A NOVEL APPROACH FOR FEEDFORWARD CONTROL OF NOISE IN DUCTS USING SIMPLIFIED MULTICHANNEL INVERSE FILTERS

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ABSTRACT

A time-domain underdetermined multichannel inverse filtering technique is proposed for the feedforward active noise control (ANC) for ducts. In the commonly used filtered-x least-mean-square (FXLMS) algorithm, the feedforward control problem is formulated as an overdetermined inverse filtering problem which leads to non-zero residual noise. In the paper, a multichannel approach is derived in light of a vector subspace viewpoint and model-matching paradigm. By introducing multiple secondary sources, the problem can be reformulated into an underdetermined system, admitting infinite number of exact solutions with zero residual noise. However, as a major shortcoming of the approach, the resulting finite-length impulse response (FIR) filters tend to be too long to admit real-time implementation. To address the problem, the least absolute shrinkage and selection operator (LASSO) algorithm is exploited to effectively reduce the controller orders. Simulation and experiment results obtained for a two-channel duct ANC system have demonstrated the efficacy of the proposed approach in attenuating broadband noise.

Index Terms— active noise control, multiple channels, inverse filtering.

1. INTRODUCTION

To complement traditional passive noise control, active noise control (ANC) that relies on out-of-phase noise to cancel the undesired noise is particularly effective in abating low-frequency noise. [1]-[3] Three generic structures of ANC systems were suggested in the past: the feedforward structure [4], [5], the feedback structure [6], [7], and the hybrid structure [8]-[11] which combines the former two. This paper examines primarily the feedforward structure.

Traditionally, the feedforward control is implemented using adaptive filtering approaches such as the celebrated filtered-x least-mean-squares (FXLMS) algorithm [1], [12]-[14]. In matrix formulation, the feedforward control problem is an overdetermined inverse filtering problem which usually leads to non-zero residual noise and hence limits the achievable ANC performance. As inspired by the multiple-input/output inverse theorem (MINT) approach [15], we propose a time-domain underdetermined multichannel inverse filtering method to eliminate the residual noise and to improve noise reduction performance.

Many audio and acoustic signal processing and control problems can be regarded as inverse filtering problems. The MINT approach was suggested to realize inverse filters for room response equalization. By adding multiple control sources, an overdetermined system can be converted to a square invertible system which admits an exact solution. One potential feature of using two control sources, for example, is that it is possible for them to absorb the incident sound wave, rather than just reflecting it as in the case of a single control source, thus suppressing resonances in the duct that may otherwise limit the performance. Despite the beauty of theory, there is one major issue that may hamper practical implementation of the MINT filters. The resulting finite impulse response (FIR) filters tend to be prohibitively long, which could pose difficulties in real-time implementation in delay-critical applications such as active control.

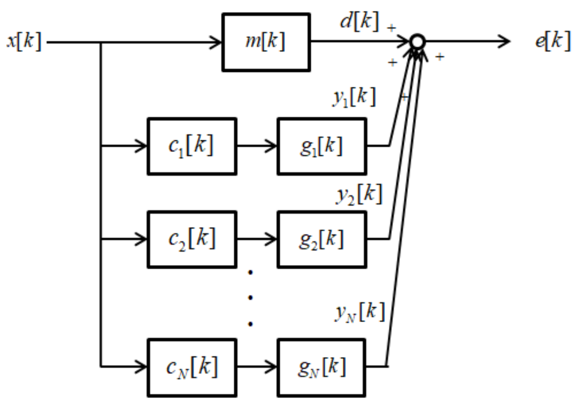
In this paper, the preceding issue is addressed with a time-domain underdetermined multichannel inverse filtering (TUMIF) approach with application in the duct ANC problem. We reformulate the MINT method into a regulated underdetermined inverse filtering problem in a general multi-channel model-matching context. Next, the inverse filtering problem is converted to an underdetermined system which admits infinite many exact solutions (with zero residual errors). The filters are formulated directly in the time domain, which avoids the noncausal artifacts frequently encountered in the frequency-domain formulation. The most basic solution method is the least squares (LS) with L2-norm penalty, also referred to as the Tikhonov regularization (TIKR) [16], [17]. However, the finite impulse response (FIR) filters obtained using the LS method tends to be very long. To combat the problem, a sparse coding (SC) [18], [19] technique, the least absolute shrinkage and selection operator (LASSO) [20] algorithm, is exploited in this work to reduce the controller complexity.

