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A MULTISCALE COMPUTATIONAL APPROACH FOR STRUCTURAL AND ACOUSTIC MEDIUM-FREQUENCY VIBRATIONS

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Abstract
The Variational Theory of Complex Rays (VTCR) is an approach that was developed in order to calculate the vibrations of slightly damped elastic structures in the medium-frequency range (Ladevèze 1996). In this paper, we extend it to acoustic problems with infinite boundary condition. Performance results obtained for two-dimensional and three dimensional Helmholtz problems highlight the advantages of the method, which yields a high accuracy with a smaller computational effort than the finite element method. Therefore, computation at higher frequencies can be reached.

INTRODUCTION

The standard Galerkin finite element method (FEM) and the boundary element method (BEM) are the most commonly used prediction methods for solving structural and acoustic problems. However these technics require many degrees of freedom as the (fastly) oscillating solutions are approximated by continuous, piecewise polynomial functions. The size of the models obtained practically restricts the applicability of these prediction techniques to the low-frequency range. Furthermore, the pollution effect (Ihlenburg and Babuska 1995, Deraemaeker et al. 1999) avoid the FEM having a robust performance with respect to the wavenumber. Indeed, when the wavenumber is increased and a fixed level of accuracy is desired, the standard FEM requires increasing the number of degrees of freedom appropriately.

Different approaches have been developed in order to circumvent this problem. These approaches include predefined reduced bases (Soize 1998), the Galerkin/least-squares FEM (Harari and Hughes 1992), the partition of unity method (Melenk and Babuska 1996), the Generalized FEM (Strouboulis et al. 2000), the residual-free bubbles (Franca et al. 1997), the ultra weak variational method (Cessenat and Despres 1998), the discontinuous enrichment method (Farhat et al. 2001) and the wave boundary element method (Perrey-Debain et al. 2003). However, these methods operate at the level of the elements, whose number must be large to represent the dynamic behavior properly. Therefore, these techniques are essentially limited to the low-frequency range. Trefftz approaches has also been developed (Desmet et al. 2001) but don't take into account the multiple scales appearing in the shape functions (which are waves which satisfy the governing differential equations exactly), and therefore lead to long computation times and are limited to simple geometry examples.

Other approaches were proposed for the high frequency regimes. Statistical Energy Analysis (SEA) (Lyon and Maidanik 1962) involves the description of the energy exchanged among various systems and yields the dynamic response of each system averaged over time and space. However, the SEA, which is characterized by a single energy level, cannot provide the spatial variation of the response within each system. Extensions of the SEA have been developed (Belov and Ryback 1975, Ichchou et al. 1997, Langley 1992, Lase et al. 1994, Langley and Cotoni 2004), but still require additional information (e.g. coupling loss factor, energy reflection coefficient, energy transmission coefficient...) in order to yield predictive results and, consequently, can compute predictive solutions only for specific geometries, such as bars or beams.

THE VTCR

The Variational Theory of Complex Rays (VTCR) was proposed in Ladevèze 1996 and in Ladevèze and Arnaud 2000 as a predictive tool for the prediction of vibration problems in the medium frequency range. Its fundamental aspects are described in Ladevèze and al. 1999 and in Riou and al. 2004.

The first characteristic feature of the VTCR is the use of a new mixed variational formulation of the problem, which was developed so that the approximations within the substructures can be independent of one another. Therefore, it is not necessary for these approximations to satisfy a priori compatibility and equilibrium conditions at the interfaces between substructures. Instead, these conditions are incorporated into the variational formulation.

The second feature which characterizes the VTCR is the introduction of two-scale approximations with a strong mechanical meaning: the solution is assumed to be properly described locally as a wave band which is the superposition of an infinite number of propagative and evanescent waves. Each wave associated with a substructure verifies the governing equation and the constitutive law over the substructure's domain. All wave directions are taken into account. The waves constitute two-scales approximations. The slowly varying scale (amplitude of the waves) alone is discretized. The rapidly varying scale (phase of the waves) is taken into account analytically. The unknowns are discretized amplitudes of the slowly varying parts of the solution with relatively long wavelengths. Therefore, no refined discretization is needed and the approximate solution is obtained by a small, computationally efficient model compared to that of the FEM or BEM.

