ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

CALCULATION OF LOGARITHMIC DECREMENT BY MORLET WAVELET OF A DECAY CURVE

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Abstract

Estimation of damping properties is one of most important tasks in structural dynamics. Despite noticeable development of finite element methods results of the modeling can be inaccurate especially for complex structures at comparatively high frequencies. Therefore experimental techniques are widely employed for estimation of energy dissipation in a structure.

Analysis of output signals measured on the structure is widely used. The consecutive post- process of responses is performed either in time or frequency domain. Unfortunately accuracy of calculations is impaired by noise components of the recorded signals.

It is proposed method to calculate logarithmic decrement corresponding to natural frequency of a structure. The method is based on Morlet wavelet transform of a measured decay curve. The derived formula enables to extract logarithmic decrement without complex signal processing and computational procedures. The technique can be employed for decay curve containing several natural frequencies. The proposed procedure should be applied to every natural frequency apart. Details of the signal processing are described. Logarithmic decrement of the pipe's natural mode is calculated as a demonstration of the method.

The technique is not sensitive to noise component presented in a measured signal. It provides high accuracy in detection of damping properties of a structure even at not satisfactory signal-to-noise ratio. The method is appropriate for modal testing of different structures that can be found in engineering practice.

INTRODUCTION

Some average (integral) estimation of dissipative properties of solid structures or elements containing fluid/gas is needed for many engineering tasks. Logarithmic decrement is related to natural logarithm of amplitude ratio for two consecutive oscillations in a decay curve [1]. It is conventionally utilized to describe rate of decay for free oscillations.

In accordance to Basil's hypothesis every natural frequency has its unique logarithmic decrement. A logarithmic decrement can be extracted from data analysis either in time domain or frequency domain [1], [2]. Impact hammer method with analysis of the decay curve or accelerance functions are commonly used in engineering practice.

These methods are very sensitive to quality of initial data. A source signal may contain many noise components or some probable natural frequencies have very close frequencies. Such procedures like signal windowing, truncation, coherence analysis have been proposed to refine the original data. However if a structure itself has a rich spectrum of natural frequencies, procedure of natural modes and shapes extraction looks implicit.

Another robust method would be useful for many practical applications. The technique based on wavelet transform is proposed as an alternative to the conventional procedures.

IMPLEMENTATION OF WAVELET TRANSFORM TO DAMPING PROPERTIES ANALYSIS

Such features of wavelet transform like time localization, easy scaling and shift make it attractive for implementation in a signal processing. A measured signal can be represented by a set of wavelets gathered from mother wavelet by scaling or shift procedure [3],[4]:

$$\psi(t)=\psi(a,b,t)=\frac{1}{\sqrt{a}}\psi_0\left(\frac{t-b}{a}\right), \tag{1}$$

where $\psi(t)$ is a wavelet transform of the measured signal, a- scale factor, b- time location, $\psi_0(t)$ is the mother wavelet. Multiplier $a^{-0.5}$ is introduced in expression (1) to normalize all wavelet functions.

Basic idea of the wavelet transform states that a signal $x(t)$ can be represented as a sum of scaled mother functions [3],[4]:

$$x(t)=\sum_k C_k\psi_k(t).$$

As a measurement is performed within limited time interval and $a, b \in R$ (R is real number domain), $a \neq 0$:

$$C(a,b) = \int_R x(t) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) dt .$$

In signal processing different types of the mother wavelet can be utilized. For this case Morlet wavelet is chosen due to many attractive features which are discussed in scientific literature. It is described as follows [5]:

$$\psi(t) = \frac{1}{\sqrt{\pi f_B}} e^{2j\pi f_c t} \cdot e^{-t^2/f_B} , \qquad (2)$$

where f_B is the frequency bandwidth, f_c is the wavelet central frequency. Morlet wavelet is well localized in real and Fourier space and represents a sinusoidal Gaussian- based function. However there is a balance between frequency and time resolution. With f_c increase frequency resolution gets higher but time localization goes down. Morlet wavelet function provides the best combination between frequency and time resolution [8]. Time resolution is increased as frequencies become higher.

