Equation-Error Model Based Active Noise Cancellation Systems

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ABSTRACT

The filtered-U recursive least mean square (FURLMS) algorithm for active noise control (ANC) system is derived according to the output-error (OE) model based adaptive infinite impulse response (IIR) filter. This paper develops the ANC system by using the equation-error (EE) model based adaptive IIR filter. Since the weighting vectors in the mean square error (MSE) of the EE model baesd filter is a quadratic function, the EE model is guaranteed to reach the global minimum of MSE by properly choosing the step size. This paper also derives the proposed EE model based ANC system is identical to the hybrid ANC system. Simulation results show the effectiveness of the proposed algorithm.

***Index Terms*—** Filtered-U recursive least mean square (FURLMS), active noise control (ANC), output-error (OE), equation-error (EE).

1. INTRODUCTION

Noise problem becomes a serious issue in nowadays due to the demand of living quality. Traditional passive noise control (PNC) methods use sound absorbing material or insulation walls to block the noises. On the other hand, the ANC based technique produces the anti-noise signal, which has the same amplitude but 180 degree phase difference of the unwanted noise, to destructively interfere the noise [1],[2]. In general, the filtered-x LMS (FxLMS) algorithm is popular for ANC systems because of its simplicity and stability. However, the FxLMS algorithm which is based on the finite impulse response (FIR) filter, requires longer filter length and thus results in massive computation load.

Using IIR filter in an ANC system is a straightforward approach to reduce the computing load. The IIR filter can match the poles and zeros of the physical plant system better than the FIR filter with sufficient low order. In [3],[4], the FURLMS algorithm, based on the concept of output-error (OE) IIR filter model [5]-[7], was first introduced to solve the acoustic feedback problem of the feedforward ANC system, shown in Fig. 1(a). The response of adaptive IIR filter is

$$\frac{Y(z)}{X(z)} = \frac{A(z)}{1 - B(z)}, \tag{1}$$

where the $Y(z)$ and the $X(z)$ are the output signal $y(n)$ and the input signal $x(n)$ in Z-domain, the $P(z)$ is the primary plant, $A(z)$ and $B(z)$ are both the transversal filters adjusted by the LMS algorithm. Since the adaptive IIR filter contains both poles and zeros, it can model the plant $P(z)$ efficiently with less filter lengths. However, because of the recursive term of the IIR filter, the FURLMS algorithm may lead the IIR filter to unstable condition. So, its stability cannot be proved [1].

Another approach to develop IIR filter is to apply the EE model. Figure 1(b) depicts the adaptive IIR filter based on the EE model [6],[8]. Obviously, the input signal of the adaptive filter $B(z)$ is replaced by the primary signal $d(n)$; therefore, the response of EE based adaptive IIR filter is

$$\frac{Y(z)}{X(z)} = A(z) + P(z)B(z). \tag{2}$$

In brief, the EE model based adaptive IIR filter is formed by two adaptive FIR filters and thus becomes linear functions. Therefore, the MSE function regarding to the weighting vectors of EE is a quadratic function [5]. This confirms that the EE model based IIR filter is stable and the adaptive processes can reach the global minimum with sufficient low step size.

In this paper, we proposed a new ANC algorithm using the EE based adaptive IIR filter. The updating processes for the weights in the IIR filters are developed based on the LMS algorithm. The EE based adaptive IIR filter can reach the global minimum. When compared with the general feedforward ANC algorithm using transversal filters, the filter length of the proposed work is sufficiently low to save computing power and to thus reduce the causality problem. Meanwhile, the IIR filter based ANC system is capable to model complex plant.

2. FURLMS ALGORITHM

The FURLMS algorithm is derived based on the OE model IIR filter. It was first developed to deal with the acoustic feedback problem of the feedforward ANC. Figure 2 shows the block diagram, where the $P(z)$ and $S(z)$ denote the primary and the secondary paths, respectively. The output signal $y(n)$ can be expressed as

$$y(n)=\sum_{i=0}^{I-1}a_i(n)x(n-i)+\sum_{j=1}^{J}b_j(n)y(n-j), \tag{3}$$

where $a_i(n)$ $(i=0,1,\ldots,I-1)$ and $b_j(n)$ $(j=1,2,\ldots,J)$ are the weighting coefficients of $A(z)$ and $B(z)$ at time n, and the I and J are the respective filter lengths. By using the steepest-descent algorithm, the generalized recursive LMS algorithm for the IIR filter becomes

$$a_i(n+1)=a_i(n)+\mu x'(n-i)e(n) \tag{4}$$

and

$$b_j(n+1)=b_j(n)+\mu\hat{y}'(n-j)e(n). \tag{5}$$

The $x'(n)$ is the reference signal filtered by the estimated secondary path $\hat{S}(z)$

