POWER NETWORK PARAMETER CORRECTION VIA SPARSE UNSUPERVISED REGRESSION

Dilan Senaratne, Jinsub Kim

School of Electrical Engineering and Computer Science, Oregon State University, Corvallis, OR 97331
senaratg@oregonstate.edu, kimjinsu@oregonstate.edu

ABSTRACT

The problem of correcting power network parameters and topology using multi-period SCADA measurements is considered. Starting from the current knowledge of parameter values, we formulate the parameter correction problem as a sparse unsupervised regression problem by exploiting the sparsity of the parameter errors. The advantage of the proposed approach is that it can localize and estimate parameter errors at the same time; there is no need for prior knowledge of error locations. Furthermore, the approach can be adapted to correct sparse errors in both parameters and topology simultaneously. We present an iterative parameter correction algorithm and demonstrate its efficacy using the IEEE 14-bus test case.

Index Terms— Power system parameter estimation, unsupervised regression, sparse error correction

1. INTRODUCTION

Accurate modeling of the power system is essential in power flow analysis. In particular, it is essential to have correct topology information and estimates of power network line parameters such as series impedance and shunt admittance. Working with incorrect network parameters may result in big errors in state estimation, suboptimal dispatch and control decisions, and so on. In practice, line parameter values available to power system operators may contain some errors due to several factors including errors in the parameter data provided by manufacturers, use of an incorrect model for parameter calculation, uninformed network changes, and severe weather conditions [1]. Therefore, it is important to equip power systems with the capability to identify and correct errors in power network parameters.

Problem Description We consider the problem of correcting parameter values based on multi-period SCADA measurements. Starting from the current knowledge of parameters denoted by $\boldsymbol{\beta}_0 \in \mathbb{R}^l$, our objective is to estimate the parameter error $\boldsymbol{\Delta\beta} \triangleq \boldsymbol{\beta} - \boldsymbol{\beta}_0$ where $\boldsymbol{\beta}$ denotes the true parameter value. We assume that $\boldsymbol{\Delta\beta}$ is a *sparse* or *compressible* vector, i.e., only few parameters contain significant errors. The sparsity of parameter errors is a reasonable assumption that has been popularly employed, either implicitly or explicitly, in power network parameter estimation literature [1–3].

We formulate the sparse parameter correction problem as a sparse unsupervised regression problem. Specifically, we aim to estimate sparse $\boldsymbol{\Delta\beta}$ from the set of SCADA measurements

$$\mathbf{y}^{(i)} = \gamma(\mathbf{x}^{(i)}; \boldsymbol{\beta_0} + \boldsymbol{\Delta\beta}) + \mathbf{v}^{(i)} \qquad i = 1, \ldots, M \qquad (1)$$

where $\mathbf{y}^{(i)} \in \mathbb{R}^m$, $\mathbf{x}^{(i)} \in \mathbb{R}^n$, and $\mathbf{v}^{(i)} \in \mathbb{R}^m$ are the SCADA measurement vector, the state vector, and the noise vector for the i-th measurement period, and $\gamma(\mathbf{x}; \boldsymbol{\beta})$ is the measurement function that maps the state vector $\mathbf{x}$ to the corresponding measurement vector when the parameter is $\boldsymbol{\beta}$. This problem is *unsupervised* in that the state vectors $\mathbf{x}^{(i)}$'s are unknown. It can be easily shown that without the sparsity constraint on $\boldsymbol{\Delta\beta}$, the above problem becomes ill-posed, i.e., $\boldsymbol{\Delta\beta}$ is fundamentally not identifiable from the measurements. Therefore, sparsity of $\boldsymbol{\Delta\beta}$ is an essential condition.

Summary of Contributions In this paper, we formulate the parameter correction problem as a sparse unsupervised regression problem as described above and develop a sparse optimization framework to find an estimate of $\boldsymbol{\Delta\beta}$. We develop an iterative reweighted algorithm to find a local optimum of the sparse optimization problem. We demonstrate the efficacy of this method by evaluating its performance in parameter and topology correction for the IEEE 14-bus test case [4].

