VIBRATION OF A VISCOELASTICALLY SUPPORTED VISCOELASTIC CANTILEVER BEAM WITH A TIP MASS SUBJECTED TO HARMONIC BASE EXCITATION

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Abstract

Vibration of a viscoelastic cantilever beam having a viscoelastic point support with a tip mass is analyzed by using the principle of relative motion within the framework of the Euler-Bernoulli beam theory. Kelvin-Voigt model is used for the material of the beam. The Lagrange equations are used to examine the steady-state response to harmonic base excitation of a viscoelastically supported viscoelastic cantilever beam with the tip mass. The constraint condition against rotation of the support end is taken into account by using the Lagrange multipliers. In the study, for applying the Lagrange equations, the trial functions denoting deflection of the beam is expressed in polynomial form. The influence of the support damping, internal damping and the tip mass on the steady-state response of the beam is investigated numerically for various external damping, internal damping and mass ratio.

INTRODUCTION

In recent years, base isolation of structures to earthquake excitation has been very important research topic. Thus, the dynamic response of beam-type structures with concentrated masses, such as chimneys of plants, communication towers, manipulator arms and many others, to base excitation are of considerable interest to the engineers designing mechanical and structural systems. As it is known vibration damping is very important in engineering practice. The damping treatments at the base of the structures can be alternative solution to surface damping treatments with viscoelastic materials for beams and plates in some cases. There are many studies on forced vibration of beams and plates ([1]-[8]).

In this study, the problem is analyzed by using the Lagrange equations with the trial function in polynomial form denoting the deflection of the beam for determining the peak values of the dynamic responses of the viscoelastically supported viscoelastic cantilever beam with a tip mass under the effect of the harmonic base excitation within the framework of Bernoulli-Euler beam theory. The constraint condition against rotation of the supported end is taken into account by using Lagrange multipliers. In numerical analysis, the steady-state response to a harmonic base excitation is determined for the first two peaks of the tip displacements for different values of internal damping, external damping and the mass ratio.

ANALYSIS

Consider a viscoelastically supported viscoelastic cantilever beam of length L, cross-section area A, moment of inertia I, modulus of elasticity E, mass of the beam per unit volume ρ with the tip mass under harmonic base excitation effect as shown in Fig.1, where k_s is the spring constant, c_s is the damping coefficient, M is the mass of the tip mass. The beam is constrained against rotation at the lower end. Also, the mass of this part is not taken into account, and it is assumed that there are no friction forces between the base and the beam system.

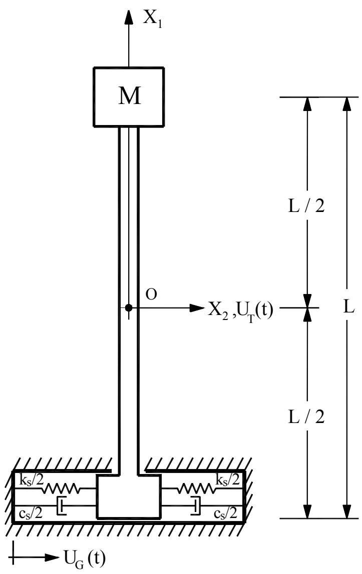

Figure 1-Viscoelastically supported viscoelastic cantilever beam with a tip mass under base excitation effect

The relative displacement is given as follows:

$$W(X,t) = U_T(X,t) - U_G(t), \qquad (1)$$

where $U_T(X,t)$ is total displacement, $U_G(t)$ is base displacement. For the beam subjected to harmonic base excitation, the base displacement can be expressed as $U_G(t) = \bar{U}_G\, e^{i\omega t}$, where $\bar{U}_G$ is amplitude of the base displacement and ω is radian frequency of the harmonic base movement. The constitutive relations for the Kelvin-Voigt model which is for the material between the stresses and strains become

$$\sigma = E\,\varepsilon + c_i\,\dot{\varepsilon} = E\left(\varepsilon + \eta_i\,\dot{\varepsilon}\right), \tag{2}$$

where ε is the longitudinal stress, c_i is the coefficients of the internal damping of the beam, η_i is the proportionality constants of the internal damping of the viscoelastic beam. Dimensions of the coefficient of the internal damping of the beam and proportionality constant of the internal damping of the beam are $\mathrm{N\,s/m^2}$ and s, respectively.