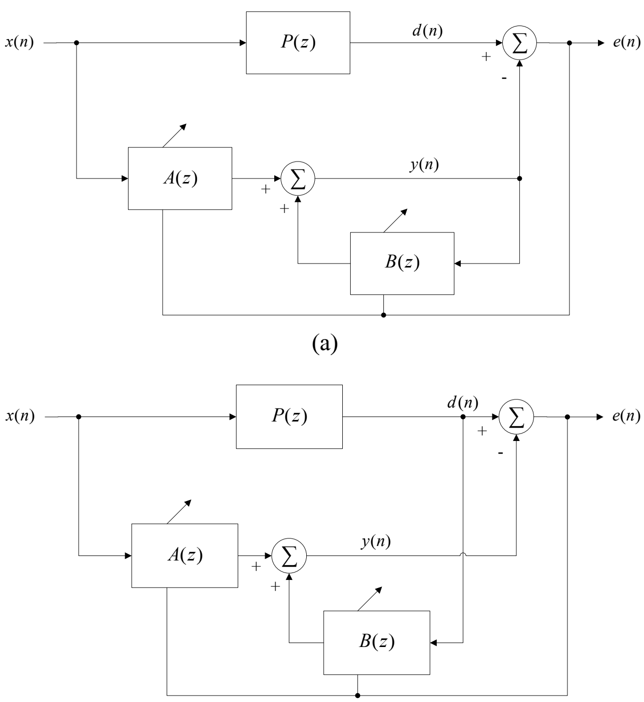
$$x'(n)=\hat{s}(n)*x(n), \tag{6}$$

and $\hat{y}'(n)$ is the filtered reference signal

$$\hat{y}'(n)=\hat{s}(n)*y(n), \tag{7}$$

where $\hat{s}(n)$ is the impulse response of $\hat{S}(z)$.

Although the FURLMS algorithm can use lower filter length than the FXLMS algorithm to achieve the noise attenuation; however, the updating processes of $b_j(n)$ include recursive terms, which may lead to unstable poles of the FURLMS algorithm and thus the stability cannot be guaranteed.



(a)

(b)

Fig. 1. Block diagram of system identifier using adaptive IIR filter with (a) output-error (OE) and (b) equation-error (EE) method.

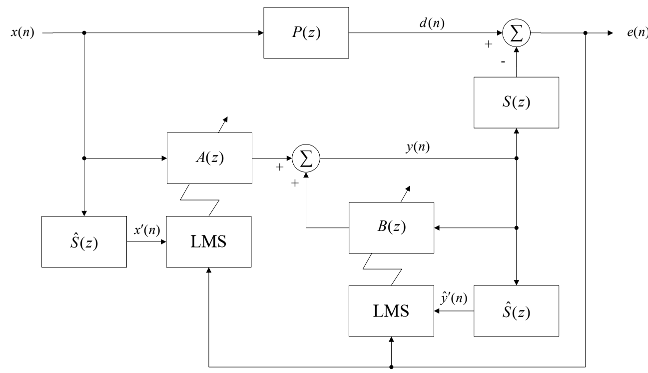


Fig. 2. The ANC system using the FURLMS algorithm.