Numerous examples have shown the effectiveness of this approach in terms of convergence rate and computational complexity. Figure 1 shows the comparison between the VTCR and the FEM solutions on a plate structure. The VTCR solution has been obtained with 60 degrees of freedom (dofs). The FEM solution used 1225 dofs (10 dofs per wavelength).This figure shows that, for a given level of accuracy, the VTCR requires very less dofs. This is due to the fact that only the slowly varying scale is discretised.

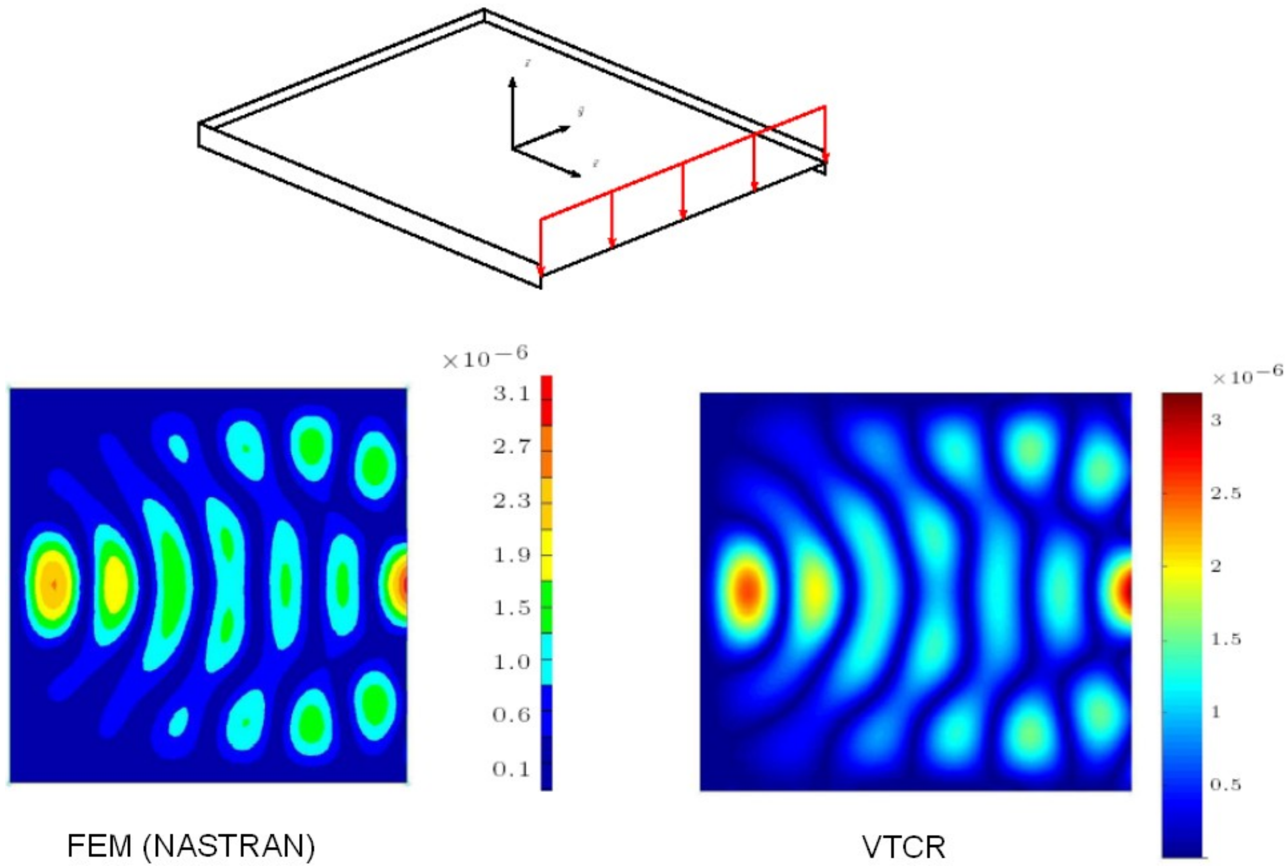

Figure 1 - Comparison between the FEM (1225 dofs) and the VTCR (60 dofs).

