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APPLICATION OF WAVELETS TO ANALYSIS OF VIBRATION DUE TO MOVING LOAD INSIDE A LAYER

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Abstract

Solution for vibration of the viscoelastic layer generated by a load moving along a beam inside the layer is presented by means of the Fourier integral. The integrand is a complicated function of many parameters and the program in the direct integration approach, to calculate the inverse transform, is much time consuming. In this paper it is shown that the inversion of the Fourier transform can be obtained using wavelet theory. The computational time for the calculation of the displacements based on a coiflets, which are the special kind of wavelets, can be reduced substantially in comparison with the direct numerical integration. In order to work with a minimum number of parameters that determine the solution of the problem in hand, the dimensionless formulation has been introduced. The properties of the solution can be investigated quantitatively on the basis of the derived formulae for the displacements using MATHEMATICA system. The calculations are conducted on the basis of coiflet filter coefficients, which can be found in the existing literature with no need of computing mother wavelet in explicit form.

INTRODUCTION

It is important to study phenomena related to ground vibrations due to moving train in order to envisage ways of reducing their impact on the built environment. To some extent the problems related to ground vibrations generated by a train moving on the surface have been considered in the papers [1],[2] and [7], but further investigations, especially in the case of train moving in a tunnel [5], are needed.

Consider the two beams connected elastically by continuously distributed springs and located in the elastic layer [4]. The distributed harmonic load moving along the lower beam generates vibrations on the layer surface. The mathematical model consists of the two equations of motion for the beams, Navier's elastodynamic equations for the viscoelastic layer medium and equations for the boundary and continuity conditions. The solution is obtained using two Fourier transforms with respect to space and time variable, respectively. Solution for the displacements is expressed by means of the single Fourier integral and the amplitude spectra are found as explicit algebraic expressions.

BASIC EQUATIONS

According to Timoshenko beam theory [3], the equations of motion for the elastically connected two beams, due to distributed vertical load P^* moving along the lower beam with velocity V^*, can be written as

$$EI\frac{\partial^4 W_2^*}{\partial x_1{}^4} + m_b{}^*\frac{\partial^2 W_2^*}{\partial t^2} + \chi^*(W_2^* - W_1^*) + W_{T2}^*(x_1,t)$$
$$= a\sigma_{zz}^*(x_1, h^* + d^*, t) + P^*(x_1,t) + P_T^*(x_1,t) \qquad (1)$$

$$EI\frac{\partial^4 W_1^*}{\partial x_1{}^4} + m_b{}^*\frac{\partial^2 W_1^*}{\partial t^2} + \chi^*(W_1^* - W_2^*) + W_{T1}^*(x_1,t) = -a\sigma_{zz}^*(x_1,h^*,t) \qquad (2)$$

where

$$P^*(x_1,t) = \frac{P_0^*}{b^*}\cos^2\frac{\pi(x_1 - V^*t)}{2b^*}[H(x_1 - V^*t + b^*) - H(x_1 - V^*t - b^*)]\cdot e^{i\Omega^* t}, \quad (3)$$

$$W_{Tj}^*(x_1,t) = \frac{m_b{}^* J_m}{K_s}\frac{\partial^4 W_j^*}{\partial t^4} - (J_m + \frac{EI\, m_b{}^*}{K_s})\frac{\partial^4 W_j^*}{\partial x_1^2 \partial t^2}, \ (j = 1,2) \qquad (4)$$

$$P_T^*(x_1,t) = \frac{J_m}{K_s}\frac{\partial^2 P^*}{\partial t^2} - \frac{EI}{K_s}\frac{\partial^2 P^*}{\partial x_1{}^2} \qquad (5)$$

In the above equations EI is the bending stiffness, m_b^* the mass per unit length, χ^* the stiffness per unit length of the springs that connect the beams, a is a characteristic length associated with the length of the structure in the x_2-direction, $2b^*$ describes the range of distributed load in x_1-direction, t the time, Ω^* the load frequency, K_s the shear stiffness , J_m the rotary inertia of mass per unit length of the beam, W_j^* the vertical displacement for beam, σ_{zz}^* the vertical stress and $H(\cdot)$ denotes Heaviside function [4].