Let us consider continuous wavelet transform of periodic signal $x(t) = \cos \omega t$ with Morlet mother wavelet:

$$C(a,b) = \frac{1}{\sqrt{\pi f_B a}} \int_{-\infty}^{+\infty} e^{j2\pi f_c \left(\frac{t-b}{a}\right)} e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} \cos \omega t dt = A(I_1 + jI_2)$$

where

$$A = \frac{e^{-\frac{2j\pi f_C b}{a}}}{\sqrt{\pi f_B a}},\ I_1 = \int_{-\infty}^{+\infty} \cos\left(\frac{2\pi f_c}{a} t\right) \cos(\omega t) e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} dt ,\ I_2 = \int_{-\infty}^{+\infty} \sin\left(\frac{2\pi f_c}{a} t\right) \cos(\omega t) e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} dt . \qquad (3)$$

Utilizing the Euler's formula and representation of the cosine function product by the cosine sum, one can get:

$$I_1 = \frac{1}{2}(I_3 + I_4),$$

$$I_3 = \int_{-\infty}^{+\infty} \cos\left[\left(\frac{2\pi f_c}{a} - \omega\right) t\right] e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} dt ,\ I_4 = \int_{-\infty}^{+\infty} \cos\left[\left(\frac{2\pi f_c}{a} + \omega\right) t\right] e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} dt .$$

It is possible to determine integrals I_3 and I_4 taking use of relation [6]:

$$\int_{-\infty}^{\infty} e^{-a^2x^2}\cos(mx)dx = \frac{\sqrt{\pi}}{a} e^{-\frac{m^2}{4a^2}} .$$

The integrals are:

$$I_3 = a\cos\left[b\left(\frac{2\pi f_c}{a} - \omega\right)\right]\sqrt{\pi f_B}\, e^{-\frac{f_B}{4}(2\pi f_c - a\omega)^2},\ I_4 = a\cos\left[b\left(\frac{2\pi f_c}{a} + \omega\right)\right]\sqrt{\pi f_B}\, e^{-\frac{f_B}{4}(2\pi f_c + a\omega)^2}.$$

Thus integral I_1 is expressed as follows:

$$I_1 = \frac{\sqrt{\pi f_B}\, a}{2}\left\{\cos\left[b\left(\frac{2\pi f_c}{a} - \omega\right)\right] e^{-\frac{f_B}{4}(2\pi f_c - a\omega)^2} + \cos\left[b\left(\frac{2\pi f_c}{a} + \omega\right)\right] e^{-\frac{f_B}{4}(2\pi f_c + a\omega)^2}\right\}. \quad (4)$$

The similar technique can be employed to calculation of integral I_2. Also the next property should be taken into account:

$$\int_{-\infty}^{+\infty} e^{-a^2x^2}\sin(mx)dx = 0.$$

The second integral takes form:

$$I_2 = \frac{\sqrt{\pi f_B}\, a}{2}\left\{\sin\left[b\left(\frac{2\pi f_c}{a} - \omega\right)\right] e^{-\frac{f_B}{4}(2\pi f_c + a\omega)^2} + \sin\left[b\left(\frac{2\pi f_c}{a} + \omega\right)\right] e^{-\frac{f_B}{4}(2\pi f_c - a\omega)^2}\right\}. \quad (5)$$

Formulas (3), (4), (5) enable to get analytical expression for the wavelet transform coefficients. It should be noted that the coefficients are complex numbers. Thus wavelet spectrum $C(a,b)$ can be represented by the amplitude $|C(a,b)|$ and phase $\arg C(a,b)$ surfaces in 3D graph. In this paper the wave spectrum is shown by projections to plane (a,b). It is possible to track change of amplitude $|C(a,b)|$ at different scale and time by iso- lines.

The modulus for Morlet wavelet can be gathered as follows:

$$|C(a,b)| = |A| \cdot \sqrt{I_1^2 + I_2^2} . \quad (6)$$

Taking into account formulas (3)~(5) one can get from (6):

$$|C(a,b)| = \frac{\sqrt{a}}{2} e^{-f_B\left(\pi^2 f_c^2 + 0,25 a^2 \omega^2\right)} \sqrt{e^{2\pi f_c f_B a\omega} + e^{-2\pi f_c f_B a\omega} + 2\cos 2b\omega} . \tag{7}$$

In conventional wavelet algorithm it is supposed that time intervals have a unit spacing. This point is kept here in effect too. Then it is convenient to take use of normalized formulas:

$$\omega_n = \frac{\omega}{f_d} ; t_n = t \cdot f_d ,$$

where f_d is the sampling frequency of an original analog signal.