Related Works Existing approaches on power network parameter estimation can be categorized into two groups. The first is online local parameter estimation methods that aim to estimate parameters associated with certain line using the π model, the wave propagation model, and local meter data [5–11]. These methods can provide an accurate real-time parameter estimate, but their limitation is that they typically require certain types of meters to be located at specific locations. On the other hand, the second group is network-wide parameter estimation approaches that leverage SCADA or PMU measurements and the state estimation model to infer line parameter values [12–17]. For instance, the state augmentation methods leverage the existing state estimation framework to estimate parameter errors by considering uncertain parameter values as additional variables of the state estimation problem [13–15]. Their main limitation is that they require prior knowledge of suspected locations of parameter errors. In order to overcome this issue, Lagrange multipliers based methods have been proposed in [2, 16, 17]; the approaches analyze the Lagrange multipliers of parameter-related constraints in state estimation to localize parameter errors.

In contrast to benchmarks [12–16], our network-wide parameter estimation approach exploits the sparsity of parameter errors to enhance the error correction performance. Another difference is that our approach can be adapted to correct errors in parameters and topology simultaneously. This aspect makes our approach more robust than existing benchmarks; most benchmarks assume and rely on the accurate knowledge of topology in correcting parameter errors.

2. PROBLEM SETUP

Notations Throughout this paper, vectors are represented by boldface lower case letter (e.g., $\mathbf{x}$). The i-th entry of the vector $\mathbf{x}$ is denoted by $\mathbf{x}_i$. The set of all n dimensional real positive vectors are denoted by $\mathbb{R}_+^n$. The sign operator is denoted by $sign$, that is

$$sign(t) \begin{cases} 1 & if\ t \geq 0, \\ -1 & otherwise. \end{cases}$$

Parameter Correction as Sparse Unsupervised Regression The measurement equation (1) can be simplified as follows by defining $g(\mathbf{x}^{(i)}; \mathbf{\Delta\beta}) \triangleq \gamma(\mathbf{x}^{(i)}; \boldsymbol{\beta}_0 + \mathbf{\Delta\beta})$:

$$\mathbf{y}^{(i)} = g(\mathbf{x}^{(i)}; \mathbf{\Delta\beta}) + \mathbf{v}^{(i)} \qquad i = 1, \ldots, M. \tag{2}$$

We assume that the parameter error vector $\mathbf{\Delta\beta}$ is sparse (or compressible), i.e., only few parameters have significant errors. Given the set of measurements $\{\mathbf{y}^{(i)}, i = 1, \ldots, M\}$, our objective is to *estimate the sparse* $\mathbf{\Delta\beta}$. Note that this problem can be seen as fitting the *range* of $g(\cdot; \mathbf{\Delta\beta})$ to the cloud of data points $\{\mathbf{y}^{(i)}, i = 1, \ldots, M\}$ while constraining $\mathbf{\Delta\beta}$ to be sparse. This regression problem is unsupervised in a sense that $\mathbf{x}^{(i)}$'s are unknown. To estimate the sparse $\mathbf{\Delta\beta}$, we aim to find $\mathbf{\Delta\beta}$ that minimizes the fitting error while regularizing the l_p norm of $\mathbf{\Delta\beta}$ with $p \in (0, 1)$:

$$\min_{\mathbf{\Delta\beta}} \left(\sum_{i=1}^{M} \min_{\bar{\mathbf{x}}^{(i)}} \|\mathbf{y}^{(i)} - g(\bar{\mathbf{x}}^{(i)}; \mathbf{\Delta\beta})\|_2^2 + \lambda \cdot \|\mathbf{\Delta\beta}\|_p^p \right) \tag{3}$$

where $\min_{\bar{\mathbf{x}}^{(i)}} \|\mathbf{y}^{(i)} - g(\bar{\mathbf{x}}^{(i)}; \mathbf{\Delta\beta})\|_2^2$ is the squared Euclidean distance from $\mathbf{y}^{(i)}$ to the range of $g(\cdot; \mathbf{\Delta\beta})$.