The displacement $\mathbf{u}^* = [u^*(x_1,x_3,t), 0, w^*(x_1,x_3,t)]$ generated inside the layer by the moving load obeys the Navier's equation of motion

$$((\lambda_{cl}^* + \mu_{cl}^*)\nabla(\nabla\cdot\mathbf{u}^*) + \mu_{cl}^*\nabla^2\mathbf{u}^* = \rho^*\frac{\partial^2\mathbf{u}^*}{\partial t^2} \qquad (6)$$

where $\mu_{cl}^* = \mu^* + \mu_d{}^* \dfrac{\partial}{\partial t}$ and $\lambda_{cl}^* = \lambda^* + \lambda_d{}^* \dfrac{\partial}{\partial t}$ are the operators used to describe the viscoelastic behavior of the medium, ρ^* is the mass density and operator nabla ∇ acts in the $x_1 x_3$ plane. The boundary and interface conditions are as follows:

$$W_2^*(x_1,t) = w^*(x_1,h^*+d^*,t),\ W_1^*(x_1,t) = w^*(x_1,h^*,t),\ u^*(x_1,h^*,t) = 0,$$
$$u^*(x_1,h^*+d^*,t) = 0,\ \sigma_{zz}^*(x_1,0,t) = 0,\ \sigma_{xz}^*(x_1,0,t) = 0,$$
$$u^*(x_1,h^*+d^*+H^*,t) = 0,\ w^*(x_1,h^*+d^*+H^*,t) = 0. \tag{7}$$

where h^*, and H^* denote upper and lower layer thickness, respectively and d^* is distance between them.

GENERAL FORM SOLUTION

The dynamic response of the structure can be found from the system of equations (1), (2), (6) and (7). In order to work with the minimum number of parameters that determine the solution of the problem in hand, the following dimensionless variables and parameters are introduced :

$$x = \frac{x_1}{a},\ z = \frac{x_3}{a},\ \tau = \frac{c_T}{a}t,\ W_j = \frac{W_j^*}{a},\ \mathbf{u} = [u,0,w] = [\frac{u^*}{a},0,\frac{w^*}{a}],$$
$$m_b = \frac{a^2 c_T^2}{EI} m_b^*,\ \chi = \frac{a^4}{EI}\chi^*,\ \mu = \frac{a^4}{EI}\mu^*,\ \mu_d = \frac{c_T \mu_d^*}{a\mu^*},\ P_0 = \frac{a^3 P_0^*}{b^* EI},$$
$$h = \frac{h^*}{a},\ d = \frac{d^*}{a},\ H = \frac{H^*}{a},\ b = \frac{b^*}{a},\ V = \frac{V^*}{c_T},\ \Omega = \frac{a}{c_T}\Omega^*,$$
$$K_a = c_T^2(\frac{J_m}{EI} + \frac{m_b^*}{K_s}),\ K_b = \frac{c_T^4 m_b^* J_m}{EI\, K_s},\ K_c = \frac{c_T^2 J_m P_0^*}{bEI\, K_s},\ K_d = \frac{P_0^*}{bK_s}. \tag{8}$$

The quantities introduced above without an asterisk are the respective dimensionless variables. The problem can be solved by application the exponential Fourier transforms, over space variable x and time τ. Applying the Fourier transforms to equations of motion and boundary conditions in dimensionless form one obtains, after solution, the transformed dimensionless displacements. The amplitude spectra of vibration at the surface can be written as

$$\mathbf{u}_f(f) = [u_f(f),0,w_f(f)] = \frac{P_0}{V}\frac{\sin bk}{k(1-(bk/\pi)^2)}\hat{\mathbf{u}}_0(k,\omega) \tag{9}$$

where $k = \dfrac{\Omega - 2\pi f}{V}$, $\omega = -2\pi f$.

At the layer surface the vertical and horizontal displacements are given by the inverse Fourier transforms. The vector function $\hat{\mathbf{u}}_0(k,\omega) = [\hat{u}_0,0,\hat{w}_0]$ in the Fourier transform domain is given explicitly in the paper [4], by means of five determinants of order 8.

To investigate vibrations at the surface one can use the point $x=0$. The double integral for displacements can be reduced to the single Fourier integral

$$\mathbf{u}(0,0,\tau)=\frac{P_0}{2\pi}\int_{-\infty}^{+\infty}\frac{\sin bk}{k(1-(bk/\pi)^2)}\hat{\mathbf{u}}_0(k,Vk-\Omega)e^{-i(Vk-\Omega)\tau}dk \tag{10}$$

For the case of Timoshenko beam the effect of the rotary inertia of mass and shear distortion become more and more important as the beam stiffness, the forced velocity and frequency increase [2].