Experiments are undertaken to validate the proposed TUMIF-based duct ANC system. Band-limited white noise is used as the noise input and a rectangular duct is employed in the experiments. In simulation, noise reduction achieved by using the two ANC algorithms, the FXLMS and the

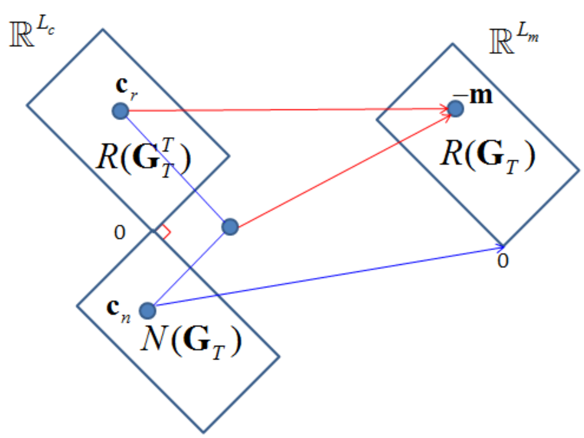
TUMIF (LASSO), are compared. The reduced-order TUMIF (LASSO) controller is implemented on a digital signal processor (DSP) in the experiments. The residual noise obtained using preceding ANC algorithms are compared in the time domain and the frequency domain in terms of the noise reduction performance of each controller.

2. THE TIME-DOMAIN UNDERDETERMINED MULTICHANNEL INVERSE FILTERING

2.1 Multichannel Inverse Filtering



(a)



(b)

Fig.1 (a) System block diagram of an N-channel feedforward ANC system (b) Vector subspaces for the full-rank underdetermined problem, $\mathbf{G}_T\mathbf{c} = -\mathbf{m}$. There exists infinite number of exact solutions.

Consider an N-channel (N >1) feedforward system, as depicted in Fig. 1 (a). Similar to the preceding one-channel feedforward ANC, the multichannel system can be written as a model-matching problem:

$$c_1[k]*g_1[k]+c_2[k]*g_2[k]+\cdots+c_N[k]*g_N[k]=-m[k] \qquad (1)$$

Frequency weighting can be readily incorporated into $m[k]$ and $g[k]$. Or, in z-domain,

$$G_1(z^{-1})C_1(z^{-1})+G_2(z^{-1})C_2(z^{-1})+\cdots+G_N(z^{-1})C_N(z^{-1})=-M(z^{-1}) \qquad (2)$$

Solutions of the equation above exist when $G_1(z^{-1})$, $G_2(z^{-1}),\cdots$, and $G_N(z^{-1})$ are co-prime. That is, $G_1(z^{-1}), G_2(z^{-1}),\cdots$, and $G_N(z^{-1})$ have no common zeros in the z-plane [15]. For simplicity, we assume that all the controllers, $\mathbf{c}_1$, $\mathbf{c}_2$, …, $\mathbf{c}_N$, have the same number of taps L_c, and all the secondary paths, $\mathbf{g}_1$, $\mathbf{g}_2$, …, $\mathbf{g}_N$, have the same number of taps L_g. (5) can be written in a matrix form:

$$-\mathbf{m}=\mathbf{G}_1\mathbf{c}_1+\mathbf{G}_2\mathbf{c}_2+\cdots+\mathbf{G}_N\mathbf{c}_N=\mathbf{G}_T\mathbf{c}_T, \qquad (3)$$

where

$$\mathbf{G}_T=\begin{bmatrix}\mathbf{G}_1 & \mathbf{G}_2 & \cdots & \mathbf{G}_N\end{bmatrix}\in\mathbb{R}^{L_m\times NL_c} \qquad (4)$$