ACOUSTIC PROBLEMS

For an acoustic problem, the boundary value problem (BVP) to solve on $\Omega=\Omega_1\cup\Omega_2$ is

Find $u\in H^1(\Omega)$ such that

$$
\begin{aligned}
&\Delta p+k^2 p=0 \text{ in } \Omega\\
&p=p_p \text{ on } \partial_p\Omega\\
&v=v_p \text{ on } \partial_v\Omega\\
&p_1=p_2 \text{ on } \Omega_1\cap\Omega_2\\
&v_1+v_2=0 \text{ on } \Omega_1\cap\Omega_2
\end{aligned}
$$

where $v=\dfrac{i}{\rho\omega}\dfrac{\partial p}{\partial n}$ is the velocity, and $\partial_p\Omega$ (resp. $\partial_v\Omega$) is the part on the boundary where the pression p_p (resp. velocity v_p) is prescribed. This BVP is transformed to the equivalent variational problem :

$$
\mathrm{Imag}\left(\begin{aligned}&\sum_{e=1,2}\int_{\partial_p\Omega_e}(p_e-p_{ep})\delta v_e^* \,ds+\sum_{e=1,2}\int_{\partial_v\Omega_e}\delta p_e(v_e-v_{ep})^* \,ds\\ &+\frac{1}{2}\int_{\Omega_1\cap\Omega_2}\left((p_1-p_2)(\delta v_1-\delta v_2)^*+(\delta p_1+\delta p_2)(v_1+v_2)^*\right)\end{aligned}\right)=0 \quad \forall\,\delta p_e\in S_{e,ad}
$$

where * is the complex conjuguate and $S_{e,ad}$ the space of functions that satisfy $\Delta p+k^2 p=0$ in Ω_e. This variational formulation can easily be extended to structures with more than two substructures (Ladevèze and Arnaud 2000).

The solution is searched in this way: $p(\mathbf{x})=\int_\theta p(\theta)e^{ik\boldsymbol{\theta}.\mathbf{x}}d\theta$, where $\mathbf{x}$ is the position vector and θ is direction of propagation of the wave. The unknown $p(\theta)$ is discretized and the discrete approximation of the solution is written as

$$p(\mathbf{x})=\Sigma_j \int_{\theta_j}^{\theta_{j+1}} p(\theta_{j+1/2}) e^{ik\boldsymbol{\theta}.\mathbf{x}} d\theta = \Sigma_j p(\theta_{j+1/2}) \int_{\theta_j}^{\theta_{j+1}} e^{ik\boldsymbol{\theta}.\mathbf{x}} d\theta$$

Only the unknown $p(\theta_{j+1/2})$ have to be computed. The wave band $\int_{\theta_j}^{\theta_{j+1}} e^{ik\boldsymbol{\theta}.\mathbf{x}} d\theta$, which corresponds to the superposition of all the waves travelling in the direction $\left[\theta_j;\theta_{j+1}\right]$, is explicitely taken into account in the computation.

Many examples have been computed to assess the performance of the VTCR. One of them is an acoustic cavity of a car on which a given velocity has been prescribed on one boundary. Figure 2 shows the solution to obtained and the comparison between the VTCR and the FEM. Regarding the number of degrees of freedom to use to have an accurate solution, the VTCR yields a very interesting convergence rate.