3. PROPOSED APPROACH

The sparse unsupervised regression problem (3) can be rewritten as follows by explicitly including $\bar{\mathbf{x}}^{(i)}$'s as optimization variables:

$$\min_{\mathbf{z}} \left\{ F(\mathbf{z}) = f(\mathbf{z}) + \lambda \|\mathbf{\Delta\beta}\|_p^p \right\} \tag{4}$$

where $\mathbf{z} \triangleq [\mathbf{\Delta\beta}, \bar{\mathbf{x}}^{(1)}, \ldots, \bar{\mathbf{x}}^{(M)}]^T \in \mathbb{R}^{l+nM}$ and

$$f(\mathbf{z}) \triangleq \sum_{i=1}^{M} \|\mathbf{y}^{(i)} - g(\bar{\mathbf{x}}^{(i)}; \mathbf{\Delta\beta})\|_2^2$$

Instead of (4), we solve its ϵ-approximation, for which we can develop a simple iterative reweighted algorithm:

$$\min_{\mathbf{z}} \left\{ F_{\alpha,\epsilon}(\mathbf{z}) = f(\mathbf{z}) + \lambda \sum_{i=1}^{l} \left[|\mathbf{\Delta\beta}_i|^\alpha + \epsilon \right]^{\frac{p}{\alpha}} \right\} \tag{5}$$

where $\epsilon \in \mathbb{R}_+$ is very small constant and $\alpha \geq 1$.

3.1. Overview of the Approach

The proposed approach consists of two steps.

1. Sparse Unsupervised Regression First, we solve (5) to find a sparse $\mathbf{\Delta\beta}$ that fits the measurement data well. To solve (5), we use the iterative reweighted algorithm to be described in Section 3.2.

2. Bias Correction The least square estimate with the l_p norm regularization is often biased. Therefore, instead of accepting $\mathbf{\Delta\beta}$ as it is, we correct the bias in this estimate as follows. We compare the magnitude of each entry of $\mathbf{\Delta\beta}$ to a small threshold δ to detect the nonzero entry locations of $\mathbf{\Delta\beta}$; let $\mathcal{S}$ denote the set of detected nonzero row indices. Then, we solve the following least squares problem with the support constraint to obtain a new estimate of $\mathbf{\Delta\beta}$:

$$\begin{array}{ll} \min_{\mathbf{\Delta\beta}, \bar{\mathbf{x}}^{(1)}, \ldots, \bar{\mathbf{x}}^{(M)}} & \sum_{i=1}^{M} \|\mathbf{y}^{(i)} - g(\bar{\mathbf{x}}^{(i)}; \mathbf{\Delta\beta})\|_2^2 \\ \text{subj. to} & \mathbf{\Delta\beta}_i = 0 \ \text{ for } i \notin \mathcal{S}. \end{array}$$

3.2. Iterative Reweighted Algorithm for Regression

Lu in [18] proposed the IRL_α minimization algorithm to solve an l_p regularized unconstrained nonlinear problem with very mild condition on regression function, which is

$$\min_{\mathbf{z}} \left\{ P_{\alpha,\epsilon}(\mathbf{z}) = f(\mathbf{z}) + \lambda \sum_{i=1}^{l+nM} \left[|\boldsymbol{z_i}|^\alpha + \epsilon \right]^{\frac{p}{\alpha}} \right\} \tag{6}$$

where $f(\mathbf{z})$ is a smooth function with L_f- Lipschitz continuous gradient. We adapted this algorithm to solve our problem (5) wherein, unlike (6), the l_p norm of only *partial entries* of $\mathbf{z}$, especially that of $\mathbf{\Delta\beta}$, are regularized; state vectors are not sparse. Therefore, we modified the algorithm in [18] to solve (5) with the sparsity regularization on a subset of variables.

