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COMPARISON BETWEEN DIFFERENT APPROACHES FOR THE LOW-FREQUENCY VIBROACOUSTIC ANALYSIS OF A PLATE COUPLED TO AN AIR OR A WATER-FILLED CAVITY

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Abstract

The validation of a modal method which uses a formulation in potential of fluid, with both the static pressure term and an appropriate static displacement potential, is presented. The validation of the method is performed by a comparison with other different approaches and experiments for the low-frequency (LF) vibroacoustic response of a coupled system composed of an elastic structure containing an internal acoustic fluid. The internal fluid can be either a light fluid (air) or a dense slightly compressible fluid (water). The approaches used for validating the modal approach are: a fully analytical approach, a medium-frequency (MF) numerical approach using a specific algorithm and normally well-adapted to MF analysis of vibroacoustic problems. These approaches are compared to each other and also to a solution coming from a commercial code named "I-deas/Rayon". In case of a water-filled cavity, they are also compared to measurements. The modal approach uses different formulations. Among these formulations, the classical formulation in internal pressure, commonly used and well-adapted for light fluids like air, is replaced by a formulation in internal displacement potential which takes into account the quasi-incompressibility of dense fluids like liquids. This latter formulation is the correct formulation because it uses the static displacement potential term. Convergence of the method can be accelerated when using the exact solution of the static displacement potential instead of a classical expression, for the case of a rectangular plate coupled to a parallelepipedic cavity. The comparison between methods was focused on the evaluation of the first resonant frequencies of the coupled system, the vibratory response of the plate and the acoustic pressure within the cavity in the frequency band [0, 5 000Hz], for both cases of air and water. The vibroacoustic system tested experimentally is a rigid cylindrical box containing a parallelepipedic cavity entirely filled with water which is defined by five infinitely-rigid walls and closed at one end by a clamped elastic homogeneous plate. For this system, an analytical solution of the overall vibroacoustic problem can be constructed as a reference solution.

INTRODUCTION

In this paper, we make a comparison between methods and experiments for the low-frequency (LF) vibroacoustic response of a steel clamped elastic rectangular plate coupled to a parallelepipedic cavity which is entirely filled with either air or water. A modal approach is validated. The modal approach is formulated in internal fluid displacement potential. The method needs to introduce both the static pressure term (which takes into account the zero-frequency stiffness effect of the fluid) and an appropriate static displacement potential (which takes into account the fluid mass effect for heavy fluids). Therefore, the method becomes accurate for either gas or liquids.

The analyzed frequency band is [0, 5 000Hz]. Within this band the overall vibroacoustic system has modal behaviour, which makes the validation of the modal method very efficient.

DESCRIPTION OF THE ANALYZED VIBROACOUSTIC SYSTEM

The analyzed vibroacoustic system is described on Figure 1 below. It is small and that is why it has modal behaviour within the frequency band [0, 5 000Hz]. The plate is excited by one point mechanical force and five points (two for the structure and three inside fluid) are observed for comparisons.

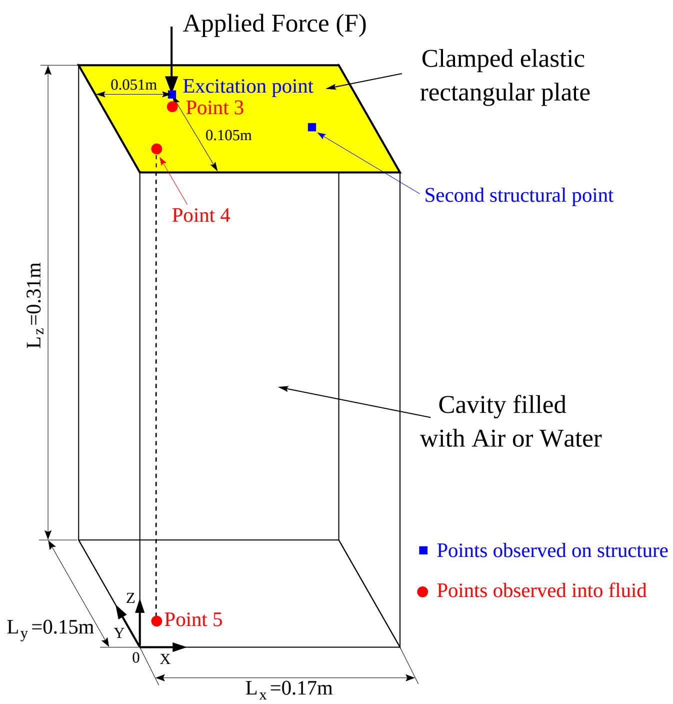

Figure 1 – Geometry of the analyzed vibroacoustic system and location of observation points