Projection of the amplitude surface $|C(a,b)|$ of a periodic signal based on a Morlet wavelet represents a stripe of almost constant intensity that is parallel to the time axis. It is practically not sensitive to b parameter change. Section of the amplitude surface by plane $b = const$ is shown in Fig.1. The line in Fig.1 has clear maximum at

$$a_0 \approx \frac{2\pi f_c}{\omega} . \tag{8}$$

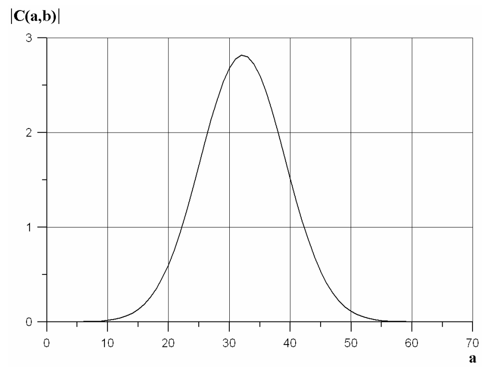


Fig.1 Modulus of coefficient ***C***

Expression (8) is an approximate ratio between scale of the Morlet transform and frequency resolution.

Modulus of the wavelet coefficient is a periodic function since expression (7) has member $2\cos 2b\omega$ under the square root sign. If $a > \frac{0.7}{f_c f_B}$ formula (7) can be simplified since members $\left(e^{-2\pi f_c f_B a\omega}\right)$ and $(2\cos 2b\omega)$ become negligible in comparison with the first term under the square root:

$$|C(a,b)| = \frac{\sqrt{a}}{2} e^{-f_B\left(\pi^2 f_c^2 + 0,25 a^2 \omega^2\right)} \sqrt{e^{2\pi f_c f_B a\omega}} = \frac{\sqrt{a}}{2} e^{-f_B(\pi f_c - 0,5 a\omega)^2} . \tag{9}$$

Instead of expression (8) it is possible to derive more precise formula for scale factor a_0 by taking derivative $\frac{d|C|}{da}$ that must equal zero:

$$a_0 = \frac{\pi f_c + \sqrt{\pi^2 f_c^2 + 1/f_B}}{\omega}. \tag{10}$$

If $f_c \gg \frac{1}{\pi\sqrt{f_B}}$ expression (10) can be approximated by formula (8). It can be represented in another form:

$$a_0 = \frac{2\pi f_c}{\omega} \cdot k(f_B, f_c).$$

At $f_B = f_c = 1$, k becomes equal 1.025, k value reaches 1.011 at $f_B = 1$ and $f_c = 1.5$. Maximum amplitude of C coefficient can be obtained by substitution of (8) into (9):

$$\max|C(a,b)| \approx \sqrt{\frac{\pi f_c}{2\omega}}. \tag{11}$$

The error of $\max|C(a,b)|$ estimation by expression (11) does not exceed 0.4 %.

If the periodic function has amplitude that is distinguished from unit (for example X), the modulus of the wavelet transform coefficient calculated by formula (7) or (9) must be multiplied by X.

CONTINUOUS WAVELET TRANSFORM OF A DECAY CURVE

Let us consider wavelet transform of decay curve. This function has zero values at $t < 0$:

$$x(t) = 1(t) A e^{-\alpha t} \cos \omega t,$$

where $1(t)$ is Heaviside's unit function, α is the logarithmic decrement. The wavelet coefficient can be calculated from integral expression:

$$\begin{aligned} C(a,b) &= \frac{1}{\sqrt{\pi f_B a}} \int_{-\infty}^{+\infty} e^{j 2\pi f_c \left(\frac{t-b}{a}\right)} e^{-\frac{1}{f_B}\left(\frac{t-b}{a}\right)^2} \cdot 1(t) \cdot A e^{-\alpha t} \cos(\omega t)\, dt = \\ &= \frac{A e^{-\frac{b^2}{f_B a^2} - 2j\frac{\pi f_c b}{a}}}{\sqrt{\pi f_B a}} \int_0^{\infty} e^{-\frac{1}{f_B a^2} t^2 + \left(\frac{2b}{f_B a^2} - \alpha + \frac{2 j \pi f_c}{a}\right) t} \cos(\omega t)\, dt = B \cdot I \end{aligned}, \tag{12}$$