According to Bernoulli-Euler beam theory, the elastic strain energy of the beam, dissipation function and the kinetic energy of the beam with the tip mass at any instant are expressed as an integral in Cartesian co-ordinates, respectively

$$U = \frac{1}{2}\int_{-L/2}^{L/2} EI(X)\left(\frac{\partial^2 W(X,t)}{\partial X^2}\right)^2 \mathrm{d}X\,, \tag{3}$$

$$R = \frac{1}{2}\int_{-L/2}^{L/2} c_i\,I(X)\left(\frac{\partial^2 \dot{W}(X,t)}{\partial X^2}\right)^2 \mathrm{d}\mathrm{X}\,, \tag{4}$$

$$T = \frac{1}{2}\int_{-L/2}^{L/2} \rho A(X)\left(\frac{\partial U_T(X,t)}{\partial t}\right)^2 \mathrm{d}X + \frac{1}{2}M\left(\frac{U_T(X_m,t)}{\partial t}\right)^2, \tag{5}$$

where $W\left(X,t\right)$ is the steady-state response of the beam relative to the harmonic excitation and X_m is the co-ordinate of the tip mass. The kinetic energy due to the rotation of the beam and the tip mass is ignored. Also, for the considered parameters, the ratio of the tip mass to the mass of the beam is not large enough to cause secondary effects such as buckling. Therefore, secondary effects are not of concern and the bending is assumed as independent of axial loadings such as the weight of the beam and the tip mass. Additive strain energy and dissipation function of viscoelastic supports are

$$F_S = \frac{1}{2}\,k_s\left(U_T\left(X_S,t\right) - U_G\left(t\right)\right)^2 \tag{6a}$$

$$F_D = \frac{1}{2}\,c_s\left(\dot{U}_T\left(X_S,t\right) - \dot{U}_G\left(t\right)\right)^2, \tag{6b}$$

where X_S is the co-ordinate of the support of the beam, $U_T\left(X_S,t\right)$ and $\dot{U}_T\left(X_S,t\right)$ are the total displacement and the total velocity of the lower end of the beam, respectively. The functional of the problem is

$$I = T - \left(U + F_S\right). \tag{7}$$

In order to apply the Lagrange equations, the trial function for $W\left(X,t\right)$ is approximated by space-dependent polynomial terms $X^0, X^1, X^2, ..., X^{N-1}$ and time-dependent generalized displacement co-ordinates $q_n\left(t\right)$. Therefore,

$$W\left(X,t\right) = \sum_{n=1}^{N} q_n\left(t\right) X^{n-1}, \tag{8}$$

The only constraint condition of the support is satisfied by using the Lagrange multipliers. The constraint condition of the beam is given as follows:

$$W'\left(X_S, t\right) = 0 \tag{9}$$

In Eq. (9) X_S denotes the location of the support, prime denotes the derivative with respect to *X*. The Lagrange multipliers formulation of the considered problem requires us to construct the Lagrangian functional as follows:

$$L = I + \alpha W'\left(X_S, t\right), \tag{10}$$

In Eq. (10), α is the Lagrange multiplier which is the support moment reaction against the rotation of the supported end of the beam in the considered problem. Lagrange equations are given as follows:

$$\frac{\partial L}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} + Q_D = 0, \qquad k = 1, 2, 3, ..., N+1, \tag{11}$$

In Eq. (11), Q_D is the generalized damping force obtained from

$$Q_D = -\frac{\partial F_D}{\partial \dot{q}_k} - \frac{\partial R}{\partial \dot{q}_k}, \qquad k = 1, 2, 3, ..., N+1 \tag{12}$$