3. PROPOSED EE BASED ANC SYSTEM

Refer to the EE based IIR filter shown in Fig. 1(b), the output signal $y(n)$ can be expressed as

$$y(n)=\sum_{i=0}^{I-1}a_i(n)x(n-i)+\sum_{j=1}^{J}b_j(n)d(n-j) \tag{8}$$

and $\quad d(n)=p(n)*x(n),$

where I and J are respective filter lengths of $A(z)$ and $B(z)$, $p(n)$ is the impulse response of $P(z)$. Therefore, we have the output of the proposed EE based IIR filter,

$$Y(z)=\left(A(z)+B(z)P(z)\right)\cdot X(z),$$

where $X(z)$ and $Y(z)$ are the Z-transform of $x(n)$ and $y(n)$. Compared with the output signal of the OE based method in Eq. (3), the EE based approach uses the primary signal $d(n)$ as the input of the filter $B(z)$, which is also a function related to reference signal $x(n)$. Therefore, the output signal $y(n)$ of the proposed work does not contain recursive term, the EE based IIR filter acts as the combination of two transversal filters $A(z)$ and $B(z)P(z)$, thus will not diverge due to the tuning of adaptive weights of the filters $A(z)$ and $B(z)$.

However, we cannot obtain $d(n)$ in a practical ANC system; therefore, the overall block diagram of EE based adaptive IIR filter ANC system can be modified as shown in Fig. 3, additional $\hat{S}(z)$ is needed to estimate $d(n)$. We synthesis the primary signal by applying the secondary path estimate after the system output signal $y(n)$. The error signal $e(n)$ is

$$e(n)=d(n)-s(n)*y(n), \tag{9}$$

the $s(n)$ is the impulse response of secondary path $S(z)$ with L coefficients. By applying the LMS algorithm, the estimated square error signal is

$$\hat{\xi}(n)=e^2(n), \tag{10}$$

and the gradient estimation is defined as

$$\nabla\hat{\xi}(n)=2\left[\nabla e(n)\right]e(n). \tag{11}$$

In Eq.(11), the gradient of the error signal $\nabla e(n)$ is

$$\nabla e(n) = \left[\frac{\partial e(n)}{\partial a_0(n)} \cdots \frac{\partial e(n)}{\partial a_{I-1}(n)} \frac{\partial e(n)}{\partial b_1(n)} \cdots \frac{\partial e(n)}{\partial b_J(n)} \right]^T$$
$$= -s(n) * \left[\frac{\partial y(n)}{\partial a_0(n)} \cdots \frac{\partial y(n)}{\partial a_{I-1}(n)} \frac{\partial y(n)}{\partial b_1(n)} \cdots \frac{\partial y(n)}{\partial b_J(n)} \right]^T . \tag{12}$$

We assume that the recursion based on the old output gradients is negligible [1]; therefore, Eq. (12) can be simplified as

$$\nabla e(n) = -\hat{s}(n) * \left[x(n)\ \ x(n-1)\ \cdots\ x(n-I+1) \right.$$
$$\left. d(n-1)\ \cdots\ d(n-J) \right]^T . \tag{13}$$

Substituting Eqs. (11) and (13) into the steep-descent algorithm, we have

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \left[\hat{s}(n) * \mathbf{u}(n) \right] e(n) , \tag{14}$$

where the $\mathbf{w}(n)$ is weight matrix defined as $\mathbf{w}(n) = \left[\mathbf{a}(n) \quad \mathbf{b}(n) \right]^T$ with weighting vectors

$$\mathbf{a}(n) \equiv \left[a_0(n)\ a_1(n)\ \cdots\ a_{I-1}(n) \right]^T , \tag{15}$$

and

$$\mathbf{b}(n) \equiv \left[b_1(n)\ b_2(n)\ \cdots\ b_J(n) \right]^T , \tag{16}$$

respectively. Besides, the $\mathbf{u}(n)$ is the input signal matrix $\mathbf{u}(n) = \left[\mathbf{x}(n) \quad \mathbf{d}(n-1) \right]^T$, with

$$\mathbf{x}(n) \equiv \left[x(n)\ x(n-1)\ \cdots\ x(n-I+1) \right]^T \tag{17}$$

and

$$\mathbf{d}(n) \equiv \left[d(n-1)\ d(n-2)\ \cdots\ d(n-J) \right]^T . \tag{18}$$