$$\mathbf{G}_m=\begin{bmatrix} g_m[0] & \cdots & 0 \\ g_m[1] & g_m[0] & \vdots \\ \vdots & g_m[1] & \\ g_m[L_g-1] & \vdots \ \ddots & \\ & g_m[L_g-1] & g_m[0] \\ & \ddots & g_m[1] \\ \vdots & & \vdots \\ 0 & \cdots & g_m[L_g-1] \end{bmatrix}\in\mathbb{R}^{L_m\times L_c}, \quad m=1,2,\cdots,N \qquad (5)$$

$$\mathbf{c}_T=\begin{bmatrix}\mathbf{c}_1^T & \mathbf{c}_2^T & \cdots & \mathbf{c}_N^T\end{bmatrix}^T\in\mathbb{R}^{NL_c} \qquad (6)$$

$$\mathbf{c}_n=\begin{bmatrix}c_n[0] & c_n[1] & \cdots & c_n[L_c-1]\end{bmatrix}^T, \quad n=1,2,\ldots,N \qquad (7)$$

Clearly, (3) is an underdetermined linear system if

$$L_m = L_c + L_g - 1 < NL_c$$

which can be rearranged into

$$L_c > \frac{L_g-1}{N-1}. \qquad (8)$$

It follows that an underdetermined system can be constructed if the controller tap lengths can be judiciously chosen to comply with the inequality of (8). Assume that $\mathbf{G}$ has full-row rank ($\dim R(\mathbf{G}) = L_m$). The existence and uniqueness of solution can be visualized in the vector subspace [23] diagram illustrated in Fig. 1(b), where the null space of $\mathbf{G}^T$, $N(\mathbf{G}_T^T)$, shrinks to a point ($\mathbf{0}$). Therefore, exact solutions exist because $\mathbf{m}\in R(\mathbf{G}_T^T\mathbf{c})$ is always true, which results in zero residual errors. Furthermore, the dimension of the null space of $\mathbf{G}_T$ $\dim N(\mathbf{G}_T) = NL_c - L_m > 0$. Therefore, infinite number of exact solutions exists, which offers extra design degrees of freedom to achieve other purposes such as sparsity.

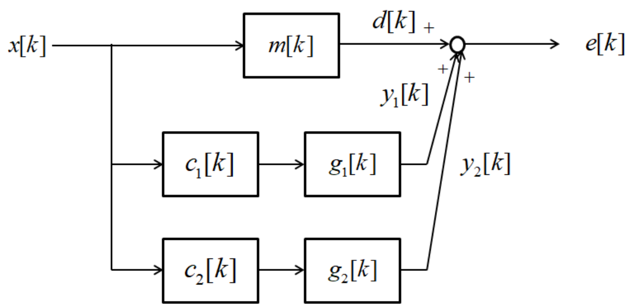


Fig.2. System block diagram of a two-channel TUMIF feedforward ANC system with single input and single error microphone

Consider a two-channel ANC system, as shown in Fig. 2. According to (12), the tap length of the controllers must satisfy

$$L_c > L_g - 1. \tag{9}$$

For a full-row-rank $\mathbf{G}_T$, the minimum-norm solution is given as

$$\mathbf{c}_{LS} = -\mathbf{G}_T^{+}\mathbf{m} = -\mathbf{G}_T^T(\mathbf{G}_T\mathbf{G}_T^T)^{-1}\mathbf{m}. \tag{10}$$

Although the preceding solution is exact, it may still result in high gain controllers at times. This issue can be mitigated at some expense of model-matching performance by using the Tikhonov regularization (TIKR) approach:

$$\mathbf{c}_{TIKR} = -(\mathbf{G}_T^T\mathbf{G}_T + \beta^2\mathbf{I})^{-1}\mathbf{G}_T^T\mathbf{m}, \tag{11}$$

where the regularization parameter β serves to tradeoff the residual error and solution norm.

