ITERATIVELY REWEIGHTED LINEAR LEAST SQUARES FOR FREQUENCY ESTIMATION IN UNBALANCED THREE-PHASE POWER SYSTEM

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ABSTRACT

Smart grid has attracted increasing attention in the past decade, and one of its common problems is the variation of the nominal frequency (50 or 60 Hz) introduced by harmonics. In this paper, a batch-mode frequency estimator that can accurately obtain the deviation from the nominal frequency is proposed. The signal model, which includes not only the fundamental frequency but also the harmonics, is first defined, and its characteristic is then studied. Employing the linear prediction (LP) property of the model, the deviated frequency is iteratively updated according to the weighted LP errors, to achieve accurate fundamental frequency estimation. Computer simulations indicate that our proposed method is more accurate and reliable than the conventional estimators in the presence of harmonics and amplitude oscillation.

Index Terms— Unbalanced three-phase power system, frequency estimation, harmonics, generalized weighted linear prediction, batch-mode

1. INTRODUCTION

Due to high efficiency and reliability, smart grid [1] has attracted much attention. Different from the traditional power system, smart grid is a mesh network [2], whose nodes can be both user terminals and power generators. Although smart grid improves the utilization efficiency of electricity, it also causes many problems in power system, such as the harmonics, unbalanced voltages and current oscillation [3]. All these problems result in the resonance and overheating of electrical equipments. Among these interferences, the third and fifth harmonics [4] are most commonly confronted and difficult to avoid, and a fluctuation of nominal frequency is then produced, leading to the asynchronism and instability of the whole electrical system. To ensure the stability, monitoring the change of frequency in the presence of the interferences is a crucial task in smart grid. Therefore, accurate frequency deviation estimation [5] in the case of harmonics or amplitude variation becomes very important.

Among numerous estimators developed in the literature, the well-known frequency estimation techniques include the complex least mean square (CLMS) [6] and augmented complex least mean square (ACLMS) [7], which can provide the real-time estimation. However, in the presence of harmonics or amplitude oscillation, the performance of these methods is not satisfactory since the gap between the estimates and true values becomes large as time goes on. Furthermore, an accurate estimator, namely, the iterative adaptive approach (IAA) [8] is developed. However, since grid search on $[0, 2\pi)$ is required, this method suffers from very high computational complexity, particularly for a large number of grid points.

In this work, an accurate and stable frequency estimator is devised. Since at a fixed time index, we employ a sliding window to collect the current and its previous measurements, our method can be regarded as a batch-mode algorithm. To deal with the harmonics, we define a new signal model, where the third and fifth harmonics are considered as the signal. Employing the $\alpha\beta$-transformation, the new signal model can be regarded as the linear combination of three complex tones. The linear prediction (LP) property of the multiple tones is first studied, while the generalized weighted linear prediction (GWLP) [9]– [11] is utilized to estimate the frequency accurately. To guarantee the accuracy of our proposed scheme, the estimates at a fixed time index t is obtained according to a batch of observations.

The rest of this paper is organized as follows. Our new signal model is first described in Section 2, and the proposed algorithm is then developed. To demonstrate the performance of the proposed method, computer simulation results are studied in Section 3 in the presence of harmonics and/or amplitude oscillation. Finally, conclusions are drawn in Section 4.

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2. PROPOSED METHOD

Without loss of generalization, in the unbalanced three-phase system, the observation at time slot t_n is modeled as [7]:

$$u_a[n] = V_a \cos(2\pi f t_n + \phi) + \eta_a[n], \tag{1}$$

$$u_b[n] = V_b \cos\left(2\pi f t_n + \phi - \frac{2\pi}{3}\right) + \eta_b[n], \tag{2}$$

$$u_c[n] = V_c \cos\left(2\pi f t_n + \phi + \frac{2\pi}{3}\right) + \eta_c[n], \tag{3}$$

where V_a, V_b and V_c denotes amplitudes corresponding to different phase components, f is the unknown fundamental frequency whose nominal value is 50 (or 60) Hz, $t_n = n/F_s$, F_s denotes the sampling frequency in Hz, ϕ denotes the initial phase. Here the system is balanced when $V_a = V_b = V_c$, and unbalanced otherwise. The $\eta_a[n]$, $\eta_b[n]$ and $\eta_c[n]$ are independent and identically distributed noise sequence following white Gaussian distribution with unknown equivalent variance σ^2 [12]. The task is to find the unknown f from observations $\{u_a[n]\}_{n=0}^{N-1}$, $\{u_b[n]\}_{n=0}^{N-1}$ and $\{u_c[n]\}_{n=0}^{N-1}$.