where $\boldsymbol{B} = \dfrac{\boldsymbol{A}e^{-\frac{b^2}{f_B a^2} - 2j\frac{\pi f_c b}{a}}}{\sqrt{\pi f_B a}}$, $\boldsymbol{I} = \int\limits_0^\infty e^{-\frac{1}{f_B a^2}t^2 + \left(\frac{2b}{f_B a^2} - \alpha + \frac{2j\pi f_c}{a}\right)t} \cos(\omega t)dt$.

Table integrals in reference [7] allow to calculate integral $\boldsymbol{I}$:

$$\int\limits_0^\infty e^{-ax^2-cx}\cos(bx)dx = \frac{1}{4}\sqrt{\frac{\pi}{a}}\left\{e^{\frac{(c+jb)^2}{4a}} erfc\left(\frac{c+jb}{2\sqrt{a}}\right) + e^{\frac{(c-jb)^2}{4a}} erfc\left(\frac{c-jb}{2\sqrt{a}}\right)\right\},\ \text{at } Re\,a > 0 .$$

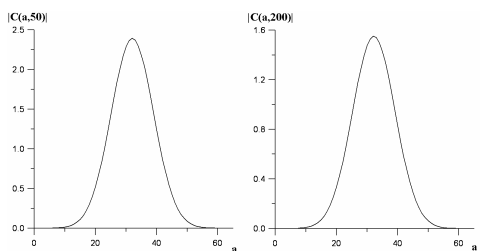
So the analytical solution is:

$$\begin{aligned} I = 0{,}25a\sqrt{\pi f_B}\Bigg\{ & e^{0{,}25\, f_B a^2\left[\alpha - \frac{2b}{f_B a^2} - j\left(\frac{2\pi f_c}{a} - \omega\right)\right]^2}\left(1 - erf\left(0{,}5a\sqrt{f_B}\left(\alpha - \frac{2b}{f_B a^2} - j\frac{2\pi f_c}{a} + j\omega\right)\right)\right) + \\ & + e^{0{,}25\, f_B a^2\left[\alpha - \frac{2b}{f_B a^2} - j\left(\frac{2\pi f_c}{a} + \omega\right)\right]^2}\left(1 - erf\left(0{,}5a\sqrt{f_B}\left(\alpha - \frac{2b}{f_B a^2} - j\frac{2\pi f_c}{a} - j\omega\right)\right)\right)\Bigg\} . \end{aligned} \tag{13}$$

The amplitude of the wavelet transform coefficient can be found from formulas (12) and (13):

$$|C(a,b)| = |B| \cdot |I| , \tag{14}$$

where $|\boldsymbol{B}| = \dfrac{\boldsymbol{A}e^{-\frac{b^2}{f_B a^2}}}{\sqrt{\pi f_B a}}$.



a) *b)*

Fig.2 Modulus of coefficient ***C*** *at a)* ***b=50****, b)* ***b=200***

Fig.2 represents sections of the wavelet coefficient amplitude surface by planes $\boldsymbol{b = const}$ at $\boldsymbol{f_B = f_c = 1}$, $\boldsymbol{\omega = 0{,}2}$, $\boldsymbol{\alpha = 0{,}003}$. One can see shape similarity of Fig.1 and 2. However with increase of parameter $\boldsymbol{b}$ the amplitude of all points of the last surface monotonically decreases (in a limit - up to zero).

Section of the amplitude surface by plane $\boldsymbol{a = const}$ at the same modeling parameters as above is represented in Fig.3. The curve has clear maximum. At the scale factor $\boldsymbol{a}$ determined by expression (10) the maxima is observed at $\boldsymbol{b_0 = 45}$. Incremental trend of the considered curve at small $\boldsymbol{b}$ numbers can be explained by the fact that the wavelet function approaches boundary $\boldsymbol{t = 0}$ (corresponds to $\boldsymbol{b = 0}$) and begins to spread out of the boundary $\boldsymbol{t > 0}$, where the original signal equals zero. With $\boldsymbol{a}$ reduction the value $\boldsymbol{b}$, at which the maximum of amplitude of the wavelet coefficient for considered cross-section takes place, decreases.

