ATTENUATION OF SOUNDS PROPAGATING THROUGH A BUBBLE SCREEN

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Abstract

The attenuation of sound propagating through a bubble screen was investigated both theoretically and experimentally in this study. By adding an additional virtual mass force into the momentum equation of the bubbly liquid, a modified Helmholtz equation valid for the sound propagation in the bubbly liquid at finite gas-volume fraction was reformulated. The transmission and reflection coefficients for the sound propagating through a bubble screen were derived. Experimental study was made to verify the accuracy of the theory.

1. INTRODUCTION

It has been known for a long time that the speed of sound propagating in the water is affected by the presence of the air bubbles. Bubbles of different sizes are produced by the wave breaking occurring in deep water and shallow water zones of the oceans. The air bubbles of relative small size may be transported by the nearshore current from the shallow water region to the deeper water region. Hence, for the propagation of sound waves at the nearshore region we require the information of acoustic properties in the mixture of water and air bubbles, which is often called bubbly water, or in general, bubbly liquid. The sound waves propagating in the bubbly liquid or fog was treated by Foldy (1945) as a problem of the multiple scattering of waves by a random distribution of isotropic scatterers. Equations for the average value of the wave function were derived. Cartensen and Foldy (1947) obtained experimental data on the transmission and reflection of sound by screens of bubbles. Van Wijngaarden (1968) derived equations for describing one-dimensional unsteady flow in bubble-liquid mixtures. In the present study the attenuation of sound propagating through a bubble screen at arbitrary gas volume fraction was studied both theoretically and experimentally. By adding an additional virtual mass force to the momentum equation of the bubbly liquids, we reformulate a wave equation, which is valid for the sound propagation in the bubbly liquids at finite gas volume fraction. The transmission and reflection coefficient of the sound propagating through a bubble screen was then derived based on this equation and the interface boundary conditions between the pure liquid and the bubbly liquid. Experiments were also carried out to validate the accuracy of the proposed theory. In the experiments the sound frequency was restricted to the range from 10 to 100 kHz due to the restriction of the facilities.

2. THEORETICAL ANALYSIS

The local fraction of volume occupied by the gas is given by $\beta = 4/3\pi R^3 n$, where R denotes the instantaneous radius of the bubbles and n is their number per unit volume. According to Commander and Prosperetti (1989) the continuity equation for the bubbly liquids can be expressed as

$$\frac{1}{\rho_l c^2}\frac{\partial p_m}{\partial t} + \nabla \cdot \underline{u}_m = \frac{1}{1-\beta}\frac{\partial \beta}{\partial t} \tag{1}$$

where c is the sound speed in the liquid and the subscripts m, l and b denote the physical variables of the bubbly liquid, the pure liquid and the bubbles, respectively. If the viscosity effects are ignored, the terms quadratic in $\underline{u}_m$ are assumed to be small and $\rho_m \approx (1-\beta)\rho_l$, the momentum equations of the bubbly liquids can be approximated by

$$(1-\beta)\rho_l \frac{\partial \underline{u}_m}{\partial t} = -\nabla p_m \tag{2}$$

If two spheres of the same size with radius a move upwards at the same speed V and the distance between the bubble centers is $2h$, then the kinetic energy of the fluid is $T = \frac{\pi\rho a^3}{3}(1+\frac{3}{16}\frac{a^3}{h^3})V^2$ (Streeter, 1948). The total virtual mass force in a unit volume is $\underline{F} = \frac{1}{2}\beta\rho_l\left[1+\frac{3}{16}\left(\frac{R}{h}\right)^3\right]\frac{\partial(\underline{u}_b - \underline{u}_m)}{\partial t}$. Biesheuvel and Wijngaarden (1984) showed that in the bubbly liquid when the buoyancy equals to the viscous force, then

$\underline{u}_b = 3\underline{u}_m$. The above equation becomes $\underline{F} = \beta\rho_l\left[1+\frac{3}{16}\left(\frac{R}{h}\right)^3\right]\frac{\partial \underline{u}_m}{\partial t}$. If the virtual mass force is taken into account, the previous momentum equation, Eq. (2) is revised as

$$\left[1+\frac{3}{16}\left(\frac{R}{h}\right)^3\beta\right]\rho_l\frac{\partial \underline{u}_m}{\partial t} = -\nabla p_m \tag{3}$$

There are three unknowns, namely, p_m, $\underline{u}_m$ and β contained in Eqs. (1) and (2) or (3). Hence, we require one more equation to solve for the unknowns. This is the bubble equation.