MODAL METHOD FORMULATED IN DISPLACEMENT POTENTIAL

When the structure is excited by a set of external forces F^e, the variational formulation of the internal fluid-structure problem, using (u,ϕ) as variables (u being the displacement field of the structure and ϕ the internal displacement potential of the fluid) with both *static pressure* $p_0 = -\frac{\rho_F c_F^2}{V_F}\int_{\Sigma_F} u.n_F dS$ and *static displacement potential* ϕ_0 terms, is expressed as follows (see Refs. [1] to [5]):

$$\begin{aligned}&\int_{\Omega_s}\sigma_{ij}(u)\varepsilon_{ij}(\partial u)dV + \frac{\rho_F c_F^2}{V_F}\int_{\Sigma_F} u.n_F dS.\int_{\Sigma_F}(\partial u).n_F dS - \omega^2\int_{\Omega_s}\rho_s u.(\partial u)dV\\ &-\omega^2\int_{\Sigma_F}\rho_F\phi.(\partial u).n_F dS = \int_{\Sigma_e}F^e.(\partial u)dS\end{aligned} \qquad (1)$$

for the structure and

$$\int_{\Omega_F}\nabla\phi.\nabla(\partial\phi)dV - \frac{\omega^2}{c_F^2}\int_{\Omega_F}\phi.(\partial\phi)dV - \int_{\Sigma_F}u.n_F.(\partial\phi)dS = 0 \qquad (2)$$

for the internal acoustics.

Within LF frequency domain, the coupled problem (1) and (2) above can be projected on two separated modal bases: one for the structure *in-vacuo* (of eigenmodes $\varphi_\beta,\ \forall\beta=\{1,\cdots,N_S\}$) and the second for the internal acoustics of a rigid-walled cavity (of eigenmodes $\psi_\alpha,\ \forall\alpha=\{1,\cdots,N_A\}$).

By using the modal projections: $u.n_F = \sum_{\beta=1}^{N_S} u_\beta.\varphi_\beta$, for the normal displacement of the structure and $\phi = \phi_0 + \sum_{\alpha=1}^{N_A}\kappa_\alpha.\psi_\alpha$, for the internal displacement potential, the variational formulation (1) and (2) above leads to a linear matrix reduced system of dimension $(N_S\times N_A)^2$, to be solved:

$$\begin{aligned}&\lambda^S\mu_\gamma^S u_\gamma + \frac{\rho_F c_F^2}{V_F}\sum_{\beta=1}^{N_S}\left(\int_{\Sigma_F}\varphi_\gamma dS\right)\left(\int_{\Sigma_F}\varphi_\beta dS\right)u_\beta =\\ &\omega^2\left\{\mu_\gamma^S u_\gamma + \int_{\Sigma_F}\rho_F\phi_0\varphi_\gamma dS + \sum_{\beta=1}^{N_A}C_{\gamma\beta}\kappa_\beta\right\} + F_\gamma,\end{aligned} \qquad \forall\gamma = \{1,\cdots,N_S\} \qquad (3\text{-a})$$

$$\lambda_\gamma^A \mu_\gamma^A \kappa_\gamma = \omega^2 \left\{ \mu_\gamma^A \kappa_\gamma + \sum_{\beta=1}^{N_S} C_{\beta\gamma} u_\beta \right\}, \quad \forall \gamma = \{1, \cdots, N_A\} \tag{3-b}$$

where: $C_{\beta\gamma} = \rho_F \int_{\Sigma_F} \varphi_\beta \psi_\gamma dS$ is the coupling term between structural mode φ_β and acoustic mode ψ_γ, $\lambda_\gamma^S, \mu_\gamma^S$ the squared eigenfrequency and generalyzed mass of mode φ_γ, $\lambda_\gamma^A, \mu_\gamma^A$ the squared eigenfrequency and generalyzed mass of mode ψ_γ and the F_γ the generalyzed force of mode φ_γ.