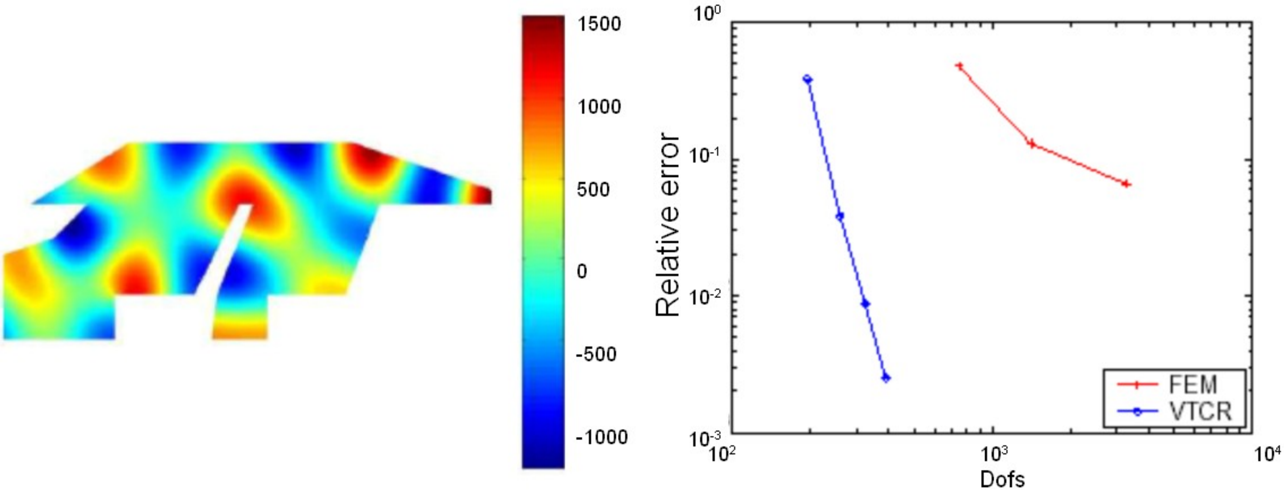

Figure 2 - Comparison between the FEM and the VTCR on an acoustic car cavity example.

The extension of the VTCR to infinite problems is easily taken into account. The infinite condition of Sommerfeld can be approximated by a finite condition on an absorbing boundary. To this end, the absorbing conditions given in Bayliss et al. 1982 can be used (among others), and the local equation to satisfy $v=\frac{i}{\rho\omega}\left(ikp-\frac{p}{2r}\right)$ is taken into account in the variational formulation by the new quantity $\int \delta p\left(v-\frac{i}{\rho\omega}\left(ikp-\frac{p}{2r}\right)\right)^* ds$.

Figure 3 shows the scattering of a circle sumbitted to an incident wave (this problem has an exact analytical solution). The solution given by the VTCR (136 dofs) is on the right and is very similar the the exact solution.

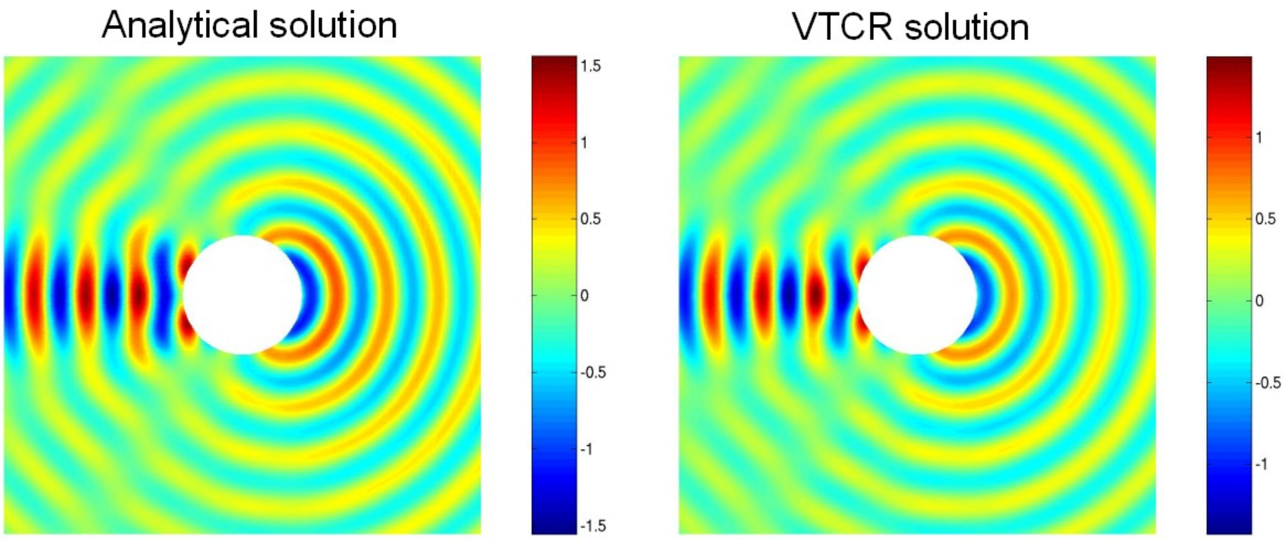

Figure 3 - Scattering of a circle. Comparison between the exact solution (left) and the VTCR solution (right, 136 dofs).

CONCLUSIONS

The approach proposed here, called the "Variational Theory of Complex Rays", was originally introduced in order to calculate the vibrations of slightly damped elastic structures in the medium-frequency range. It is a general multiscale approach with a strong mechanical basis. This paper emphasizes the extension of this theory to the analysis of acoustic cavities, with finite or infinite boundaries. The key points (introduction of evanescent waves in the shape functions space, conditioning of the matrices, p or h vision, extension to 3-D problems) will be discussed.

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