In the further analysis we shall consider only decreasing part of cross-section of the amplitude surface by plane $a = const$ since this part of the graph is a matter of interest to calculate logarithmic decrement α.

Part of the wavelet transform which corresponds to decay of free oscillations has coefficients $b > b_0$, where $|C(a,b)|$ reaches maximum at time location b_0, $a = a_0$, where a_0 is determined from expression (10). General formula (14) for the amplitude of the wavelet coefficient can be simplified for these parameters a and b. The approximate relation is:

$$|C(a,b)| \approx 0{,}25A\sqrt{a_0}\,e^{0{,}25\, f_B a_0^2 \alpha^2}\left(1 - erf\left(0{,}5a_0\sqrt{f_B}\left(\alpha - \frac{2b_0}{f_B a_0^2}\right)\right)\right)e^{-\alpha b} = Ze^{-\alpha b}, \quad (15)$$

where $a_0 = \dfrac{2\pi f_c}{\omega}k(f_B, f_c)$, $Z = 0{,}25A\sqrt{a_0}\,e^{0{,}25\, f_B a_0^2 \alpha}\left(1 - erf\left(0{,}5a_0\sqrt{f_B}\left(\alpha - \dfrac{2b_0}{f_B a_0^2}\right)\right)\right)$.

It should be noted that Z does not depend on b if $a_0 = const$. Deriving equation (15), it is taken into account that at $a = a_0$ and $b > b_0$: $\dfrac{2\pi f_c}{a} - \omega \approx 0$, and modulus of the second exponential term in equation (13) is significantly less than the second one. The calculation error by formula (15) does not exceed 12%.

Analysis of equation (15) discovers that at $a = a_0 = const$ the section of the amplitude surface at $b > b_0$ is an exponentially decreasing curve. Exponential degree is a negative of the product of logarithmic decrement and wavelet time location.

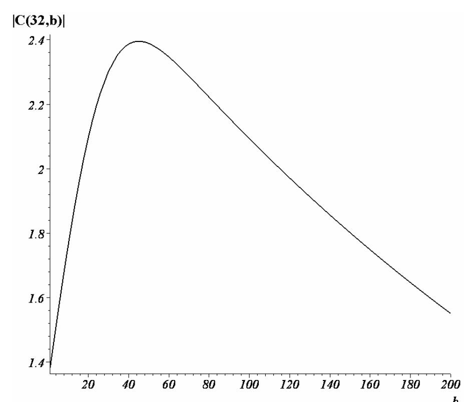


*Fig.3 Modulus of coefficient **C** depending on parameter **b***

CALCULATION OF THE LOGARITHMIC DECREMENT BY AMPLITUDE OF THE WAVELET COEFFICIENT

To calculate logarithmic decrement from amplitude of the wavelet coefficient let us consider system of 2 equations:

$$\begin{cases} |C(a_0, b_1)| = Ze^{-\alpha b_1}, \\ |C(a_0, b_2)| = Ze^{-\alpha b_2}. \end{cases} \quad (16)$$

As decaying part of the wavelet transform is considered at $b_1 > b_0$ and $b_2 > b_0$, solution of equations (16) can be found as follows:

$$\alpha = \frac{1}{b_2 - b_1} ln \frac{\left|C(a_0, b_1)\right|}{\left|C(a_0, b_2)\right|}. \tag{17}$$

Expression (17) is invariant to the time location. If the measured signal has time shift ***b'***, solution (17) is the same. However it is necessary to point out that equation (17) is valid at $b > b_0 + b'$ in this case.

The solution for logarithmic decrement can be expanded for discrete wavelet transform. Thus it is possible to extract logarithmic decrement from measured decay curve performing next operations.

1. Discrete wavelet transform with Morlet mother function should be executed.
2. Detect value of the wavelet scale factor a_0 by shape of the amplitude surface of the wavelet coefficient.
3. Calculate wavelet time scale coefficient $b_0 + b'$ at which effects at the signal boundaries cease to influence on the amplitude surface.
4. Choose two time locations b_1 and b_2 and respective amplitudes of the wavelet coefficients $\left|C(a_0, b_1)\right|$ and $\left|C(a_0, b_2)\right|$. The logarithmic decrement α is calculated by formula (17).