By introducing $q_{N+1} = \alpha$, and following non-dimensional parameters

$$x = \frac{X}{L},\ w\left(x,t\right) = \frac{W\left(X,t\right)}{L},\ u_T\left(x,t\right) = \frac{U_T\left(X,t\right)}{L},\ u_G\left(t\right) = \frac{U_G\left(t\right)}{L},$$
$$\kappa = \frac{k_s L^3}{EI},\ \gamma = \frac{c_s L}{\sqrt{\rho A E I}},\ \eta = \frac{\eta_i}{L^2\sqrt{\rho A / EI}},\ \lambda^2 = \frac{\rho A \omega^2 L^4}{EI},\ m_r = \frac{M}{\rho A L}, \tag{13}$$

and considering that when the base excitation is expressed as $U_G\left(t\right) = \bar{U}_G\ e^{i\omega t}$, then the time-dependent generalized co-ordinates are expressed as follows:

$$q_n(t) = \overline{q}_n \, e^{i\omega t} \tag{14}$$

In Eq. (14), $\overline{q}_n$ is a complex variable containing a phase angle. After the application of the Lagrange equations by taking into account Eq. (1), a set of linear algebraic equations is obtained which can be expressed in the following matrix form:

$$[A]\{\overline{q}\} + i\eta\lambda[B]\{\overline{q}\} + i\lambda\gamma[C]\{\overline{q}\} - \lambda^2[D]\{\overline{q}\} - m_r\lambda^2[E]\{\overline{q}\} = \{f\} \tag{15}$$

where $[A]$, $[B]$, $[C]$, $[D]$ and $[E]$ are coefficient matrices and $\{\overline{q}\}$ is vector of unknowns. Elements of the generalized force $\{f\}$ are expressed as

$$f_k = u_G\lambda^2 \int_{-1/2}^{1/2} x^{k-1}\,\mathrm{d}x + m_r u_G \lambda^2 (x_m)^{k-1} \quad k = 1, 2, 3, ..., N+1 \tag{16}$$

where m_r is the ratio of the mass of the tip mass to the mass of the beam. The maximum displacements of the base and the tip of the beam are given, respectively

$$w_b = \sum_{n=1}^{N} q_n(-0.5)^{n-1}, \;\; w_t = \sum_{n=1}^{N} q_n(0.5)^{n-1} \tag{17}$$

The dimensionless horizontal reaction force at the base is given below

$$R = (\kappa + i\gamma\lambda)\,w(-0.5) \tag{18}$$

NUMERICAL RESULTS

The steady state response of a viscoelastic cantilever beam with a tip mass to a periodic base motion $U_G(t) = u_G e^{i\omega t} L$, viscoelastically supported at the base, is calculated numerically. In the numerical calculations, dimensionless amplitude of the base movement is taken as $u_G = 0.1$. In the study performed by Kocatürk [6] steady state response with respect to base movement of viscoelastically supported cantilever beam was analyzed. In the study performed by Kocatürk et al. [7] steady state response with respect to base movement of viscoelastically supported viscoelastic cantilever beam was analyzed. In the present study, only a tip mass is added to the free end of the viscoelastic cantilever beam. Because a convergence study was made and it was found that 12 polynomial terms were sufficient for the desired numerical accuracy, in the calculation of the results of the present study, 12 terms of the polynomial series are used, namely the size of the determinant is 13x13.

Figures 2a, 3a and 4a show that with the variation of the damping parameter γ of the support, a damping parameter can be obtained for which the first and second peak values of the tip displacements respectively are minimum. The peak values of the tip displacements occur at different values of λ while changing the damping parameter γ. However, the frequency parameter λ remains between the frequency parameters λ obtained for $\gamma = 0$ and $\gamma = \infty$. Therefore, in Figures 2, while changing γ for obtaining minimum peak value of the tip displacements for the considered mode, the frequency parameter λ also changes a little. It is seen in Figure 2a that, regardless of the damping parameters of the support, there are some points of intersection of the transverse deflection curves of the tip of the beam and support reactions of the beam. Similar figures can be obtained for the other values of κ for the tip displacements and for the reaction forces. However, as seen from Figures 2b, c, d, 3b, c, d and 4b, c, d, when the internal damping is different from zero the curves obtained for various damping values of support do not intersect each other at the same point. It is seen from Figures 2b, c, d, 3b, c, d and 4b, c, d that, existence of internal damping reduces and smoothes the displacement responses significantly. Also, it is seen from these figures that, when both support damping and internal damping are existent, then the most effective vibration damping is obtained. For the considered parameters, when the internal damping parameter is $\eta = 0.1$, all the tip displacement responses w_t within the considered frequency range becomes smaller than 1 as seen from Figures 2d, 3d and 4d. As an expected result, Figures 2-4 show that, with the increase of the tip mass, the frequencies, in which the peak displacements occur, decrease.

