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NOISE OF STORMS

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Abstract

The stormy area in the ocean radiates sound into the ocean and in the atmosphere in a low frequency band. This noise radiation can be observed at a distances of thousands of miles from the source. The effect of sound radiation is connected apparently to the interaction of the counter-propagating sea-surface waves that produces a sound radiation of the doubled frequency of the surface wave oscillation. The radiated sound due to diffraction effects is trapped by the horizontal atmospheric wave-guide that provides the long distance propagation. Theoretical model of the sound generation by the sea - surface waves is developed on the basis o f the advanced theoretical approach The spectrum of sound radiation is presented as a function of the sea – surface wave features. The diffraction effects are taken into account to evaluate the trapping of the radiation by a horizontal atmospheric wave-guide, important for the long-range propagation of the noise.

INTRODUCTION

The sound radiation into the atmosphere by the gravity surface wave nonlinear interaction has a specific feature due to differences in the boundary conditions at the water-air interface for sound in the water and in the atmosphere. The ocean surface is acoustically absolutely compliant (has a low impedance) for radiation of sound into the water. As a result, dipole sound sources are produced, as it were by forces applied to the ocean-air boundary appear on that surface. These sources produced by the counter - propagating waves radiate the sound into the water, the theory of this effect is developed by Longuet - Higgins [1]. For the radiation of sound into the atmosphere, on the other hand, the ocean surface is a surface of volume-velocity sources (monopoles) because of its high impedance [2].The sound in water produces the boundary condition at the sea surface for the sound generated into the atmosphere To obtain the correct source strength for the radiation of atmospheric microbaroms the radiation into the ocean first need to be considered.

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EQUATION OF NONLINEAR INTERACTION

Nonlinear interaction of the surface gravity waves produces the sound in water and atmosphere. Let the z coordinate increases vertically upward, and let the plane z=0 coincide with the free surface of the water. Then the counter-propagating gravity waves can be described by the equations

$$\varphi_1 = ae^{k_1 z}\cos[k_1 x - \omega_1 t] \tag{1}$$

$$\varphi_1 = ae^{k_1 z}\cos[k_1 x - \omega_1 t] \tag{2}$$

here a is the amplitude of the gravity wave velocity potential, k_i is the wave number and ω_i is the circular frequency of the gravity wave, i=1,2, .
Consider the generation of sound by these counter-propagating surface waves. It is supposed that the sound wave is propagating downward (into the ocean), and can be represented as

$$\varphi_3 = \varphi_3(t)e^{-i\omega_3 t - iq_{1z}z + iq_{1x}x} \tag{3}$$

q_1 is the sound wave number determined by the dispersion relation

$$q_1 = \frac{\omega_3}{c_1}, q^2{}_1 = q^2{}_{1x} + q^2{}_{1z} \tag{4}$$

is the sound wave circular frequency and c_1 is the sound speed in water.
Sound wave equation can be taken in linear approximation,

$$\frac{\partial^2 \varphi_3}{\partial t^2} - c_1{}^2 \Delta\varphi = 0, \tag{5}$$

the main nonlinear effects of the problem are described by the boundary condition on the sea – surface.
Consider the triplet of the nonlinear resonant interacting waves

$$\varphi_3 = \varphi_1 + \varphi_2 + \varphi_3 \tag{6}$$

the interaction takes place if the resonance conditions in time and space are fulfilled

$$\omega_1 + \omega_2 = \omega_3,$$

$$k_2 - k_1 = q_{1x,} \tag{7}$$

q_{1x} is the horizontal components of sound wave-number.

The equation of nonlinear interaction of the surface waves and sound obtained from the nonlinear boundary conditions on the sea - surface is.[3]

$$\frac{\partial^2\varphi}{\partial t^2}+g\frac{\partial\varphi}{\partial z}=-(\frac{\partial^2\varphi}{\partial t^2}+g\frac{\partial\varphi}{\partial z})\varsigma\frac{\partial\varphi}{\partial z}-\frac{1}{2}(\nabla\varphi)^2+\frac{\partial}{\partial t}(\frac{\partial\varphi}{\partial z})^2+g\frac{\partial\varsigma}{\partial x_i}\frac{\partial\varphi}{\partial x_i},\ z=0,\ (8)$$

g is the gravitational acceleration.
The substitution of sum [6] into this equation leads to the set of three equations.
Consider the equation for the sound wave.
The left-hand side of this equation can be modified at the approximation of the slow varying amplitudes. Then the nonlinear interaction equation is [3]

$$-\omega^2{}_3=-(\frac{\partial^2\varphi}{\partial t^2}+g\frac{\partial\varphi}{\partial z})\varsigma\frac{\partial\varphi}{\partial z}-\frac{1}{2}(\nabla\varphi)^2+\frac{\partial}{\partial t}(\frac{\partial\varphi}{\partial z})^2+g\frac{\partial\varsigma}{\partial x_i}\frac{\partial\varphi}{\partial x_i}.\ -\qquad(9)$$