Initialize: $\alpha \geq 1$, $0 < L_{min} < L_{max}$, $\tau > 1$, $p \in (0, 1)$, $c, \epsilon, K_{max}, \zeta \in \mathbb{R}_+$, $\mathbf{z}^0 \in \mathbb{R}^{l+nM}$ is given. Set $k = 0$

1) Choose $L_k^0 \in \left[L_{min}, L_{max}\right]$ arbitrarily, set $L_k = L_k^0$

1a) solve the weighted l_α minimization problem

$$\mathbf{z}^{k+1} = \underset{\mathbf{z}\in\mathbb{R}^{l+nM}}{arg\,min}\Big\{ f(\mathbf{z}^k)+\nabla f(\mathbf{z}^k)^T(\mathbf{z}-\mathbf{z}^k)+\frac{L_k}{2}\|\mathbf{z}-\mathbf{z}^k\|_2^2 + \frac{\lambda p}{\alpha}\sum_{i=1}^{l}\mathbf{s}_i^k|\boldsymbol{\Delta\beta}_i|^\alpha \Big\} \quad (7)$$

where $\mathbf{s}_i^k = \Big[|\boldsymbol{\Delta\beta}_i^k|^\alpha + \epsilon\Big]^{\frac{p}{\alpha}-1}$, for $i = 1, \ldots, l$

1b) If

$$F_{\alpha,\epsilon}(\mathbf{z}^k) - F_{\alpha,\epsilon}(\mathbf{z}^{k+1}) \geq \frac{c}{2}\|\mathbf{z}^{k+1} - \mathbf{z}^k\|_2^2 \quad (8)$$

then go to step 2. Otherwise, go to 1c).

1c) set $L_k \leftarrow \tau L_k$ and go to Step 1a)

2) If $k < K_{max}$ and $\|\mathbf{z}^{k+1} - \mathbf{z}^k\|_\infty \geq \zeta$, set $k \leftarrow k+1$ and go to Step 1; otherwise, return $\mathbf{z}^* \leftarrow \mathbf{z}^{k+1}$ and terminate.

3.3. Solving Sub-problem (7)

When α is 1 or 2, we can derive a closed-form solution for sub-problem (7) in a similar way as it was done in [19]. After removing the constant terms, equation (7) can be written as follows.

$$\mathbf{z}^{k+1} = \underset{\mathbf{z}\in\mathbb{R}^{l+nM}}{arg\,min}\left\{\frac{1}{2}\|\mathbf{z} - \mathbf{u}^k\|_2^2 + \frac{\lambda p}{\alpha L_k}\sum_{i=1}^{l}\mathbf{s}_i^k|\boldsymbol{\Delta\beta}_i|^\alpha\right\}$$

where $\mathbf{u}^k = \mathbf{z}^k - \frac{1}{L_k}\nabla f(\mathbf{z}^k)$. As the objective function is separable, $\mathbf{z}^{k+1}$ can be computed per entry as follows: $\mathbf{z}_i^{k+1}$ is

$$\begin{cases} \underset{\Delta\beta_i\in\mathbb{R}}{argmin}\left\{\frac{(\Delta\beta_i - \mathbf{u}_i^k)^2}{2} + \frac{\lambda p}{\alpha L_k}\mathbf{s}_i^k|\Delta\beta_i|^\alpha\right\}, & \text{if } 1 \leq i \leq l \\ \mathbf{u}_i^k, & \text{o.w} \end{cases} \quad (9)$$

Note that when $\alpha = 2$, the minimization for $1 \leq i \leq l$ is simply minimization of a second-order polynomial. When $\alpha = 1$, it is well known that this minimization has a closed-form solution involving a soft-thresholding operator:

$$\underset{\Delta\beta_i\in\mathbb{R}}{argmin}\left\{\frac{(\Delta\beta_i - \mathbf{u}_i^k)^2}{2} + \frac{\lambda p}{L_k}\mathbf{s}_i^k|\Delta\beta_i|\right\} = soft\left(\mathbf{u}_i^k, \frac{\lambda p \mathbf{s}_i^k}{L_k}\right) \quad (10)$$

where $soft(u, a) \triangleq sign(u)\max\{|u| - a, 0\}$.

3.4. Convergence Analysis

Following the same steps as the convergence analysis of the iterative reweighted algorithm in [18], we can derive the following convergence property of our iterative reweighted algorithm. The proof is omitted due to the page limit.