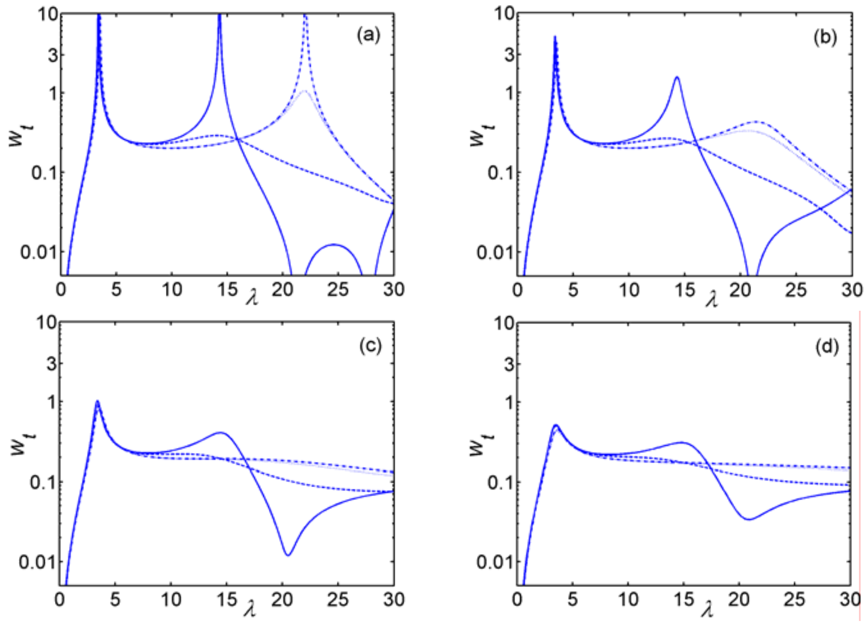

Figure 2-The tip displacements for the variation of λ for various values of γ, $\kappa = 100$, $m_r = 0$, (a) $\eta = 0$, (b) $\eta = 0.01$, (c) $\eta = 0.05$, (d) $\eta = 0.1$, $\gamma = 0$ ——, $\gamma = 5$ – – –, $\gamma = 50$ ⋯⋯, $\gamma = 1000$ –·–·–.