For a bubble in radial motion, an equation accounting the liquid compressibility was given by Keller and Miksis (1980) in the form

$$(1-\frac{R'}{c})RR'' + \frac{3}{2}(1-\frac{R'}{3c})R'^2 = \frac{1}{\rho_l}(1+\frac{R'}{c}+\frac{R}{c}\frac{d}{dt})(p_l - p) \tag{4}$$

where $R' = dR/dt$ and p_l is the liquid pressure at the bubble interface, which relates to the internal pressure of the bubble, p_b, by $p_b = p_l + \frac{2\sigma}{R} + 4\mu\frac{R'}{R}$, where σ and μ denote the surface tension and the dynamic viscosity of the liquid, respectively. Equation (4) is nonlinear and difficult to solve. To linearize this equation we assume that $R(t) = a[1+\varepsilon(t)]$, $p_b = p_o(1-\phi\varepsilon)$, $p_o = \overline{p} + 2\sigma/a$ and $q = p_m - \overline{p}$, where a is the equilibrium radius of the bubble, p_o denotes the equilibrium pressure inside the bubble, ε is the perturbed amplitude and is assumed to be proportional to $e^{i\omega t}$, ϕ is the phase shift, $\overline{p}$ is the equilibrium pressure in the liquid and q represents the perturbed pressure in the liquid. Substituting the above terms into Eq. (4) and after ignoring the higher-order terms, we obtain the expressions for ε, ω_0 and b, where ω_0 is the natural frequency and b is the damping constant of the bubble oscillation. By combining the first law of thermodynamics, the state equation of the ideal gas, and the continuity equation for the gas inside the bubble, a governing equation for the temperature field can be obtained. This will in turn determine the internal pressure of the bubble and the phase shift ϕ.

Taking the time derivative of Eq. (1) and taking the divergence of Eq. (2), we get

$$\frac{1}{c^2}\frac{\partial^2 p_m}{\partial t^2}-\frac{\nabla^2 p_m}{1-\beta}=\rho_l\left[\frac{1}{(1-\beta)^2}\left(\frac{\partial\beta}{\partial t}\right)^2+\frac{1}{1-\beta}\frac{\partial^2\beta}{\partial t^2}\right] \tag{5}$$

Substituting Eq. (1) into Eq. (5) and assume that $p_m-\overline{p}=q$ is proportional to $\exp(i\omega t)$, then the reduced wave equation or the Helmholtz equation for wave propagation in the bubbly liquid is

$$\nabla^2 q+k_m^2 q=0\,,\quad k_m^2=(1-\beta)\left(\frac{\omega^2}{c^2}+\frac{4\pi\omega^2}{1-\beta}\frac{na}{\omega_o^2-\omega^2+2ib\omega}\right) \tag{6}$$

where k_m is the wave number in the bubbly liquid. Similarly, taking time derivative of Eq. (1), and taking the divergence of Eq. (3), we have

$$\frac{1}{c^2}\frac{\partial^2 p_m}{\partial t^2}-\frac{\nabla^2 p_m}{1+\frac{3}{16}\left(\frac{a}{h}\right)^3\beta}=\rho_l\left[\frac{1}{(1-\beta)^2}\left(\frac{\partial\beta}{\partial t}\right)^2+\frac{1}{1-\beta}\frac{\partial^2\beta}{\partial t^2}\right] \tag{7}$$

Compared with the previous case, where the bubble interaction was not taken into account, Eq. (7) differs from Eq. (6) only in the coefficient of the diffusion term.