However, in Eqs. (8) and (14), the primary noise $d(n)$ is not available. Therefore, we use the estimate signal $\hat{d}(n)$ instead,

$$\hat{d}(n) = e(n) + \hat{s}(n) * y(n) . \tag{19}$$

So, the updating equations for adaptive filter $A(z)$ and $B(z)$ are

$$\mathbf{a}(n+1) = \mathbf{a}(n) + \mu \mathbf{x}'(n) e(n) \tag{20}$$

and

$$\mathbf{b}(n+1) = \mathbf{b}(n) + \mu \hat{\mathbf{d}}'(n) e(n) , \tag{21}$$

where $\mathbf{x}'(n) \equiv \hat{s}(n) * \mathbf{x}(n)$ and $\hat{\mathbf{d}}'(n) \equiv \hat{s}(n) * \hat{\mathbf{d}}(n)$. Hence, the anti-noise signal of the proposed EE based ANC system is shown in Eq. (8), and the adaptive processes to tune the weights in $A(z)$ and $B(z)$ are depicted in Eqs. (20) and (21).

Based on Eqs. (8), and (21), we can find the proposed EE model adaptive IIR filter based ANC system is the same with the hybrid ANC system. The hybrid ANC system [1], which includes both the feedforward and the feedback structures, generates canceling signal based on the signals obtained from both the reference and error sensors. Since the proposed EE model IIR filter has the same canceling signal and adaptive processes with the hybrid ANC system, we can claim that the EE model IIR filter based ANC system is identical to the hybrid ANC system.

In order to derive the optimal weights, we define the input signal matrix of the proposed system as

$$\mathbf{h}(n) = \left[\mathbf{x}(n) \quad \hat{\mathbf{d}}(n) \right]^T . \tag{22}$$

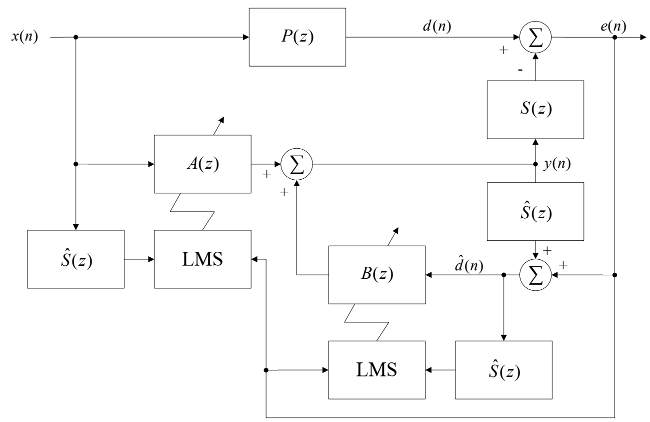


Fig. 3. Block diagram of proposed EE based adaptive IIR filter ANC system.

Therefore, according to Eqs. (10), (15) and (22), the expect value of the MSE function can be calculated as

$$\begin{aligned} \xi(n) &= E\left[e^2(n) \right] = E\left[d(n) - \mathbf{w}^T(n)\mathbf{h}'(n) \right]^2 \\ &= E\left[d^2(n) \right] + \mathbf{w}^T(n)\mathbf{R}'\mathbf{w}(n) - 2\mathbf{w}^T(n)\mathbf{p}' \\ &= E\left[d^2(n) \right] + \mathbf{a}^T(n)\mathbf{R}_{\mathbf{x}'\mathbf{x}'}\mathbf{a}(n) + \mathbf{b}^T(n)\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'}\mathbf{b}(n) \\ &\quad + 2\left[\mathbf{a}^T(n)\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{b}(n) - \mathbf{a}^T(n)\mathbf{p}_{d\mathbf{x}'} - \mathbf{b}^T(n)\mathbf{p}_{d\hat{\mathbf{d}}'} \right], \end{aligned} \tag{23}$$

where $\mathbf{R}' = E\left[\mathbf{h}'(n)\mathbf{h}'^T(n) \right]$ is the autocorrelation function of input signal filtered by the secondary path $\hat{S}(z)$, and $\mathbf{p}' = E\left[d(n)\mathbf{h}'(n) \right]$ is the cross-correlation function between $d(n)$ and input signal. To further extend the equation, the rest of the correlation matrix can be defined as $\mathbf{R}_{\mathbf{x}'\mathbf{x}'} \equiv E\left[\mathbf{x}'(n)\mathbf{x}'^T(n) \right]$, $\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'} \equiv E\left[\hat{\mathbf{d}}'(n)\hat{\mathbf{d}}'^T(n) \right]$, $\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'} \equiv E\left[\mathbf{x}'(n)\hat{\mathbf{d}}'^T(n) \right]$, $\mathbf{p}_{d\mathbf{x}'} \equiv E\left[d(n)\mathbf{x}'(n) \right]$, and $\mathbf{p}_{d\hat{\mathbf{d}}'} \equiv E\left[d(n)\hat{\mathbf{d}}'(n) \right]$. Based on Eq. (23), the gradients of the MSE function corresponding to filter $A(z)$ and $B(z)$ are