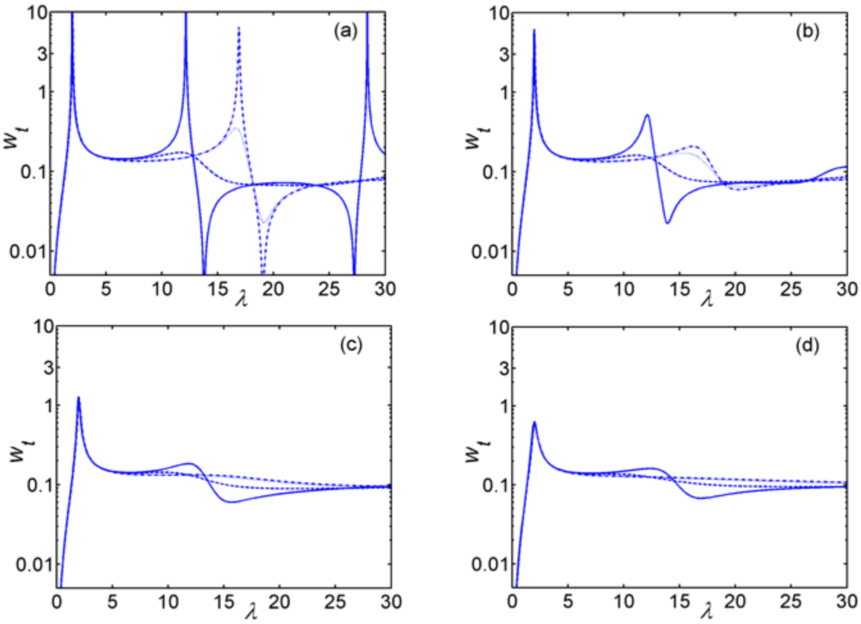

Figure 3-The tip displacements for the variation of λ *for various values of* γ, $\kappa = 100$, $m_r = 0.5$, *(a)* $\eta = 0$, *(b)* $\eta = 0.01$, *(c)* $\eta = 0.05$, *(d)* $\eta = 0.1$, $\gamma = 0$ —— , $\gamma = 5$ – – – , $\gamma = 50$ ⋯⋯, $\gamma = 1000$ –·–·– .

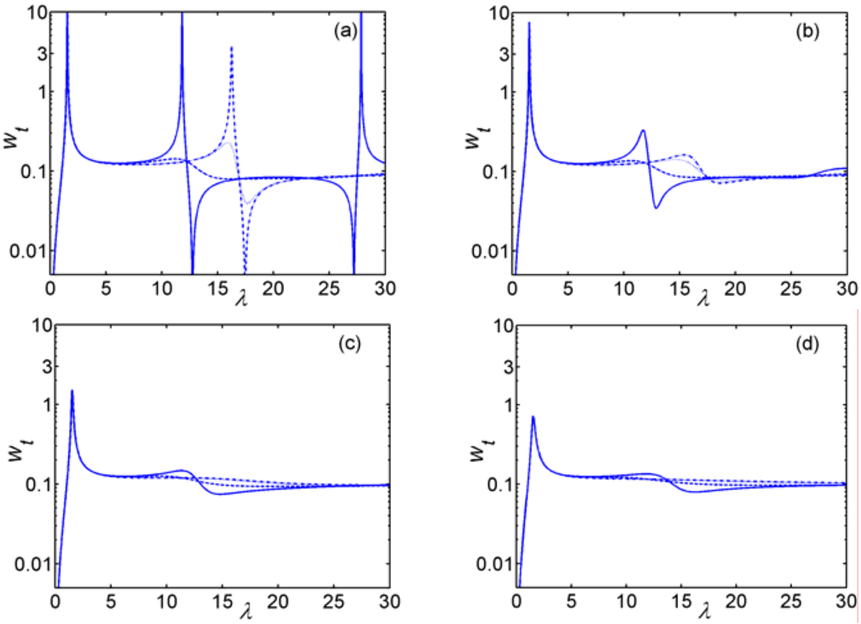

Figure 4-The tip displacements for the variation of λ *for various values of* γ, $\kappa = 100$, $m_r = 1.0$, *(a)* $\eta = 0$, (b) $\eta = 0.01$, (c) $\eta = 0.05$, (d) $\eta = 0.1$, $\gamma = 0$ —— , $\gamma = 5$ – – – , $\gamma = 50$ ⋯⋯, $\gamma = 1000$ –·–·– .

CONCLUSIONS

By using the Lagrange equation, the steady state response of a viscoelastically supported viscoelastic cantilever beam with a tip mass to a sinusoidally varying base movement has been studied. To use the Lagrange's equation with the trial function in the polynomial form and to satisfy the constraint condition by the use of Lagrange multipliers is a very good way for studying the structural behavior of the present problem. For the same accuracy level, it needs considerably fewer degrees of freedom than the finite element method [8] and energy based finite difference method as it was demonstrated by Kocatürk et al. [4].

By the application of the above mentioned solution technique, the response curves for tip displacements to a sinusoidally varying base movement are determined numerically for viscoelastically supported viscoelastic cantilever beam with a tip mass. The effect of the viscosity of the support, viscosity of the material of the beam and tip mass of the cantilever beam on the response curves is investigated and shown in the figures.

All of the obtained results are very accurate and may be useful for designing structural and mechanical systems under base movement.

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