The sound propagation is assumed to be one-dimensional. The bubble screen locates at $0\le x\le s$. The amplitudes of the transmitted and reflected sounds can be determined from the interface boundary conditions, which require the continuity of the pressure and the velocity at $x=0$ and $x=s$. We define the transmission coefficient, C_t, as the ratio of the intensity of the transmitted sound to that of the incident sound and the reflection coefficient, C_r, as the ratio of the intensity of the reflected sound to that of the incident sound.

3. EXPERIMENTAL SET-UP

Measurements were carried out at the Underwater Acoustics Laboratory, Department of Hydraulic and Ocean Engineering, National Cheng Kung University. Experimental set-up for measuring the sound attenuation in the presence of the bubble screen is shown in Fig. 1. Passing air from a gas tank into a rectangular rod-shaped air stone produced the bubble screen. The bubbles vary from 0.05 mm to 2.5 mm in diameter with a uniform size distribution. The bubble size was justified by taking the bubble flow picture using CCD camera. The CCD camera is DALSA CA-D6 with the speed of 955 frames per second and a resolution of 256 pixel × 256 pixel. The hydrophone (B&K 8104) contains a piezoelectric sensing element and can be used both as the projector and sound signal receiver. The input signal was produced using a function generator (HP 33120A). We use this function generator to provide source signals and to trigger the oscilloscope (HP 54600B). The signal was then passed to the gating system (B&K 4440). The gating system was used to convert the continuous signal into a signal pulse with adjustable time duration. The signal pulse from the gating system was then amplified using the power amplifier (B&K 2713), prior to drive the sound projector. The signal received by the receiver (B&K 8104) was amplified by the charge amplifier (B&K 2635). The signal was shown on the oscilloscope and stored in the personal computer for further data analysis.

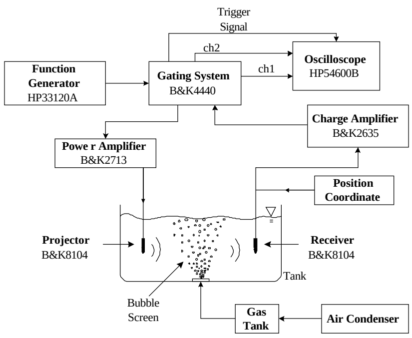

Fig. 1. Experimental set-up for measuring the sound attenuation due to the bubble screen.

The resistance void fraction sensor was used to measure the gas volume fraction of the bubbly liquid contained in the bubble screen. The resistance void fraction sensor, as shown schematically in Fig. 2, consists of two separated electrodes. Bubbly water can flow freely between these two electrodes. The void fraction is computed from the increases in resistance between the two electrodes due to the presence of bubbles in comparison with bubble-free water.

Let the specific resistance of the water alone be r_2, the specific resistance of the air alone r_1 and the resultant void fraction b, then the specific resistance r of the bubbly flow mixture is given by $r = \frac{2r_1 + r_2 + \mathrm{b}(r_1 - r_2)}{2r_1 + r_2 - 2\mathrm{b}(r_1 - r_2)} r_2$ (Su and Cartmill, 1994). Because the resistance of air is much greater than that of the water, one can assume that the resistance in the bubble goes to infinity. The above expression can be approximated by $\mathrm{b} = \frac{(r/r_2) - 1.0}{(r/r_2) + 0.5}$. Thus, the void fraction is directly related to the ratio of the resistance of the bubbly water to that of the bubble-free water. From the schematic diagram of the circuits for the void fraction sensor shown in Fig. 2, the relation between the input voltage, V_i, and the measured voltage, V_o, on the voltmeter can be determined as $R_A / R_{\mathrm{w}} = V_o /(V_i - V_o)$, where R_A represents the resistance between the two electrodes and R_{w} the variable resistance. For the sake of convenience, we can adjust R_{w} to have a value, such that when the medium between the two electrodes is water alone, V_o is equal to $0.5V_i$. To satisfy this, R_{w} must be equal to R_A. Hence, the above equation can be rewritten as $r / r_2 = V_o /(V_i - V_o)$. Substituting this equation into the previous equation, it yields $V_o = V_i \cdot (\mathrm{b} + 2)/(4 - \mathrm{b})$. Hence, the void fraction can be determined from this equation as we read the voltage shown on the voltmeter.

