SMART DSP FOR A SMARTER POWER GRID: TEACHING POWER SYSTEM ANALYSIS THROUGH SIGNAL PROCESSING

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ABSTRACT

The future Smart Grid represents an extraordinary opportunity to transform the ways we currently approach energy into a new era of low-carbon, renewable, and efficient solutions which will ultimately have a significant impact on both the environment and economy. This effort requires close collaboration of experts from the Power, Digital Signal Processing (DSP) and Machine Learning (ML) communities, with the common language between these diverse disciplines an important first step in this endeavour. To promote seamless transition of ideas, we here establish a duality between the Clarke transform, a workhorse in Power Grid analysis, and principal component analysis (PCA), a staple subspace method in DSP/ML. Upon highlighting the limitations of the Clarke transform in off-nominal unbalanced power system conditions, we illuminate a DSP-enabled class of self-balancing solutions, referred to as the Smart Transforms, based on adaptive complex widely linear modelling. Examples on system frequency estimation support the approach.

Index Terms— DSP Education, Smart Grid, Principal Component Analysis, Clarke Transform, Widely Linear Modelling

1. INTRODUCTION

The deployment of distributed generation, mainly through small photovoltaic systems, and the electrification of transport and heat are causing significant changes in the distribution network [1]. There will be more power circulating through the distribution network and it will have less-obvious power flow directions. Consequently, there will be more chances of reaching network constraints, such as the maximum current and maximum voltage deviation, together with potentially greater issues with unbalanced conditions such as brownouts and blackouts. The conventional approach to resolve this problem such as network reinforcement (e.g. upgrading transformers) is very costly and therefore network designers need to be able to push the operation of the network closer to the physical limits and will require tools that are designed to analyse conditions that are less ideal [2].

Current analysis tools such as the Clarke and related transforms derive from the antecedent technology area of Circuit Theory, and were designed for systems that operate in balanced conditions [3, 4, 5, 6]. Given the dynamically unbalanced nature of future power grids, there is a need for a fresh perspective of three-phase power signal analysis so that power systems can be analysed using modern tools and in off-nominal conditions. This will inevitably open the door for the deployment of more sophisticated algorithms, which have been extensively studied in the signal processing and machine learning communities, to be used for the most critical tasks such as fault detection and frequency estimation.

The first step towards achieving this goal is to establish a common language between the Power and Data Analytics communities, a subject of this work. Consequently, we set out to show the dualities between existing methods established in both power and data driven communities as a first step towards "Smart DSP for a Smarter Power Grid". More specifically, the focus of this work is to elucidate the duality between the Clarke transform, a fundamental tool in power system analysis, and principal component analysis (PCA). Further opportunities for wider deployment of signal processing and machine learning are shown by highlighting the nature of three-phase signals in terms of subspaces and trajectories. Finally, through adaptive complex widely linear modelling, we introduce a class of solutions, referred to as the Smart Clarke and Park transforms, which tackle the limitations of classic Clarke/Park transforms for frequency estimation in unbalanced system conditions and can be seamlessly integrated in graduate courses on Smart Grids or subspace based spectral estimation.

2. BACKGROUND

Consider a sampled three-phase voltage measurement vector, $\boldsymbol{s}_k$, which at a discrete time instant, k, is given by

$$\boldsymbol{s}_k = \begin{bmatrix} v_{a,k} \\ v_{b,k} \\ v_{c,k} \end{bmatrix} = \begin{bmatrix} V_a \cos(\omega k + \phi_a) \\ V_b \cos(\omega k + \phi_b - \frac{2\pi}{3}) \\ V_c \cos(\omega k + \phi_c + \frac{2\pi}{3}) \end{bmatrix}, \tag{1}$$

where V_a, V_b, V_c are the amplitudes of the phase voltages $v_{a,k}$, $v_{b,k}$, $v_{c,k}$, while $\omega = 2\pi f$ is the fundamental angular frequency, with f the fundamental power system frequency. The off-nominal angles for phase voltages are denoted by ϕ_a, ϕ_b, and ϕ_c. The system is said to be in 'balanced' conditions if $V_a = V_b = V_c$ and $\phi_a = \phi_b = \phi_c$. From the three-phase voltage, $\boldsymbol{s}_k$, and upon employing the identity $\cos(x) = (e^{jx} + e^{-jx})/2$, we arrive at its complex–valued phasor representation in the form

$$\boldsymbol{s}_k = \frac{1}{2}\left(\boldsymbol{v}e^{j\omega k} + \boldsymbol{v}^* e^{-j\omega k}\right), \tag{2}$$

where $\boldsymbol{v} = [\bar{V}_a, \bar{V}_b, \bar{V}_c]^\mathsf{T}$ and

$$\bar{V}_a = \frac{V_a}{\sqrt{2}} e^{j\phi_a},\ \bar{V}_b = \frac{V_b}{\sqrt{2}} e^{j(\phi_b - \frac{2\pi}{3})},\ \bar{V}_c = \frac{V_c}{\sqrt{2}} e^{j(\phi_c + \frac{2\pi}{3})}. \tag{3}$$