Employing the $\alpha\beta$-transformation [13], the three sequences on (1)–(3) can be combined into a complex-valued voltage signal, which is expressed as:

$$u_n = s_n(\omega) + q_n, \tag{4}$$

where $s_n(\omega) = A\exp\{j(\omega n + \phi)\} + B\exp\{-j(\omega n + \phi)\}$, $\omega = 2\pi f/F_s$ is the discrete frequency in rads^{-1}, q_n denotes the complex noise term, and

$$A = \frac{\sqrt{6}(V_a + V_b + V_c)}{6}, \tag{5}$$

$$B = \frac{\sqrt{6}(2V_a - V_b - V_c)}{6} - j\frac{\sqrt{2}(V_b - V_c)}{4}, \tag{6}$$

$$\Re\{q_n\} = \sqrt{\frac{2}{3}}\big(\eta_a[n] - \frac{1}{2}\eta_b[n] - \frac{1}{2}\eta_c[n]\big), \tag{7}$$

$$\Im\{q_n\} = \frac{\sqrt{2}}{2}\big(\eta_b[n] - \eta_c[n]\big), \tag{8}$$

with $\Re\{\cdot\}$ and $\Im\{\cdot\}$ denoting the real and imaginary operators. Based on (7)–(8), $E\{\Re\{q_n\}\Im\{q_n\}\} = 0$ where $E\{\cdot\}$ denotes the expectation operator, and hence q_n are uncorrelated [14]. Here the estimation problem is to obtain ω from $\{u_n\}_{n=0}^{N-1}$.

As we have known that in smart grid, the balanced third harmonic and fifth harmonic of the fundamental frequency ω arise commonly and cannot be avoided. Therefore, combining the harmonic components, we define a new signal model which is

$$v_n = s_n(\omega) + \mu_1 s_n(3\omega) + \mu_2 s_n(5\omega) + q_n, \tag{9}$$

where μ_1 and μ_2 are the coefficients of third and fifth harmonics, respectively.

As the new v_n is composed of 6 complex tones, according to [15], it can be uniquely expressed as a linear combination of its previous 6 samples, which is

$$\sum_{k=0}^{6} a_k v_{n-k} = 0, \quad a_0 = 1, \tag{10}$$

where $\{a_k\}_{k=0}^{6}$ are the LP coefficients with $a_k = a_{6-k}$. Alternatively, we can find the frequencies from the roots of

$$\sum_{k=0}^{2} a_k \left(e^{j\omega_m k} + e^{j\omega_m(6-k)}\right) + a_3 e^{3j\omega_m} = 0, \tag{11}$$

where $\omega_m = (2m+1)\omega$ with $m = 0, 1, 2$. It is worth to point out that the fundamental frequency ω corresponds to the root with smallest value.

To estimate ω at time index n, denoted by $\omega^{(n)}$, we employ current and the previous $(L-1)$ samples, which are $\{v_l\}_{l=n-L+1}^{n}$. In our study, a sliding window is utilized to obtain these L samples. Then following the main idea of the GWLP approach [9], the LP error is

$$\mathbf{e}^{(n)} = \mathbf{X}_1^{(n)} + \mathbf{X}_2^{(n)}\tilde{\mathbf{a}}^{(n)}, \tag{12}$$

where $\tilde{\mathbf{a}}^{(n)} = [\tilde{a}_1\ \tilde{a}_2\ \tilde{a}_3]^T$ and

$$\mathbf{X}_1^{(n)} = \begin{bmatrix} v_n + v_{n-6} \\ v_{n-1} + v_{n-7} \\ \vdots \\ v_{n-L+7} + v_{n-L+1} \end{bmatrix}, \tag{13}$$

$$\mathbf{X}_2^{(n)} = \begin{bmatrix} v_{n-1} + v_{n-5} & v_{n-2} + v_{n-4} & v_{n-3} \\ v_{n-2} + v_{n-6} & v_{n-3} + v_{n-5} & v_{n-4} \\ \vdots & \ddots & \vdots \\ v_{n-L+6} + v_{n-L+2} & v_{n-L+5} + v_{n-L+3} & v_{n-L+4} \end{bmatrix}. \tag{14}$$