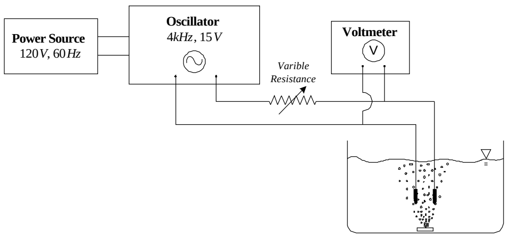

Fig. 2. Schematic diagram of the resistance void fraction sensor.

4. RESULTS AND DISCUSSION

There are three different theoretical solutions shown in the figures presented in this section. They are obtained by (i) the classical theory, which assume the air-volume fraction is very small, (ii) the modified theory in which the void fraction b is taken to be arbitrary, however, the interaction between bubbles is not taken into account, and (iii) the present theory, in which the void fraction is taken to be arbitrary and the bubble interaction is taken into account. We will begin with small air-volume fraction. Fig. 3 shows the comparison of the theoretical solutions and the experimental data for the transmission coefficient as sound propagating through a bubble screen of $\mathrm{b} = 0.08\%$, $R = 1\,mm$, $s = 1.5\,cm$ and $h = 5\,mm$, where h denotes the distance between the centers of two adjacent bubbles. The value for h was read out from the bubble picture, which was taken by the CCD camera for the determination of the bubble size. We note from Fig. 3 that the three different theories predicted the same results and the experimental data coincide with the theoretical results very well.

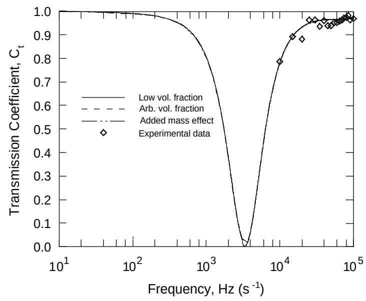

Fig. 3. Comparison of the theoretical solutions and experimental data for a transmission coefficient as sound propagating through a bubble screen of b *=0.08%, R =1mm, s =1.5cm, h =5mm.*

As the void fraction b increases from 0.08% to 11.40%, with the values for other variables being unchanged, the corresponding results were shown in Fig. 4. The difference in the theoretical results become apparent. The experimental data are in better agreement with the present theory. The difference in the theoretical results depends on the sound frequency. At the higher frequency range, the difference is about 17%, while at the lower frequency range, the difference can be as high as 24%. From Figs. 3 and 4, we note that as the void fraction increases, the transmission of sound through the bubble screen decreases. Furthermore, the frequency range, in which the sound transmission coefficient is small, becomes wider.

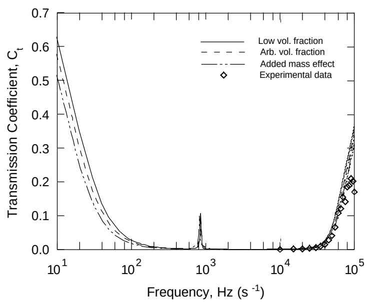

Fig. 4. Comparison of the theoretical solutions and experimental data for the transmission coefficient as sound propagating through a bubble screen of b *=11.4%, R =2.5mm, s =1.5cm, h =2.6mm.*

5. CONCLUSIONS

Based on the study we may conclude the following.

(1) When the sound propagates through a bubble screen, due to the resonance effect, the sound with natural frequency of the bubbles gives the minimum transmission coefficient and the maximum reflection coefficient.

(2) As the air-volume fraction increases, the sound transmission coefficient through the bubble screen decreases and the frequency range, in which the sound transmission coefficient is small, becomes wider.

(3) At low gas-volume fraction, our theoretical results and experimental data coincide with those obtained from the classical theory. At higher gas-volume fraction, our theoretical results agree also with our experimental data very well. However, the transmission coefficients obtained from the classical theory showed a large deviation from the experimental data.

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