RADIATION OF SOUND INTO THE ATMOSPHERE

Substituting the equations for the counter propagating surface waves into the right hand side of we obtain the expression for the sound radiation into the water produced by the surface counter propagating waves in a form

$$\varphi_3=\frac{4}{\omega_3}k_1k_2a_1a_2\sin[-\omega_3t+q_{1x}x-q_{1z}z].\qquad(10)$$

This wave produced the perturbation of the surface with the vertical component of the velocity in a form (z=-0)

$$w_3=\frac{\partial\varphi_3}{\partial z}=-4\frac{k_1k_2a_1a_2}{\omega_3}\cos[-\omega_3t+q_{1x}x-q_{1z}].\qquad(11)$$

This boundary displacement produces in the atmosphere sound wave that corresponds to the given boundary displacement velocity at z=+0

$$\varphi_a=\frac{4}{\omega_3}k_1k_2a_1a_{2.}\sin[-\omega_3t+q_{21x}x+q_{2z}].\qquad(12)$$

The horizontals component q_{2x} of the wave number of the sound wave in the atmosphere is equal to that in water $q_{2x=}q_{1x}$ and is marked later as q_x, it is determined by the horizontal space scale of the interacting gravity waves by the Eq.(7). The vertical component of the wave number in the atmosphere is

$$q_{2z} = \sqrt{\left(\frac{\omega_3}{c_2}\right)^2 - q^2{}_x} \tag{13}$$

and in water

$$q_{1z} = \sqrt{\left(\frac{\omega_3}{c_1}\right)^2 - q^2{}_x}\,. \tag{14}$$

Due to inequality $c_1 >> c_2$ evanescent waves in the water are transformed into the progressive wave in the atmosphere and makes contribution into the infrasound radiation into the atmosphere.

The pressure amplitude of the sound wave in air in the case of $q_x = 0$ is

$$p_a = -\rho_a \frac{\partial \varphi_a}{\partial t} = 4\rho_a \upsilon_1 \upsilon_2 \frac{c_2}{c_1}, \tag{15}$$

where $\upsilon_1 = k_1 a_{1,}, \upsilon_2 = k_2 a_2$.

This result can be compared to the pressure amplitude of sound radiation into the water obtained from the equation (3)

$$p_w = 4\rho_w \upsilon_1 \upsilon_2 \tag{16}$$

The ratio of sound pressure in the atmosphere to that in water can be expressed by

$$\frac{p_a}{p_w} = \frac{\rho_a c_2}{\rho_w c_1}, \tag{17}$$

in the exact correspondence to that obtained by Waxler [4].

The physical reason of this result is connected with the acoustics impedance difference between air and water, the acoustic pressure produced in the air by sound in the water is smaller by the factor $\frac{\rho_a c_2}{\rho_w c_1}$ than in the water. Once the interface is in motion the air and the water in the source region must have roughly the same velocity fields. However the changes in pressure required to support changes in velocity are proportional to fluid density and therefore the sound pressure in the water is greater than in the air by a factor determined by the ration of water density. to that of the air.

CONTINOUS SURFACE WAVE SPECTRUM

Consider the interaction of components of a continuous spectrum. of the surface waves. Make the Fourier transform of sea - surface elevation $\xi(x,t)$, x is the horizontal coordinate,

$$\xi(\mathbf{x},t) = \mathrm{Re}\int \hat{\xi}(\mathbf{k}) Exp[i(\mathbf{kx} - \omega(\mathbf{k})t]d\mathbf{k}. \qquad (18)$$

If one supposes that the surface wave field is a translation invariant Gaussian process, than the averaged values obey to the equation

$$< \hat{\xi}(\mathbf{k})\hat{\xi}(\mathbf{l}) >= F(\mathbf{k})\delta(\mathbf{k}-\mathbf{l}). \qquad (19)$$

Here **k,l** are the two-dimensional vectors in the horizontal plane, and $F(\mathbf{k})$ is the wave vector spectral density.