2.2 Order Reduction of the Inverse Filters

The preceding underdetermined system allows for exact solutions with zero residual error, or perfect noise attenuation. However, this mathematical beauty disguises somewhat a major pitfall that might hamper the practical implementation of the TUMIF approach. That is, the FIR filters calculated using the underdetermined formulation tend to be very long. In what follows, we shall propose a method on the basis of SC to address the issue.

The newly emerging SC methods and compressive sensing methods are versatile techniques to deal with underdetermined problems with sparse solutions. In the SC paradigm, the multichannel ANC problem can be posed as a constrained optimization problem:

$$\min_{\mathbf{c}} \|\mathbf{c}\|_0 \quad st.\ \mathbf{G}_T\mathbf{c} = -\mathbf{m}, \tag{12}$$

where $\|\mathbf{c}\|_0$ donates the 0-norm or cardinality of the solution vector **c**. To increase robustness against noise, an inequality constraint can be used

$$\min_{\mathbf{c}} \|\mathbf{c}\|_0 \quad st.\ \|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2 < \eta, \tag{13}$$

where η is the noise threshold and $\|\cdot\|_2$ donates the vector 2-norm.

Such a problem can be solved by relaxation methods [24]. In the work, one relaxation method, the LASSO algorithm, are exploited to reduce filter orders. Instead of the 0-norm in the cost function of (16), relaxation methods use the 1-norm in the cost function to obtain a constrained convex optimization problem [25]-[27]:

$$\min_{\mathbf{c}} \|\mathbf{c}\|_1 \quad st.\ \|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2 < \eta \tag{14}$$

which can also be reformulated into an equivalent unconstrained convex optimization problem named the least absolute shrinkage and selection operator (LASSO):

$$\min_{\mathbf{c}} \ \frac{1}{2}\|\mathbf{G}_T\mathbf{c} + \mathbf{m}\|_2^2 + \lambda\|\mathbf{c}\|_1 \tag{15}$$

where λ is a sparsity- promoting regularization parameter. The constrained and unconstraint optimization problems in (22) and (23) can be solved numerically by using the software packages such as the CVX developed by Boyd and his colleagues [28].

3. EXPERIMENTAL RESULTS

Experiments are carried out in an anechoic room (Fig.3) to validate the proposed two-channel feedforward ANC system, where a wooden duct is employed for the test. The conventional FXLMS algorithm is adopted as a benchmarking method. With impulse response measurements of the primary path and the secondary path, the controllers are calculated by using the aforementioned TIKR algorithm, the LASSO algorithm, and the FXLMS algorithm, where the first two methods are the TUMIF-based approaches. The length of the secondary paths L_g is 501. For the traditional FXLMS approach, the length of controller is selected to be 200. However, for the TUMIF approaches to satisfy the inequality of (13), the controller length L_c is selected to be 600. Lastly, for the LASSO algorithm, the controller length is substantially reduced to 400, with the aid of the proposed sparse coding technique.

In the experiments, white noise band-limited to 4 kHz is employed as the primary noise. The LASSO controllers designed under 8 kHz sampling rate are implemented on the TMS320C6713 of Texas Instrument®. Control loudspeaker 2 is used in evaluating the FXLMS control.