2.1. Clarke and Park transform

The Clarke transform, also known as the $\alpha\beta$ transform, aims to change the basis of the original 3D vector space where the three-phase signal $\boldsymbol{s}_k$ resides, to yield the Clarke–transformed $v_{0,k}, v_{\alpha,k}, v_{\beta,k}$ voltages in the form

$$\begin{bmatrix} v_{0,k} \\ v_{\alpha,k} \\ v_{\beta,k} \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}}_{\text{Clarke matrix}} \underbrace{\begin{bmatrix} v_{a,k} \\ v_{b,k} \\ v_{c,k} \end{bmatrix}}_{\boldsymbol{s}_k}. \tag{4}$$

The quantities $v_{\alpha,k}$ and $v_{\beta,k}$ are referred to as the α and β sequences, while the term $v_{0,k}$ is called the zero-sequence, as it is null when the three-phase signal, $\boldsymbol{s}_k$, is balanced, to yield the reduced 2D interpretation

$$\begin{bmatrix} v_{\alpha,k} \\ v_{\beta,k} \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}}_{\text{reduced Clarke matrix } \mathbf{C}} \begin{bmatrix} v_{a,k} \\ v_{b,k} \\ v_{c,k} \end{bmatrix}. \tag{5}$$

The Park transform, also known as the dq transform, multiplies the Clarke $\alpha\beta$ voltages in (5) with a time-varying unit–determinant rotation matrix, $\mathbf{P}_\theta$, to produce the Park voltages, $v_{d,k}, v_{q,k}$, i.e.

$$\begin{bmatrix} v_{d,k} \\ v_{q,k} \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta_k) & \sin(\theta_k) \\ -\sin(\theta_k) & \cos(\theta_k) \end{bmatrix}}_{\text{Park matrix } \mathbf{P}_\theta} \begin{bmatrix} v_{\alpha,k} \\ v_{\beta,k} \end{bmatrix}. \tag{6}$$

where $\theta_k = \omega k$. Notice that the Clarke $\alpha\beta$ voltage in (5) can be conveniently expressed as a complex variable, $\bar{s}_k \stackrel{\text{def}}{=} v_{\alpha,k} + jv_{\beta,k}$, and can, therefore, be compactly described as

$$\bar{s}_k = \boldsymbol{c}^{\mathsf{H}} \boldsymbol{s}_k, \qquad \boldsymbol{c} \stackrel{\text{def}}{=} \sqrt{\frac{2}{3}} \big[1, e^{-j\frac{2\pi}{3}}, e^{j\frac{2\pi}{3}}\big]^{\mathsf{T}}. \tag{7}$$

2.2. Principal Component Analysis

Principal component analysis (PCA) is a well-established tool which performs an orthogonal transformation in order to separate meaningful data from noise, or to reduce the dimensionality of the original signal space while maintaining the most important information bearing latent components in data. This is achieved by assuming that information is captured by variance in the data. Therefore, PCA projects the data onto a set of basis vectors (also called principal components) where the first principal component is defined as the direction in the original signal space with maximum variance, the second principal component is the direction with the next largest variance and is orthogonal to the first principal component, and so on. The overall solution to this problem is the eigenvector matrix of the data covariance matrix. In other words, for a general data vector, $\boldsymbol{x}_k \in \mathbb{R}^{M\times 1}$, for which the covariance matrix, $\mathbf{R}_{\boldsymbol{x}}$, is defined as

$$\mathbf{R}_{\boldsymbol{x}} \stackrel{\text{def}}{=} \operatorname{cov}(\boldsymbol{x}_k) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \boldsymbol{x}_k \boldsymbol{x}_k^{\mathsf{T}}, \tag{8}$$

the symmetric covariance matrix, $\mathbf{R}_{\boldsymbol{x}}$, admits the following eigenvalue decomposition

$$\mathbf{R}_{\boldsymbol{x}} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{\mathsf{T}} \tag{9}$$

where the diagonal eigenvalue matrix, $\boldsymbol{\Lambda} = \mathsf{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$, indicates the power of each principal component within $\boldsymbol{x}_k$, while the matrix of eigenvectors, $\mathbf{Q}_r = [\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_M]$, designates the principal directions in the data.