CONCEPT OF COIFLET BASED FOURIER INVERSION

Let φ and ψ be the scaling function and wavelet function, respectively. Applying Fourier transform defined as

$$\hat{f}(\omega)=\int_{-\infty}^{+\infty}f(\tau)e^{-i\omega\tau}d\tau\,,\quad f(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{f}(\omega)e^{i\omega\tau}d\omega \tag{11}$$

to the refinement equations

$$\varphi(\tau)=2\sum_{k=0}^{L-1}p_k\varphi(2\tau-k)\,,\ \psi(\tau)=2\sum_{k=0}^{L-1}q_k\varphi(2\tau-k) \tag{12}$$

one obtains

$$\hat{\varphi}(\omega)=P(e^{-i\frac{\omega}{2}})\hat{\varphi}(\frac{\omega}{2}),\ \hat{\psi}(\omega)=Q(e^{-i\frac{\omega}{2}})\hat{\varphi}(\frac{\omega}{2}) \tag{13}$$

where the integer $L-1$ indicates the support length of φ and ψ, $i=\sqrt{-1}$ and p_k, q_k are wavelet filter coefficients [8]. The filter functions P and Q are defined as

$$P(z)=\sum_{k=0}^{L-1}p_k z^k\ ,\ Q(z)=\sum_{k=0}^{L-1}q_k z^k\ . \tag{14}$$

One should note, that it is actually immaterial which of the transform operations in Eqs. (11) is conducted with the negative exponent, and where the factor $1/2\pi$ is placed. Applying recursion to Eq. (13.1) and using normalisation condition $\hat{\varphi}(0)=1$ leads to the product

$$\hat{\varphi}(\omega)=\prod_{k=1}^{\infty}P(e^{-i\frac{\omega}{2^k}}) \tag{15}$$

Among compactly supported orthogonal wavelets a family known as coiflets has a number of properties that make it particularly useful in numerical analysis. The coiflets constructed by Daubechies as well as that one used in the reference [8] correspond to particular integer choice of the first moment α of φ. In this paper we use the fact that the first moment of the scaling function

$$\alpha=\int_{-\infty}^{+\infty}x\varphi(x)dx \tag{16}$$

(called also shift [6]) does not have to be an integer. One of the key properties of interest for coiflet bases is the property of vanishing possibly high number moments for wavelet function and shifted moments for scaling function, which yield [6]

$$\sum_{j=0}^{L-1}(-1)^j j^k p_j = 0\,,\ \ 0 \le k \le N_\psi$$
$$\sum_{j=0}^{L-1}(j-\alpha)^k p_j = \delta_{k0}\,,\ \ 0 \le k \le N_\varphi \tag{17}$$

where δ_{k0} is the Kronecker symbol. The non-integer value of α should be used, if the integrand of the Fourier integral has singularity for the integer value. Such singularity appears for the integrand in Eq. (10). In general, the coiflets have no explicit expressions and can be described in terms of the coiflets filter coefficients p_k .To obtain the inversion of $\hat{f}(\omega)$ one must calculate the multiresolution coefficients [8]. These coefficients involve inner products with wavelet bases and are usually difficult to calculate. Monzon et al.[6] constructed coiflets filter coefficients with non-integer shifts, which are suitable for inversion of the more complicated functions $\hat{f}(\omega)$. Coiflets are meant to maximize both numbers of vanishing moments N_φ and N_ψ, while their values (N_φ and N_ψ) remain close to each other. The example of coiflet filter coefficients for non-integer α and $L = 18$, which has the maximal number of vanishing moments, is given in the paper [6] (see p. 207, Table 5, case Mb). According to the theory of multiresolution analysis one can obtain the formula for the inverse Fourier transform as [8]

$$f_m(\tau) = \frac{1}{2^{m+1}\pi}\hat{\varphi}(-\frac{\tau}{2^m})\sum_{n=n_{\min}}^{n_{\max}}\hat{f}(\omega = \frac{n+\alpha}{2^m})e^{i\frac{n}{2^m}\tau} \tag{18}$$

where

$$n_{\min} = \omega_{\min}2^m - 3N_\varphi + 2\,,\ n_{\max} = \omega_{\max}2^m - 1\ ,\ f(\tau) = \lim_{m\to\infty} f_m(\tau)\,. \tag{19}$$

The maximum and minimum value of n in Eq. (18) can be determined once the frequency bandwidth $[\omega_{\min}, \omega_{\max}]$ has been chosen. The proper range of variable ω can be found from condition of covering the main lobe of $\hat{f}(\omega)$ for which it has a significant influence on the characteristic of original function. The term $f_m(\tau)$ coincides approximately with $f(\tau)$ within the interval $[-2^m\pi, 2^m\pi]$ and becomes almost zero outside this interval. Increasing m leads to the larger range in which the inversion agrees with the original function $f(\tau)$. To investigate the similitude between $f_m(\tau)$ and $f(\tau)$ an error index can be defined as

$$f_{er}(\tau) = \frac{|f_m(\tau) - f_{m-1}(\tau)|}{\max\limits_{\tau}|f_m(\tau)|}\cdot 100\,\% \tag{20}$$

The term $f_m(\tau)$ can be accepted as inversion when the assumed level of error index is not exceeded.