Employing the GWLP, the estimate $\mathbf{a}^{(n)}$, referred to as $\hat{\mathbf{a}}^{(n)}$, is obtained by minimizing cost function

$$J(\hat{\mathbf{a}}^{(n)}) = \left(\mathbf{X}_1^{(n)} + \mathbf{X}_2^{(n)}\tilde{\mathbf{a}}^{(n)}\right)^H \left(\mathbf{W}^{(n)}\right)^{-1} \left(\mathbf{X}_1^{(n)} + \mathbf{X}_2^{(n)}\tilde{\mathbf{a}}^{(n)}\right), \tag{15}$$

where $^{-1}$ and ^{H} are matrix inverse and conjugate transpose, respectively, $\mathbf{W}^{(n)}$ denotes the weighting matrix, which is

$$\begin{aligned}\mathbf{W}^{(n)} =& \frac{1}{\sigma^2}E\left\{\mathbf{e}^{(n)}\left(\mathbf{e}^{(n)}\right)^H\right\} \\ =& \text{Toeplitz}([\mathbf{b}_9\ \mathbf{0}_{L-9}]),\end{aligned} \tag{16}$$

where $\text{Toeplitz}(\cdot)$ denotes the Toeplitz matrix [16], $\mathbf{0}_{L-15}$ is the $1 \times (L-15)$ row vector with all elements 0, and $\mathbf{b}_9$ is

$$\begin{aligned}\mathbf{b}_9 = [&2 + |a_3|^2 + 2|a_1|^2 + 2|a_2|^2, 2\Re\{a_1 + a_1 a_2^* + a_2 a_3^*\}, \\ &2\Re\{a_2 + a_1 a_3^*\} + |a_2|^2, 2\Re\{a_3 + a_1 a_2^*\}, \\ &2\Re\{a_2\} + |a_1|^2, 2\Re\{a_1\}, 1].\end{aligned} \tag{17}$$

According to (16)–(17), the weighting matrix $\mathbf{W}^{(n)}$ is expressed as a complicated function of $\mathbf{a}^{(n)}$. Therefore, to obtain $\hat{\mathbf{a}}^{(n)}$, the cost function $J(\hat{\mathbf{a}}^{(n)})$ is minimized in an iterative manner. Then the estimate in the ℓ-th iteration, denoted by $\hat{\mathbf{a}}_\ell^{(n)}$, is obtained by

$$\hat{\mathbf{a}}_\ell^{(n)} = -\left\{\left(\mathbf{X}_2^{(n)}\right)^H \left(\mathbf{W}_{\ell-1}^{(n)}\right)^{-1} \mathbf{X}_2^{(n)}\right\}^{-1} \bullet \left(\mathbf{X}_2^{(n)}\right)^H \left(\mathbf{W}_{\ell-1}^{(n)}\right)^{-1} \mathbf{X}_1^{(n)}, \quad (18)$$

where $\bullet$ denotes the matrix product and $\mathbf{W}_{\ell-1}^{(n)}$ is computed using $\hat{\mathbf{a}}_{\ell-1}^{(n)}$ and (16).

The steps of the proposed method in each batch-mode data is summarized in Table I.

Table 1: Summary of proposed algorithm

(i)	Prepare the batch observation vector $\{v_l\}_{l=n-L+1}^{n}$;
(ii)	Implement $\mathbf{X}_1^{(n)}$ and $\mathbf{X}_2^{(n)}$ using (13)–(14);
(iii)	Initialize $\hat{\mathbf{a}}_0^{(n)}$ by $-\left\{\left(\mathbf{X}_2^{(n)}\right)^H \mathbf{X}_2^{(n)}\right\}^{-1} \left(\mathbf{X}_2^{(n)}\right)^H \mathbf{X}_1^{(n)}$;
(iv)	Obtain $\mathbf{W}_{\ell-1}^{(n)}$ using $\hat{\mathbf{a}}_{\ell-1}^{(n)}$ and (16) with $\ell = 1, 2, \cdots$;
(v)	Compute the estimate at ℓ-th iteration $\hat{\mathbf{a}}_\ell^{(n)}$ using (18)
(vi)	Repeat (iv)-(v) until the relative error $\frac{\|\hat{\mathbf{a}}_\ell^{(n)} - \hat{\mathbf{a}}_{\ell-1}^{(n)}\|_2}{\|\hat{\mathbf{a}}_\ell^{(n)}\|_2} < \epsilon$ is reached, where $\|\cdot\|_2$ is ℓ_2-norm and ϵ is tolerance;
(vii)	Obtain the fundamental frequency $\hat{\omega}^{(n)}$ by finding the smallest positive root of (11).

It is worth to point out that since the proposed method is based on the GWLP, the variance and convergence analysis can also be derived similar to [9]. Moreover, as a batch-mode, the computational complexity of our proposed method is $\mathcal{O}(4KNL^3)$ with K being the number of iterations.