$$\nabla \xi_a = \frac{\partial \xi(n)}{\partial \mathbf{a}(n)} = 2\mathbf{R}_{\mathbf{x}'\mathbf{x}'}\mathbf{a}(n) + 2\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{b}(n) - 2\mathbf{p}_{d\mathbf{x}'} , \tag{24}$$

and

$$\nabla \xi_b = \frac{\partial \xi(n)}{\partial \mathbf{b}(n)} = 2\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'}\mathbf{b}(n) + 2\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{a}(n) - 2\mathbf{p}_{d\hat{\mathbf{d}}'} , \tag{25}$$

respectively. When the gradients of MSE are zero, the optimal solution $\mathbf{a}^o(n)$ and $\mathbf{b}^o(n)$ can be derived as

$$\begin{aligned} \mathbf{a}^o(n) = \mathbf{R}_{\mathbf{x}'\mathbf{x}'}^{-1} \Big[\mathbf{p}_{d\mathbf{x}'} - \mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'} \left(\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'} - \mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{R}_{\mathbf{x}'\mathbf{x}'}^{-1}\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'} \right)^{-1} \\ \cdot \left(\mathbf{p}_{d\hat{\mathbf{d}}'} - \mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{R}_{\mathbf{x}'\mathbf{x}'}^{-1}\mathbf{p}_{d\mathbf{x}'} \right) \Big], \end{aligned} \tag{26}$$

$$\mathbf{b}^o(n) = \left(\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'} - \mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{R}_{\mathbf{x}'\mathbf{x}'}^{-1}\mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'} \right)^{-1} \left(\mathbf{p}_{d\hat{\mathbf{d}}'} - \mathbf{R}_{\mathbf{x}'\hat{\mathbf{d}}'}\mathbf{R}_{\mathbf{x}'\mathbf{x}'}^{-1}\mathbf{p}_{d\mathbf{x}'} \right) . \tag{27}$$

Besides, since the MSE function (shown in Eq. (23)) is a quadratic function to the weights of $\mathbf{a}(n)$ and $\mathbf{b}(n)$, it means that the optimal weights $\mathbf{a}^o(n)$ and $\mathbf{b}^o(n)$ can be achieved by using sufficiently small step size in the adaptive processes.

4. SIMULATION RESULTS

According to above mathematical analysis, we verify the effectiveness of the proposed work by using simulations. The first simulation is to derive the optimal weights. To reduce the computational complexity, the reference signal $x(n)$ is supposed to be a sinusoidal signal with frequency $f = 400Hz$, and the sampling rate is $f_s = 4000Hz$. The primary path $P(z)$ is set as a 90 degree phase delay and the secondary path $S(z) = z^{-\Delta} = z^{-10}$, 10 samples delay, both with unity gains. The estimate secondary path model $\hat{S}(z)$ is assumed to be identical to $S(z)$. Since the system complexity has been reduced, we can just use $A(z) = a_0$ and $B(z) = b_1$ to cancel the undesired noise $d(n)$. Based on these information, we can calculate the correlation matrix $\mathbf{R}_{x'x'} = 0.5$, $\mathbf{R}_{\hat{\mathbf{d}}'\hat{\mathbf{d}}'} = 0.5$, $\mathbf{R}_{x'\hat{\mathbf{d}}'} = -0.5\sin(2\pi f / f_s)$, $\mathbf{p}_{dx'} = 0.5\sin(\Delta \times f)$ and $\mathbf{p}_{d\hat{\mathbf{d}}'} = 0.5\cos(\Delta \times f + f)$. Therefore, the optimal solution of the proposed system can be obtained, where $a_0^o(n) = 0.7265$ and $b_1^o(n) = 1.2361$ by Eqs. (38) and (39). Substituting the optimal solution to Eq. (28), we will have the minimum MSE equals to zero. We test the proposed EE model IIR based ANC system to verify the optimal weights of the adaptive IIR filter. The initial values for a_0 and b_0 are 0 and the step size μ is 0.1. Figure 4 plots the coefficients of the adaptive filter will converge to the calculated optimal solution within hundred iterations.