The sound pressure amplitude, expressed by equation (15), can be presented in terms of the surface displacement ξ

$$p_a = 4\rho_a \xi_1 \xi_2 \omega_1 \omega_2 \frac{q_{1z}}{q_{2z}}.$$

To obtain the radiated sound intensity find first the autocorrelation function Γ(**x**,**x′**,τ)

$$\Gamma(\mathbf{x},\mathbf{x}',\tau) =< p(\mathbf{x},z,t)p(\mathbf{x}',z',t+\tau) >= \qquad (20)$$

$$= 16\rho^2{}_a \int d\mathbf{k}d\mathbf{l}d\mathbf{k}'d\mathbf{l}' < D(k,l,q)\hat{\xi}(\mathbf{k})\hat{\xi}(\mathbf{l})\hat{\xi}(\mathbf{k}')^* \hat{\xi}(\mathbf{l}')^* D(k',l',q')^* > e^{-2\omega_3\tau},$$

here $D(k,l,q = 4\frac{\omega_1\omega_2 q_{zx}}{q_{2z}}, D'(k',l',q') = 4\frac{\omega_1\omega_2 q_{zx}}{q_{2z}}.$

The equation for the autocorrelation function can be simplified using the features of the sea - surface wave field supposed to be a Gaussian process. Indeed in this case the product of four stochastic variables is splitting into the three products of two δ - correlated variables and the wave vector spectral density. Taking into account also the condition of resonance we obtain

$$\Gamma(\mathbf{x},\mathbf{x}',\tau) = 32\rho^2{}_a \int d\mathbf{k}d\mathbf{l}d\mathbf{k}'d\mathbf{l}' F(\mathbf{k})F(\mathbf{l})D(k,l,q)D(k',l',q')^* > e^{-2\pi f\tau},$$

$$f = \frac{\omega_3}{2\pi}. \qquad (21)$$

Introducing the directional spectral density function

$$\Phi(f,\theta) = \frac{f^3}{32\pi^4 g^2} F(\mathbf{k}) \qquad (22)$$

we obtain the equation for the autocorrelation function in terms of the directional spectral density function

$$\Gamma(\mathbf{x},\mathbf{x}',\tau) = 32\rho^2{}_a \int df d\theta \Phi(\frac{f}{2},\theta) D(k,l,q) \int df d\theta \Phi(\frac{f}{2},\theta+\pi) D(k,l,q)^* e^{-i2\pi f\tau}. \quad (23)$$

The power spectrum of sound radiation is the Fourier transform of the autocorrelation function

$$S(\nu,t) = \frac{1}{2\pi}\int \Gamma(\tau,t) e^{-i\nu t} d\tau. \quad (24)$$

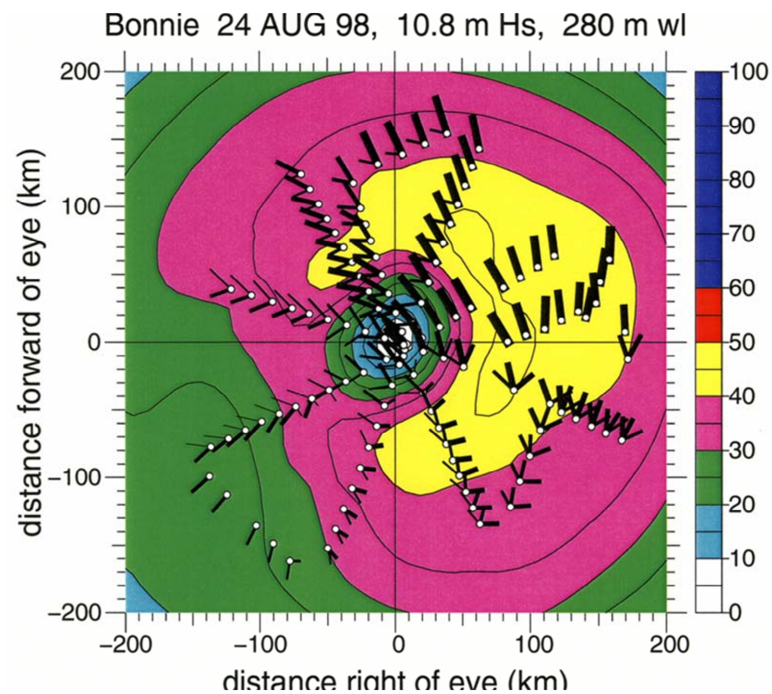

Fig.1. Spatial variatiom of the wind speed and the significant wave height H_{s} in the Hurricane Bonnie, August 24 1998. (E. Walsh et.al. [5]). H_{s} is presented by the length of the segments and the wave length by the thickness of it. The colour indicates the wind speed distribution.

As a function of θ Φ(f/2),θ) is usually strongly peaked at angles near those of the direction of the prevailing winds, and the product of the directional spectral density functions is a measure of the density of the counter - propagating waves at frequency

f. The largest component of a wave field consists of waves propagating in the direction of the prevailing wind. Thus the evaluating of correlation function requires knowing the tails of the wave number distribution, perturbed by the underwater effects. These data was obtained from the E. Walsh experiment [5], presented in the Fig.1.

CONCLUSIONS

The nonlinear theory of infrasound generation by the sea-surface perturbation produced by stormy weather is developed. The small part of the generated infrasound can be captured by the atmospheric waveguide [6] and propagates over a long distances in the atmosphere. The preliminary estimation gives the infrasound amplitude of amount of tenth microbarom at a distance of 1000 km.

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