3. SIMULATION RESULTS

In this section, computer simulations are conducted to evaluate the performance of the proposed estimator. The signal is generated according to (1)–(3) with a duration of 0.1s, which is, length of $\{u_n\}$ is $N = 500$. The parameters are set to $f = 50.2$ Hz, $F_s = 5000$ Hz and $\phi = \pi/6$, therefore we have $\omega = 0.02008\pi$ rads^{-1} according to the definition in (4). The proposed algorithm is compared with the CLMS and ACLMS approaches, whose initial values are set to as the nominal frequency 50 Hz. As a batch-mode method, the proposed method starts from $t_n = L/F_s$ and L is chosen as 90 according to a number of empirical experiments. The proposed method employ the stopping criterion that tolerance $\epsilon = 10^{-6}$ is reached. The signal-to-noise ratio (SNR) [17] is set to 15 dB and all results are based on the average of 1000 independent runs.

First, the contamination effect of third and fifth harmonics is studied. In this test, we choose $V_a = V_b = V_c = 1$ and $V_a = V_b = V_c = 1.1$ for the times before and after $t = 0.03s$, respectively. A balanced 20% third harmonic and a balanced 10% fifth harmonic of the fundamental frequency ω are included into the voltages from $t = 0.03s$. Figure 1 shows that although the fluctuation phenomenon cannot be fully removed, our estimator gives the smallest oscillation amplitude around the true value among all the algorithms. Furthermore, once the harmonics appear, the other two methods fail to estimate the frequency accurately and the derivations become larger as time t goes on.

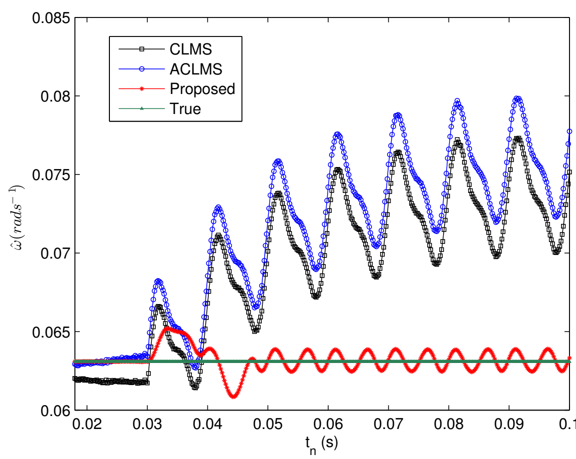

Figure 1. Frequency estimates under harmonic contamination

Second, the mean square frequency error (MSFE), defined as $E\{(\omega - \hat{\omega})^2\}$, is employed to investigate the estimation accuracy of the proposed, CLMS and ACLMS approaches. Here we only consider one batch data with $N = 90$ and $V_a = V_b = V_c = 1$, and the other parameters are the same as those in the previous test. The MSFE result for SNR varying from -10 to 20dB is shown in Figure 2, indicating that our method is superior to the other two methods since its MSE is very close to the CRLB. The reason the proposed method fails to achieve the CRLB is that it does not consider the relationship between the fundamental frequency and the harmonics, which, can be employed to further improve the performance. The situation of amplitude variation [7] is then introduced to investigate the general performance of the proposed method.

The changes of voltages and the corresponding estimation results are shown in Figures 3 and 4, respectively. It is seen in Figure 3 that the amplitudes of three voltages are $V_a = V_b = V_c = 1$ before $t = 0.02s$, and $V_a = 1.05$ and $V_b = V_c = 1.1$ among $0.02s \le t < 0.06s$; subsequently, V_c becomes 0 after $t = 0.06s$. It can be observed in Figure 4 that the proposed algorithm performs better than the CLMS and ACLMS methods for abrupt changes, due to the reason that the batch-mode method utilizes more information.