Linear system (3-a) and (3-b) can be put under a symmetric matrix system defined as:

$$\begin{bmatrix} -\omega^2 \{M_s + M_{ad}\} + j\omega C_s + K_s + K_{ad} & -\omega^2 C \\ -\omega^2 C^T & -\omega^2 M_A + j\omega C_A + K_A \end{bmatrix} \begin{bmatrix} U \\ \Phi \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \tag{4}$$

in which classical damping matrices of structure C_s and of internal acoustics C_A have been introduced. U and Φ are the vectors of unknown generalized displacement of structure and unknown generalized internal potential to be computed.

Classical expression of static displacement potential

One way to introduce ϕ_0 term is to use the classical expression given by (7.55)-page 144 of [1]:

$$\phi_0 = \sum_{\beta=1}^{N_S} \sum_{\alpha=1}^{N_A} \frac{1}{\mu_\alpha^A} \left[C_{\beta\alpha} \psi_\alpha \right] u_\beta. \tag{5}$$

ϕ_0, defined above, is not exact because it is truncated on the number of acoustic modes retained in the projection. We will see the influence of the modal truncation on the convergence of the method in case of a heavy fluid like water.

Particular case of static displacement potential

For a rectangular plate coupled to a parallelepipedic cavity, an exact analytical solution of ϕ_0 can be constructed. This solution accelerates the convergence of the resonant frequencies of the plate (loaded by the heavy fluid) and accurately improves the vibroacoustic response of the system. This solution for ϕ_0 can be split into two terms such that:

$$\phi_0 = \phi_0^0 + \phi_0^1. \tag{6}$$

ϕ_0^0 represents the potential of incompressible fluid, for which an exact analytical expression can be constructed explicitly, and ϕ_0^1 is a particular solution. The analytical expressions of these two terms are given in Appendix A of [5].

OTHER METHODS USED FOR THIS PROBLEM

Analytical solution

A fully analytical solution of the overall vibroacoustic problem of an elastic rectangular plate coupled to a parallelepipedic cavity containing an acoustic fluid (light or dense) can be constructed explicitly. One can find this solution is Appendix C of [6]. Two simulations were performed using this method: a first one when the cavity contains air and a second with water. The results provide the reference solution used to test and validate other methods, and particularly the modal approach.

MF numerical computation

A computation using an adapted MF method was performed on the system in order to predict the internal noise and vibration levels of the plate within the frequency range [100, 5 000Hz]. This method uses a direct approach by finite elements and it is fully detailed in [6, 8, 9]. It is normally adapted to MF domain but, in the present case, it was also applied with success to a system having a LF dynamic behaviour. The method was used for both an air and a water-filled cavity.

Solution from “I-deas/Rayon” code

“I-deas/Rayon” code is a commercial code for internal and external vibroacoustic applications. It uses a modal approach formulated in (γ, p) (γ acceleration field of structure and p internal pressure). This formulation is fully described in [10]. Solutions from this code were obtained for both air and water in order to finally test the accuracy of an existing commercial code on the proposed test case.

COMPARISON OF METHODS

Results of vibroacoustic response for air

Figure 2 below shows the comparison between methods of the internal pressure at a point inside the fluid for an air-filled cavity. Results coming from the modal method using the classical (u, p) formulation are also compared to other approaches. All the methods agree with each other, and (u, p) and (u, ϕ) formulations are equivalent for air (a light fluid).

Results for water when using a (u, p) formulation

Figure 3 below shows the comparison between methods and experiments of the internal pressure at a point inside the fluid for a water-filled cavity. Results coming from the modal method using the classical (u, p) formulation and those coming from I-deas/Rayon code are not correct compared to measurements, and also to analytical solution and MF numerical computation.