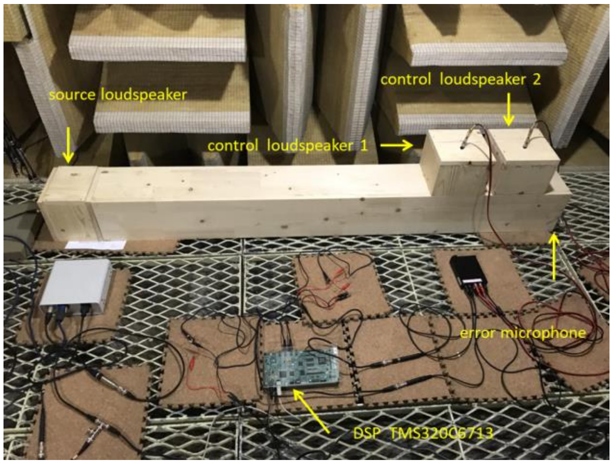
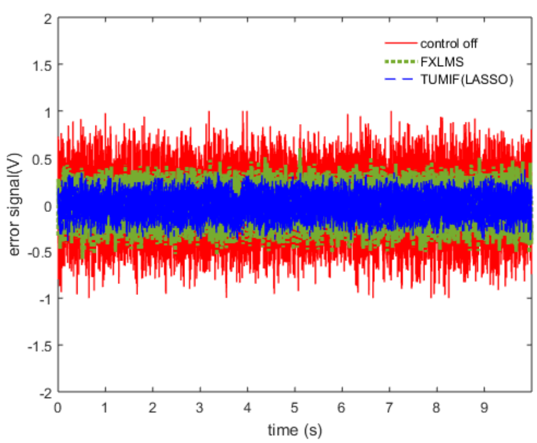
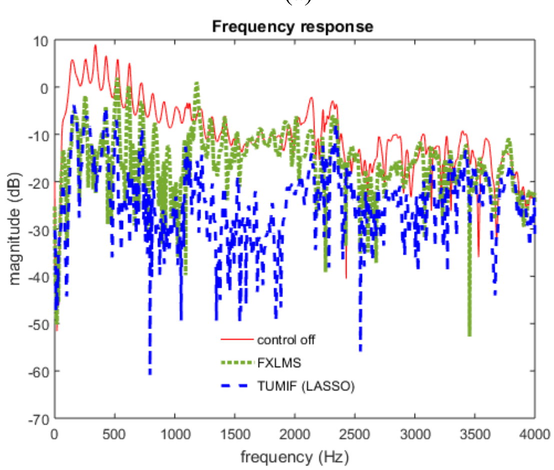


Fig.3 The experimental arrangement of the duct ANC in the anechoic room.

The noise attenuation attained by the FXLMS and TUMIF approaches is compared in the time domain and the frequency domain in Figs.4(a) and (b). The TUMIF(LASSO) controllers have attained 5-20 dB attenuation in the frequency range 100-3000 Hz, whereas the FXLMS algorithm has attained only 5-10 dB noise attenuation in the range 100-1100 Hz. In addition, the TUMIF controllers have achieved significant noise reduction in 1100-2000 Hz, but the FXLMS controller is virtually ineffective in this frequency range.



(a)



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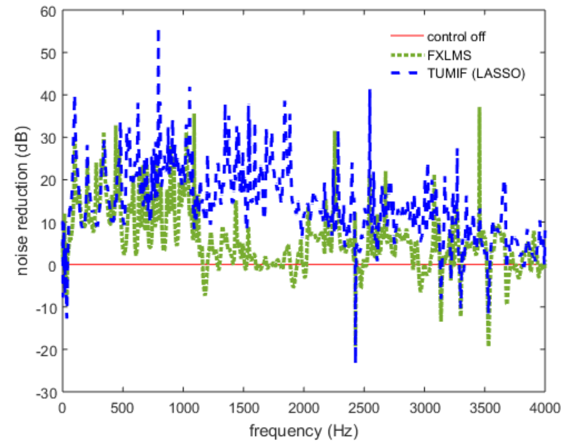

(c)

Fig.4 Experimental results obtained using the FXLMS, TUMIF (LASSO), approaches. (a) Error response in the time domain, (b) error response in the frequency domain, and (c) noise reduction = (control off) - (control on). In the FXLMS, a 200-tapped filter with the step size 0.1 is implemented.

4. CONCLUSIONS

Feedforward ANC systems have been developed on the basis of the TUMIF approach. The key to the usefulness of the TUMIF approach lies in the underdetermined system resulting from the increased number of control channels. However, the improved noise attenuation performance comes at a price of increased computational complexity of long filters. As a main contribution of this work, the LASSO algorithm enables effective reduction of the controller orders, which makes practical DSP implementation possible. Experiments have demonstrated that the TUMIF-based controllers outperform the FXLMS controller in suppressing broadband random noise at the frequency range 1100-2000 Hz

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