The procedures can be employed for decay curve containing several natural frequencies. The sequence above should be applied to every natural frequency apart.

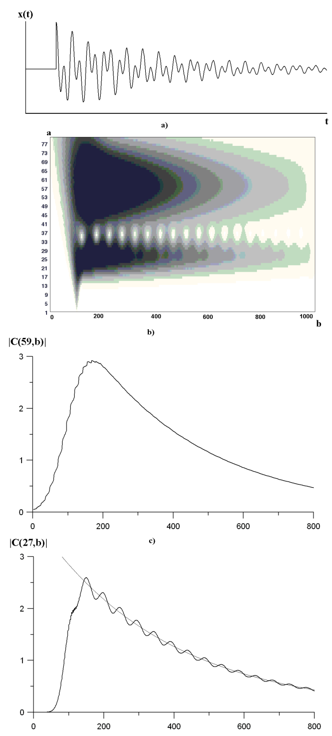


Fig.4 Test case: a) time decay curve b) projection of the wavelet transform c) section of the wavelet transform at ω=0.11 d) section of the wavelet transform at ω=0.24

DEMONSTRATION OF THE PROPOSED PROCEDURE

Fig.4a shows signal comprising two decaying components with parameters: $\alpha_1 = 0.003$; $\omega_1 = 0.11$; $A_1 = 1$; $\alpha_2 = 0.0027$; $\omega_2 = 0.24$; $A_2 = 1.1$. Projections of the corresponding amplitude surface are represented in Fig.4b. One can clearly see two areas pertained to maximums of wavelet coefficients at $a_1 = 59$ and $a_2 = 27$. They correspond to circular frequencies $\omega_1 = 0.11$ and $\omega_2 = 0.24$. Sections of the surface by plane at these frequencies are shown in Fig.4c and 4d respectively. Every curve resembles Fig.3. Oscillating form of the curve is explicitly highlighted in Fig.4d. It is connected with the fact that modulus of sum of 2 cosine functions is a cosine function too. Thus energy of oscillation with frequency $\omega_1 = \mathbf{0.11}$ is distributed throughout scale factors $\boldsymbol{a}$ and contributes to oscillation of the amplitude curve for higher component $\omega_2 = \mathbf{0.24}$.

Calculation of the logarithmic decrement by formula (17) from Fig.4c gives $\alpha_1 = \mathbf{0.003004}$. It is very close to the modeling magnitude ***0.003***. Curve in Fig.4d decreases at $b > 150$. The decaying part of the curve is approximated by an exponential function where coefficients are gathered by the least square method. Approximation of the curve in Fig.4d by exponential function and further implementation of formula (17) gives magnitude $\alpha_2 = 0{,}002698$. It is in a good compliance with the test value ***0.0027***.

Accuracy of the method if measured signal contains intensive noise components

The proposed method is robust to action of noise. Let us consider the previous decay curve (see paragraph "Continuous wavelet transform of a decay curve") with noise that is added by generator of random numbers. The noise component is described by expression:

$$x(t) = 1(t - b')(1 + rnd(1))e^{-\alpha(t-b')}\cos[\omega(t - b')],$$

where $b' = 100$, $\omega = 0{,}2$, $\alpha = 0{,}003$, $rnd(a)$ - random number within interval $0...a$. Approximating the wavelet section at $a = a_0 = 32$ and $b > 200$ by an exponential curve, one can get the logarithmic decrement $\alpha = 0{,}003074$. The calculation error in this case is less than 2.5%. Similar results are gathered if noise is represented by another dependence (Fig.5):

$$x(t) = 1(t - b')[e^{-\alpha(t-b')}\cos[\omega(t - b')] + rnd(1)],$$

where noise parameters are the same as for the case above.

To extract the logarithmic decrement from the wavelet transform, the wavelet section at $a = a_0 = 32$ has been approximated for time locations $b = 200...600$. The lower boundary of the interval is detected from condition $b > b_0$, the upper limit is chosen to provide satisfactory signal-to-noise ratio. Expression (17) gives value of the logarithmic decrement 0,003028. It means that the computational error is less than 1%.