NUMERICAL RESULTS

Using Eq. (18) the following formula can be derived for numerical calculations:

$$w_m(\tau) = \mathrm{Re}\left[\frac{1}{2^{m+1}\pi}\left(\prod_{n=1}^{n_p} P_n(\tau)\right)\left(\sum_{n=n_{\min}}^{n_{\max}} f_w e^{i\frac{n}{2^m}\tau}\right)\right] \tag{21}$$

where [3]

$$f_w = \frac{1000P_0}{2\pi}\frac{\sin(b(\Omega-\omega)/V)}{b(\Omega-\omega)(1-(b(\Omega-\omega)/\pi V)^2)}\hat{w}_0(k=\frac{\Omega-\omega}{V},-\omega) \tag{22}$$

$$P_n(\tau) = \sum_{j=0}^{L-1} p_j e^{i\frac{j}{2^{n+m}}\tau}, \ \omega = \frac{n+\alpha}{2^m}, \ n_{\min} = \omega_{\min} 2^m - 3N_\varphi + 2, \ n_{\max} = \omega_{\max} 2^m - 1,$$

$$\omega_{\min} = -\omega_{\max}. \tag{23}$$

The numerical analysis is performed for $\omega_{\max} = 1, n_p = 10, N_\varphi = 6$, ($L-1 = 3N_\varphi - 1$). The case of Euler-Bernoulli beams, i.e. $W_{Tj}^* = 0$ and $P_T^* = 0$ is assumed in further calculations. In general, the value of m in Eq. (21) should increase from $m \ge 5$ to $m \ge 10$, with increasing the load velocity V, load frequency Ω and layer thickness h. With increasing parameters V, Ω or h the displacements in wider range $\tau \in [-2^m\pi, 2^m\pi]$ should be examined. For $\Omega = 0.4, V = 0.4$ and $h = 2$ even $w_5(\tau)$ leads to good approximation. For $h > 3$ one should take $w_{10}(\tau)$ or higher term. In order to illustrate the influence of the moving load on the vibration level of the layer surface, the parametric study has been performed. The numerical analysis is based on the program prepared in MATHEMATICA system. The dimensionless material parameters used here can be obtained from the literature [1], [5] and [7]. As a basis in calculations, the following values of parameters are taken for the numerical analysis: $\nu = 0.3$, $\mu = 4.923\times10^{-2}$, $\mu_d = 2.659\times10^{-3}$, $d = 2$, $H = 2$, $b = 2$, $m_b = 1.81\times10^{-2}$, $\chi = 2.56\times10^{-1}$, $P_0 = 8\times10^{-5}$.

The surface vibration in Fig. 1 is shown, depending on the layer depth h. In general, increasing h leads to decreasing level of surface vibration. For the assumed parameter set, two critical layer thickness can be seen: $h_{cr1} = 3.43$ and $h_{cr2} = 4.27$. For layer thickness $h < h_{cr1}$ the biggest amplitude of waves occur for $\tau = 0$. For the case $h_{cr1} < h < h_{cr2}$ the amplitude of waves moving in front of the load($\tau < 0$) is much bigger than that behind the load($\tau > 0$). For $h > h_{cr2}$ the amplitude of waves moving in front of the load is smaller than that behind the load.