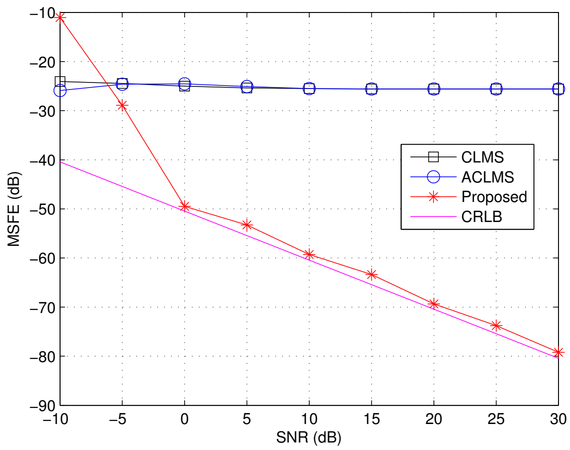

Figure 2. MSFE versus SNR

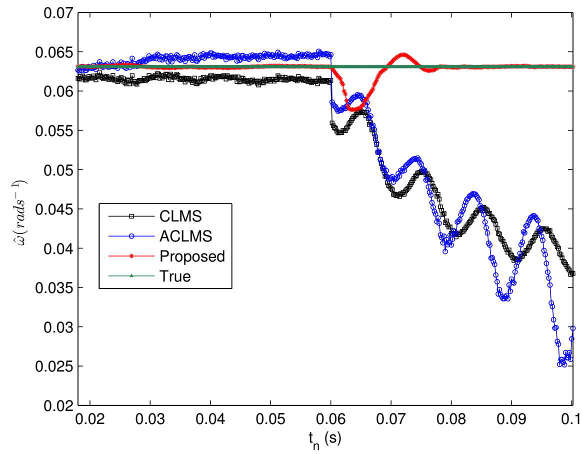

Figure 4. Frequency estimates in unbalanced system

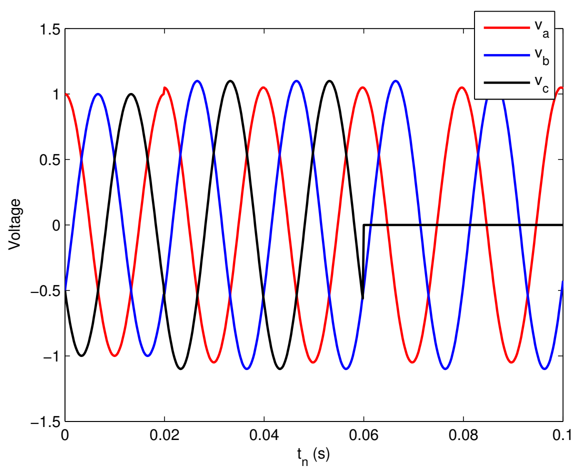

Figure 3. Three channel signals in unbalanced system

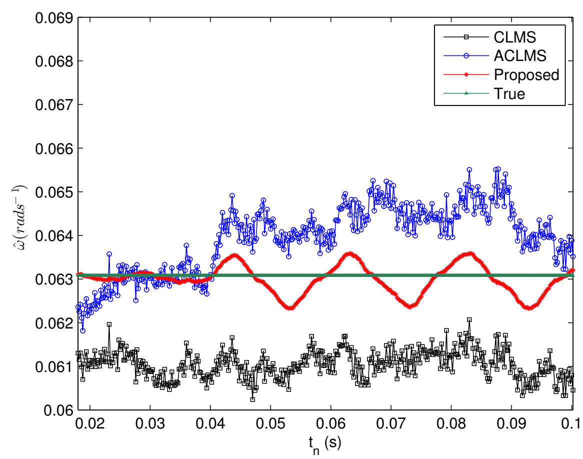

Figure 5. Frequency estimates under amplitude variation

Finally, the situation of time-varying amplitude, another type of amplitude variation, is also studied. The amplitude voltages are set to $V_a = 1 + 0.05\sin(2\pi ft), V_b = 1 + 0.1\sin(2\pi ft)$ and $V_c = 1 + 0.15\sin(2\pi ft)$ from $t = 0.03s$, and the other parameters are the same as those in the first test. It is shown in Figure 5 that although three methods oscillate, only the proposed method fluctuates around the true value with smallest oscillation amplitude. This is because that employing the property of the trigonometric function, the signal with time-varying amplitude is similar to (9) and hence, the proposed method can provide a satisfactory performance. To summarize, for amplitude variation and harmonics, our proposed algorithm always provides more accurate and reliable estimates than the other algorithms. Even with some contamination or oscillation exists, the estimate of our method usually fluctuates around the true value with the smallest variation.

4. CONCLUSION

In this paper, an accurate and reliable batch-mode frequency estimator is developed, which can monitor the variation of frequency accurately. The signal model of the three-phase system is redefined based on harmonics and the GWLP method is employed to estimate the frequency accordingly. Computer simulations are conducted in the scenarios of amplitude variation and harmonics, which shows that our proposed method is superior to CLMS and ACLMS. In the future, we will utilize the relationship between the fundamental and harmonics, to improve the performance of our method.

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