Experimental verification of the proposed method

Detection of logarithmic decrement by using wavelet transform was performed for pipeline elements (see Fig.6). Projection of the wavelet surface can be seen in Fig.7a. The amplitudes reach maximums at $a = 29$. The Morlet wavelet is applied at frequencies $f_B = f_C = 1$ and $f_d = 20\,kHz$. Correspondingly, the third natural frequency of bending oscillation is calculated at 707Hz. Another method of natural frequencies extraction gives the 3rd mode at 703Hz [9]. Section of the wavelet transform at $a = a_0 = 29$ is represented in Fig.7b. Approximation of the curve by the least square method brings the logarithmic decrement value $\alpha = 121$.

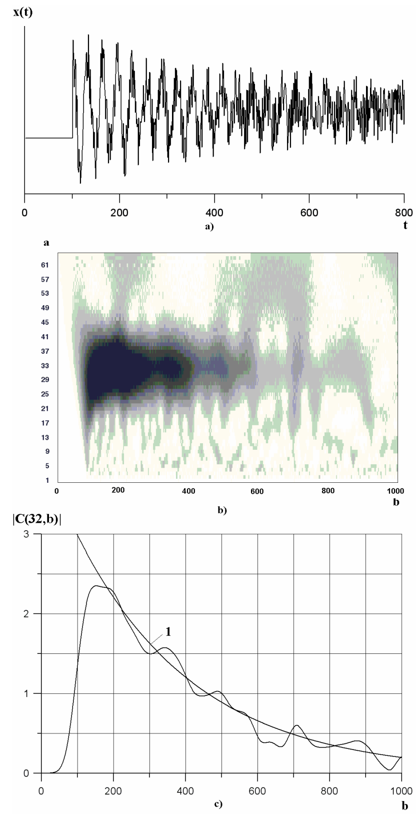


Fig.5 Signal processing for the decay curve with noise component: a) time decay curve b) projection of the wavelet transform c) section of the wavelet transform at $\boldsymbol{a_0}$*=32*

SUMMARY

It is proposed method to calculate logarithmic decrement corresponding to natural frequencies of a structure with linear viscous friction that does not depend on amplitude of excitation. The method is based on Morlet wavelet transform of a measured decay curve. The derived formula enables to extract logarithmic decrement without complex signal processing and computational procedures. The proposed technique is not sensitive to noise component presented in a measured signal. It provides high accuracy in detection of damping properties of a structure even at not satisfactory signal-to-noise ratio. These peculiarities of the proposed method make it attractive for practical implementation.

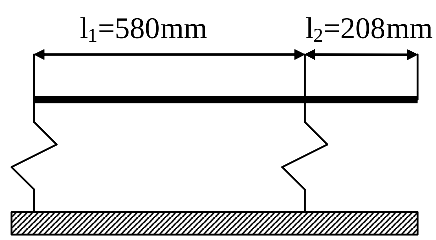


Fig.6 Scheme of the tested pipeline

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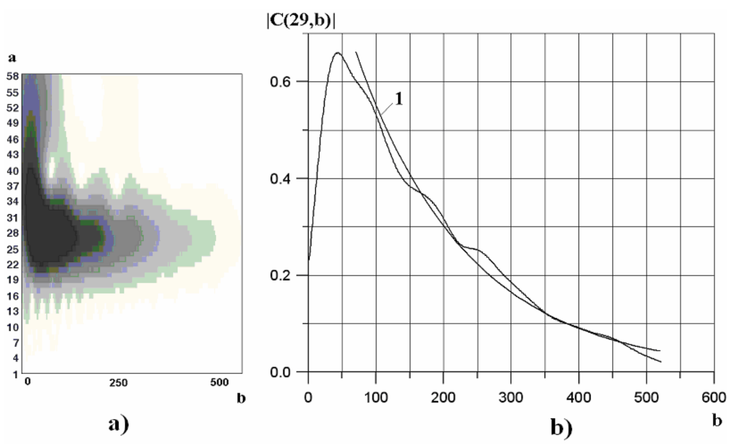

Fig.7 a) Projection of the Morlet wavelet for the measured decay curve b) Approximation (curve 1) of the wavelet section at $\boldsymbol{a_0}$*=29 for* $\boldsymbol{b>b_0}$