Theorem 1. *Assume that the function $f(z)$ has Lipschitz continuous gradients with some finite Lipschitz constant L_f. Let $\{z^k\}$ be the sequence generated by the above algorithm. Let z^* be any limit point of $\{z^k\}$. Then z^* is a first-order stationary point of (5)*

3.5. Correction of Parameter and Topology Errors

The proposed method can be adapted to handle errors in parameters and topology simultaneously. This can be achieved by redefining the parameter vector $\boldsymbol{\beta}$ to include the parameter values of all transmission lines, both energized and de-energized (redefine $\boldsymbol{\beta}_0$ and $\boldsymbol{\Delta\beta}$ accordingly as well). The entries in the true parameter vector $\boldsymbol{\beta}$ for *de-energized* lines will be zero indicating that the effective admittance of the de-energized lines is zero. In that sense, the power network topology is encoded into the support of $\boldsymbol{\beta}$, and $\boldsymbol{\Delta\beta}$ can represent topology errors and parameter errors at the same time.

For instance, suppose that our current knowledge of topology is that a certain line is de-energized although the line is actually energized. The parameter values for this line in $\boldsymbol{\beta}_0$ will be all zeros as we are perceiving the line as disconnected. However, the corresponding parameter values in the true parameter vector $\boldsymbol{\beta}$ will be nonzero as the line is actually connected. Thus, a topology error contributes to $\boldsymbol{\Delta\beta} = \boldsymbol{\beta} - \boldsymbol{\beta}_0$ in a similar way as parameter errors, and the error can be estimated and corrected by our approach.

4. EXPERIMENTAL RESULTS

We evaluated the proposed method using the IEEE 14-bus test case [4] in the following two scenarios.

Scenario 1 First, we assumed that we have measurements of power flows (both directions) and power injections from 80% of branches and buses. For measurement noise, we used Gaussian noise with zero mean and standard deviation $\sigma = 0.005$. In each Monte Carlo run, ten parameter entries were randomly chosen[1] among all nonzero parameters, and we changed their values by 20% error to create our current knowledge of parameter values, i.e., $\boldsymbol{\beta}_0$. In other words, our current knowledge of parameters has errors at ten parameters, and the error magnitude (in percentage) is set to 20%.

Scenario 2 In the second scenario, there exist topology errors and parameter errors at the same time. We assumed that two transmission lines are perceived as de-energized in our current topology information even though they are actually energized. In this scenario, we evaluate the adapted version of our method for simultaneous correction of topology and parameter errors as described in Section 3.5. We assumed

[1]Parameter values associated with the line between bus 7 and bus 8 were excluded. Because, an error in these parameters are fundamentally not identifiable based on SCADA measurements and state estimation model; this is due to that bus 8 is connected only to 7, but no other buses.

that we have measurements of power flows and power injections from 80% of branches and buses; we assumed no line flow meter on the two lines associated with the topology information errors in order to keep the problem non-trivial. Five parameter entries were randomly chosen similar to Scenario 1, and 20%-magnitude errors were introduced to the selected entries to form the erroneous current knowledge of the parameters.

Sparse Unsupervised Regression Algorithm For the initial point of our iterative algorithm, we perturbed the nominal operating state for state entries (i.e., magnitude one and phase angle zero) with Gaussian noise of zero mean and 0.01 standard deviation, and we sampled zero mean independent Gaussian random variables with standard deviation 0.001 for $\boldsymbol{\Delta\beta}$ entries. The algorithm parameters were set as follows: $p = 0.9$, $\alpha = 1$, $\epsilon = 10^{-5}$, $L_{min} = 10^{-8}$, $L_{max} = 10^{8}$, $c = 10^{-4}$, $\tau = 3$, $K_{max} = 10^{4}$, $\zeta = 10^{-12}$, $\delta = 0.001$ and $L_0^0 = 1$. For each k, L_k^0 is set to

$$L_k^0 = max\left\{L_{min}, min\left\{L_{max}, \frac{\mathbf{\Delta z}^T\mathbf{\Delta b}}{\|\mathbf{\Delta z}\|_2^2}\right\}\right\} \tag{11}$$

where $\mathbf{\Delta z} = \mathbf{z}^k - \mathbf{z}^{k-1}$ and $\mathbf{\Delta b} = \nabla f(\mathbf{z}^k) - \nabla f(\mathbf{z}^{k-1})$. This is the setting suggested for the iterative reweighted algorithm in [18] from which we derived our iterative algorithm.

