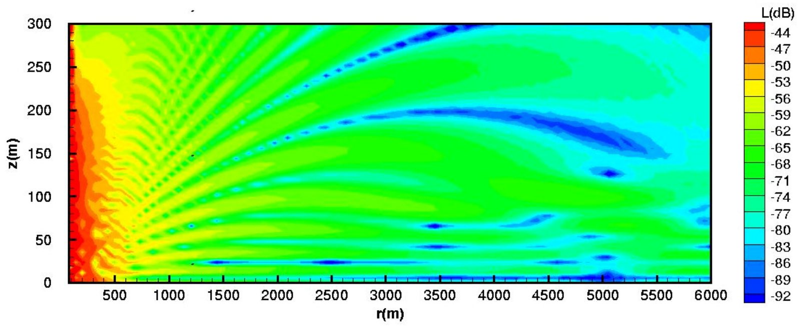


Figure 3 – Sound propagation above a sandy ground in presence of a wind gradient

SUMMARY

The wide-angle parabolic equation (WAPE) is extensively used for outdoor propagation. This paper has shown that the Higher-Order Parabolic Equation (HOPE) is an interesting alternative to the WAPE. It opens up the aperture angle of propagation and improves the results at very long range. The HOPE is based on the approximation of a square root operator with a Padé expansion. The CPU time rises in proportion to the Padé order *n*, however it remains small taking advantage of the marching algorithm in the WAPE implementation.

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