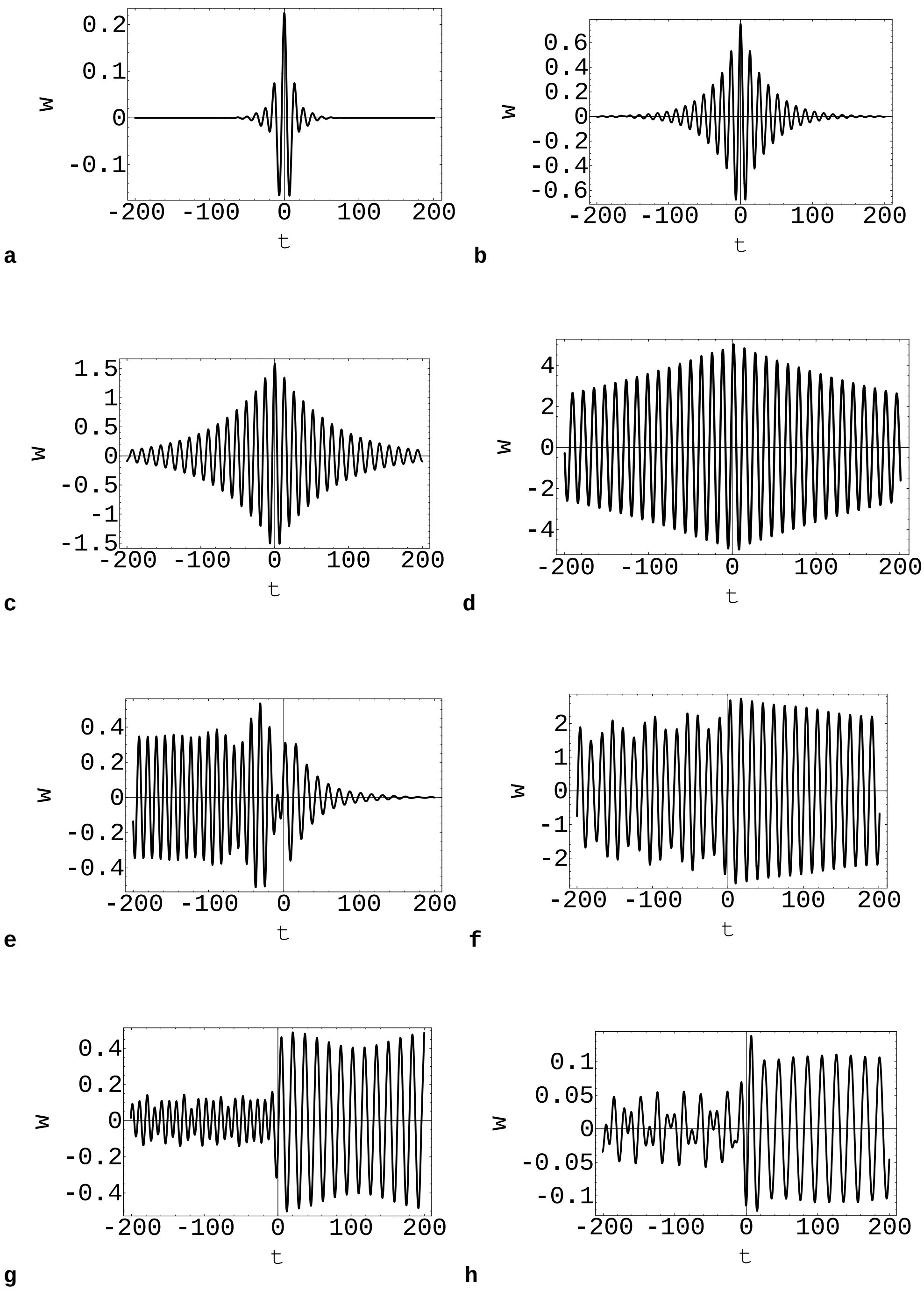

Figure 1. Vertical displacement of the layer surface for $V = 0.4$ *and* $\Omega = 0.4$*, depending on the layer depth*: a) $h = 2$, b) $h = 3.3$, c) $h = 3.4$, d) $h = 3.43$, e) $h = 4$, f) $h = 4.27$, g) $h = 5$, h) $h = 8$.

CONCLUSIONS

Vibration of the layer generated by the distributed load moving along the beam inside the layer is considered using wavelet theory. A starting point to model the railway track is the Timoshenko beam theory. Wavelet approach is better than direct numerical integration because the execution time required is much smaller and the analysis of error is possible. In order to work with the minimum number of parameters that determine the solution of the problem in hand, the dimensionless formulation has been introduced. The properties of the solution can be investigated quantitatively on the basis of the derived formulae, Eqs. (9) and (10), for the amplitude spectra and displacements using MATHEMATICA system. Wavelet approach seems to be the efficient method to overcome some shortcomings of the Fourier transform method. The solution for displacement depends on the considerable number of parameters, especially on the upper layer thickness, moving load speed and the load frequency. Excluding the critical depths, the effectiveness of the increase of layer thickness for mitigating surface vibration can be seen in presented figures. As a result of critical depths, the structures under which the trains move can be subjected to large dynamic stresses and the wave radiation effects become important for the development of high speed train tracks in tunnels. The coiflet based approach can be considered as efficient and powerful technique in solving many structural dynamics problems.

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