As expected, the algorithm performance depends on the regularization parameter λ. To choose a proper λ, we started with a sufficiently large λ and decreased λ until the least squares estimation residue from the bias correction step passes the corresponding χ^2 test with the false alarm rate 0.99 (the same as the one described in [20]); the fact that the residue passes the χ^2 test implies that all error locations were taken into account in the bias correction step. We chose the *largest* λ, with which the estimation residue passes the χ^2 test. In this way, we can prevent our approach from falsely including unnecessarily many parameter error locations in the bias correction stage. For the experiments, we chose λ from a predefined set: (10, 1, 0.1, 0.05, 0.01).

Evaluation Metric We consider the following performance metrics:

$$Mean\ squared\ error = E\left[\|\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}\|_2^2\right]$$

$$Normalized\ L_\infty\ error = E\left[\frac{1}{C}\|\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}\|_\infty\right]$$

$$Normalized\ mean\ absolute\ error = E\left[\frac{1}{C}\left(\frac{\|\boldsymbol{\beta} - \hat{\boldsymbol{\beta}}\|_1}{l}\right)\right]$$

where $\hat{\boldsymbol{\beta}}$ denotes the parameter estimate (obtained according to the estimate of $\boldsymbol{\Delta\beta}$) and $C \triangleq \|\boldsymbol{\beta}\|_1/\|\boldsymbol{\beta}\|_0$ is the average absolute value of nonzero parameters.

Results Table 1 provides the performance metrics of the proposed sparse unsupervised regression approach and the normalized Lagrange multiplier method proposed in [2] for Scenario 1. The results are based on 10 Monte Carlo runs, and both approaches used measurements from 100 periods, i.e., $M = 100$. The table also provides standard deviation σ of 10 error values that were used for computing the mean metric. While the Lagrange multiplier method performed reasonably well, our proposed approach outperformed it and resulted in a smaller mean and standard deviation in every error metric we considered. We observed that when the measurement redundancy is higher or the number of parameter errors is much lower, both approaches demonstrated quite similar performance. However, when the measurement redundancy is not high and the number of parameter errors is not so small (as in Scenario 1), our approach exploiting the sparsity of parameter errors outperformed as shown in Table 1. Roughly speaking, the normalized l_∞ error gives the maximum parameter error among all parameter entries, in percentage. Even with ten parameter errors, our approach estimated the parameters with normalized l_∞ error equal to 0.007, which means that the worst parameter error among all parameter entries is roughly about 0.7 percent in the percentage error.

	Proposed Approach	**Lagrange**
MSE	1.60×10^{-3} (σ= 1.80×10^{-3})	1.46×10^{-2} (σ= 1.75×10^{-2})
L_∞ error	7.00×10^{-3} (σ= 5.70×10^{-3})	1.81×10^{-2} (σ= 1.06×10^{-2})
MAE	3.20×10^{-4} (σ= 1.66×10^{-4})	1.1×10^{-3} (σ= 7.94×10^{-4})

Table 1. Performance metric for Scenario 1

(No.Measurements)	**100**	**200**
MSE	8.00×10^{-3} (σ= 1.98×10^{-2})	8.91×10^{-4} (σ= 3.63×10^{-4})
L_∞ error	1.19×10^{-2} (σ= 1.65×10^{-2})	5.20×10^{-3} (σ= 1.50×10^{-3})
MAE	5.72×10^{-4} (σ= 5.10×10^{-4})	2.95×10^{-4} (σ= 5.08×10^{-5})

Table 2. Performance metric for Scenario 2

Table 2 illustrates the results for Scenario 2 for sparse unsupervised regression algorithm. The proposed method was able to correct 5 parameter errors and two line connectivity errors at the same time. When measurements from 200 periods were used, the proposed method resulted in the worst parameter error (among all parameter entries) about 0.52 %.

5. CONCLUSION

A novel sparse unsupervised regression method is proposed to correct line parameters and topology by exploiting the sparsity of the parameter error vector. By incorporating sparsity, we envision that the proposed method would perform better than existing benchmarks especially when the number of parameter errors is moderate. A thorough comparative study of the proposed method and other benchmarks has to follow.

6. REFERENCES

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