Therefore, a correct formulation for modal method with heavy fluids is to use the (u,ϕ) formulation presented above. However, the convergence of the modal method is very slow (in general) for heavy fluids.

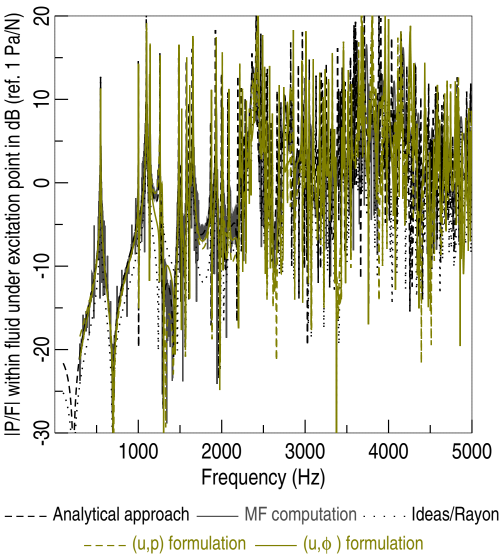

Figure 2 – Comparison of acoustic pressure levels inside fluid at Point 3, for an air-filled cavity.

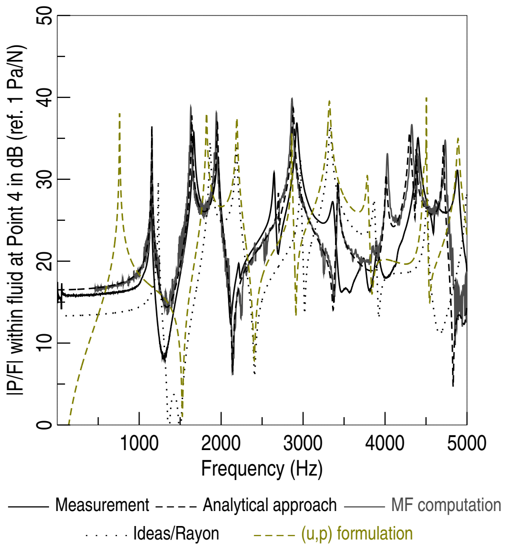

Figure 3 – Comparison of acoustic pressure levels inside fluid at Point 4, for a water-filled cavity.

Convergence of eigenfrequencies of the plate for modal method

Figure 4 below shows the convergence (toward theoretical frequencies) of the first eigenfrequencies (six) of the vibroacoustic system "Clamped elastic rectangular plate coupled to a water-filled cavity", with respect to the number of acoustic modes and to the expression of the static displacement potential introduced in the modal method.

As one can see on this figure, the convergence is very slow when using the classical expression of ϕ_0. A perfect convergence of the frequencies needs a large number of acoustic modes (> 1500).

Although, convergence of these frequencies can be accelerated accurately when using the exact expression $\phi_0^0 + \phi_0^1$ for the static displacement potential.