The second simulation test the performance to cancel broadband noise with measurement error. The primary path $P(z)$ and the secondary path $S(z)$ is obtained from [2] using FIR filters with filter lengths 128 and 64, respectively. The reference signal $x(n)$ is supposed to be a broadband noise with bandwidth 0-1000 Hz and variance 0.1. Besides, we assumed that there is a measurement error (disturbance) $v(n)$ at error signal, thus

$$e(n) = d(n) - s(n) * y(n) + v(n), \qquad (28)$$

where $v(n)$ contains 120Hz, 240 Hz, 360 Hz, 480Hz and 600Hz narrowband noise signals with each amplitude 0.1. We compared the performance of OE and EE model based ANC systems on this condition. The sampling frequency is 4kHz and the filter lengths for $A(z)$ and $B(z)$ are both 60 for OE and EE models. The step sizes are 0.01 and 0.001 for OE and EE based adaptive IIR filters, respectively.

Figure 5 displays the simulation results. The green lines depict the error signal $e(n)$. Besides, the blue and red lines show the noise cancellation results by using OE and EE based adaptive IIR filters. Obviously, the OE based IIR filter can only cancel the broadband noise. It is because the OE based ANC system is based on a feedforward ANC structure, which can only cancel the noise component related to the reference signal $x(n)$. However, the EE based ANC system is equivalent to the hybrid ANC system, which can not only reduce the noise component related to reference signal, but the narrowband noise disturbance signals obtained at the error sensors as well.

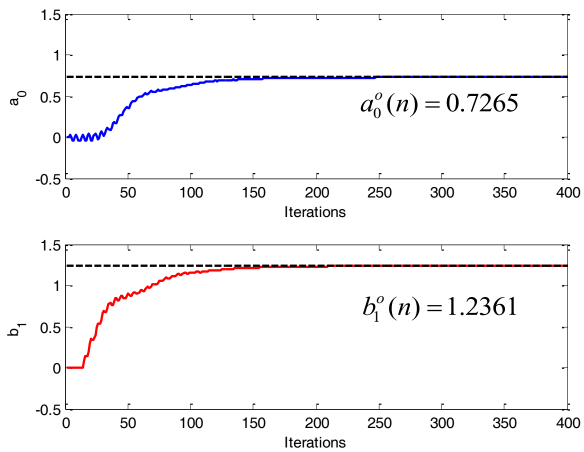


Fig. 4. Weight tracks of proposed EE based adaptive IIR filter ANC system.

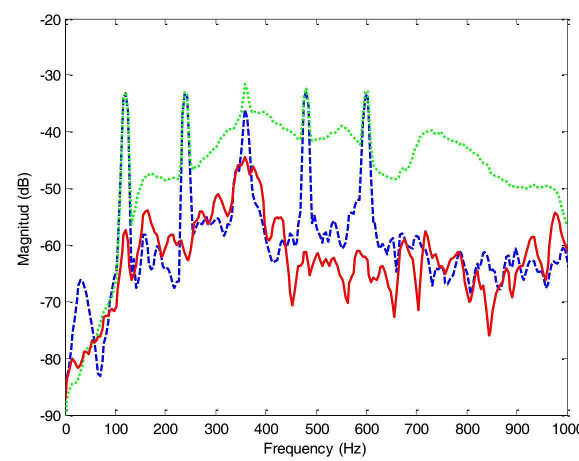


Fig. 5 Simulation Results by using OE and EE based adaptive IIR filters, green: noise signal, blue: OE and red: EE.

5. CONCLUSION

This work proposed using the EE model adaptive IIR filter. Based on the mathematical analysis of the proposed work, we showed that the EE model based ANC system is equivalent to the hybrid ANC system. Since the EE based IIR filter is composed by two transversal filters, the adaptive processes will not result in unstable poles problem. Besides, the weights of the EE based ANC system in MSE is a quadratic form, we can find small enough step size to tune the weights to achieve the global minimum of MSE.

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