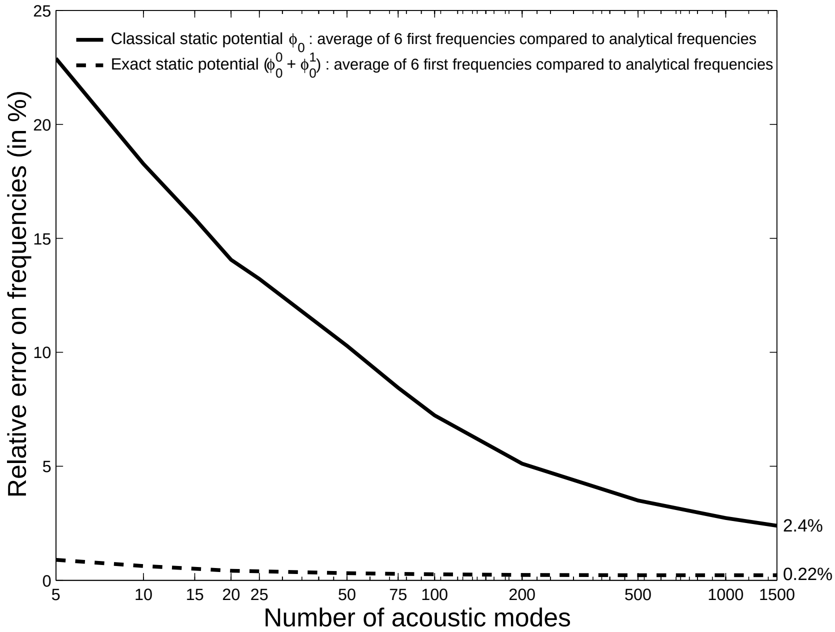

Figure 4 – Influence of the number of acoustic modes and of expression of static displacement potential on convergence of the first eigenfrequencies of the analyzed vibroacoustic system.

Convergence of vibroacoustic response for modal method - Overall comparison

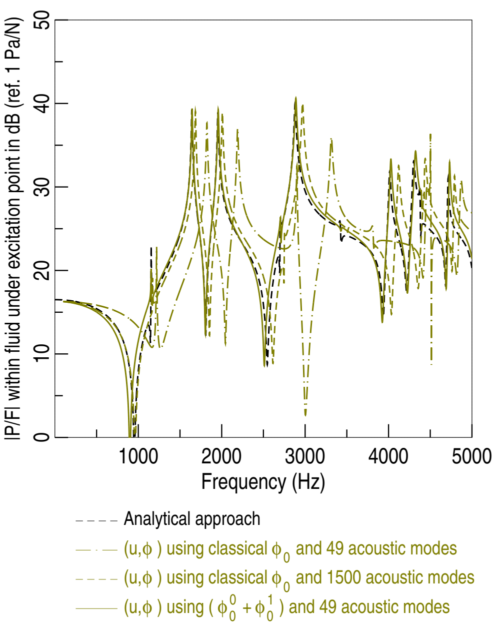

Figure 5 – Convergence of acoustic pressure levels inside fluid at Point 3, for a water-filled cavity.

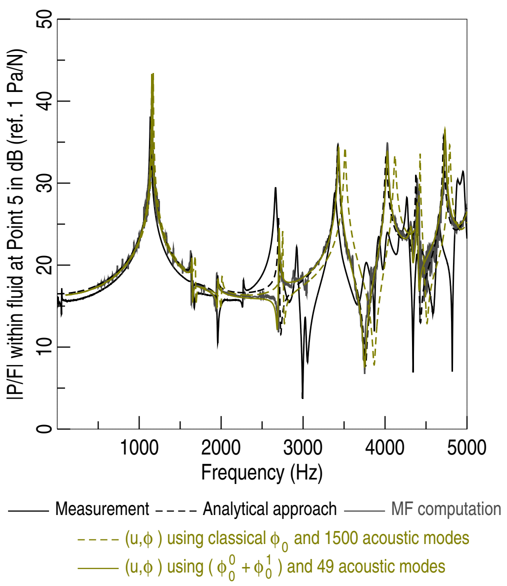

Figure 6 – Overall comparison of acoustic pressure levels inside fluid at Point 5, for a water-filled cavity.

Figure 5 above shows the convergence (toward analytical solution) of the acoustic pressure within the cavity obtained by modal method, with respect to the number of acoustic modes and to the expression used for the static displacement potential.

As one can see on this figure, the convergence of the modal method is perfectly obtained when using the exact expression $\phi_0^0 + \phi_0^1$ with a small number of acoustic modes (only 49). Results coming from modal method when using the classical expression of ϕ_0 with 1500 acoustic modes are also very good (except on the evaluation of resonant frequencies). Although with 49 acoustic modes, convergence of acoustic response has not yet been obtained.

Figure 6 above shows the overall comparison between methods and experiments for the internal pressure for another point within the cavity.

CONCLUSION

In this paper we have presented the comparison between different approaches (direct methods: *analytical* and *MF numerical*, and not direct: *modal method*) on the LF vibroacoustic behaviour of a plate coupled to an internal fluid which can be either light (i.e. air) or dense slightly compressible (i.e. water). We have shown the difficulty to deal with heavy fluids when a modal approach is used. This method needs to introduce additional terms for heavy fluids.

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