3. NEW PERSPECTIVE

We now provide a modern interpretation of the Clarke transform as a principal component analyser under balanced conditions.

3.1. Clarke transform as a PCA

Without loss of generality, we consider normalised versions of the phasors (relative to $\bar{V}_a$), and define $\delta_i \stackrel{\text{def}}{=} \bar{V}_i/\bar{V}_a$, $i \in \{a,b,c\}$, with $\delta_a = 1$, to yield $\bar{\boldsymbol{v}} = \begin{bmatrix} 1, & \delta_b, & \delta_c \end{bmatrix}^{\mathsf{T}}$, where

$$\delta_b = \frac{V_b}{V_a} e^{j(\phi_b - \phi_a - \frac{2\pi}{3})}, \quad \delta_c = \frac{V_c}{V_a} e^{j(\phi_c - \phi_a - \frac{2\pi}{3})}. \tag{10}$$

For unbalanced systems, the covariance matrix, $\mathbf{R}_{\boldsymbol{s}}^u$, reflects the effects of off–nominal amplitude/phase conditions, modelled by the complex–valued imbalance ratios, δ_b and δ_c, and is given by

$$\mathbf{R}_{\boldsymbol{s}}^u = \frac{1}{2} \begin{bmatrix} 1 & |\delta_b|\cos(\angle\delta_b) & |\delta_c|\cos(\angle\delta_c) \\ |\delta_b|\cos(\angle\delta_b) & |\delta_b|^2 & |\delta_b||\delta_c|\cos(\angle\delta_b - \angle\delta_c) \\ |\delta_c|\cos(\angle\delta_c) & |\delta_b||\delta_c|\cos(\angle\delta_b - \angle\delta_c) & |\delta_c|^2 \end{bmatrix}. \tag{11}$$

In balanced conditions, the covariance matrix of the normalised three–phase balanced power signal, $\boldsymbol{s}_k$, reduces to

$$\mathbf{R}_{\boldsymbol{s}} = \frac{1}{2} \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}. \tag{12}$$

After performing the eigen-decomposition on the above covariance matrix, i.e. $\mathbf{R}_{\boldsymbol{s}} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{\mathsf{T}}$, this yields

$$\mathbf{Q}^{\mathsf{T}} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}, \; \boldsymbol{\Lambda} = \frac{1}{4} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}. \tag{13}$$

Remark 1: *The matrix of eigenvectors, $\mathbf{Q}^{\mathsf{T}}$, is identical to the Clarke transformation matrix in* (4). *Therefore, all the variance in balanced three-phase power system voltages can be explained through the two eigenvectors associated with the non-zero eigenvalues (principal directions) of the Clarke–transform–matrix. This offers a modern Data Analytics interpretation of the Clarke transform as a Principal Component Analyser which projects a three–phase voltage signal in $\mathbb{R}^3$ onto a 2D subspace spanned by the two largest eigenvectors of the data covariance matrix.*

4. OPPORTUNITIES

The relationship between the Clarke transform and PCA in Remark 1 only exists in balanced conditions. This is because, as the nature of the data changes with the type and prominence of imbalances, PCA adapts to the data, whereas the Clarke transform remains 'static' and 'data agnostic'. Next, we show that PCA is the correct subspace transform even under unbalanced system conditions, which opens up new opportunities for the analysis of three-phase power systems. This will inevitably have an impact on the way we teach data analytics and signal processing to students in the power community [7, 8].

4.1. Location of subspace

Consider the eigen-decomposition of a symmetrically conjugate unbalanced ($\delta_b = \delta_c^*$) covariance matrix, $\mathbf{R}_{\boldsymbol{s}}^u = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{\mathsf{T}}$, to yield

$$\mathbf{Q}^{\mathsf{T}} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{4p} & \frac{-\sqrt{2}}{4p} \\ 1 & p & p \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}, \tag{14}$$

$$\boldsymbol{\Lambda} = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2(|\delta_b|^2 - p^2) & 0 \\ 0 & 0 & 1 + 2p^2 \end{bmatrix}. \tag{15}$$

where $p = |\delta_b|\cos(\angle\delta_b)$. This is realistic as typical voltage sags exhibit symmetrically conjugate imbalances [9]. Notice that for the balanced case, characterised by $p = -\frac{1}{2}$, the above expressions reduce to the equation in (13). Furthermore, observe that the data always lies on a 2D subspace, owing to a zero eigenvalue in (15). Equation (14) allows us to understand how the location of the 2D subspace where the data resides changes as a function of the imbalance. For example, Fig 1 (A) and (B) show the effect of off-nominal

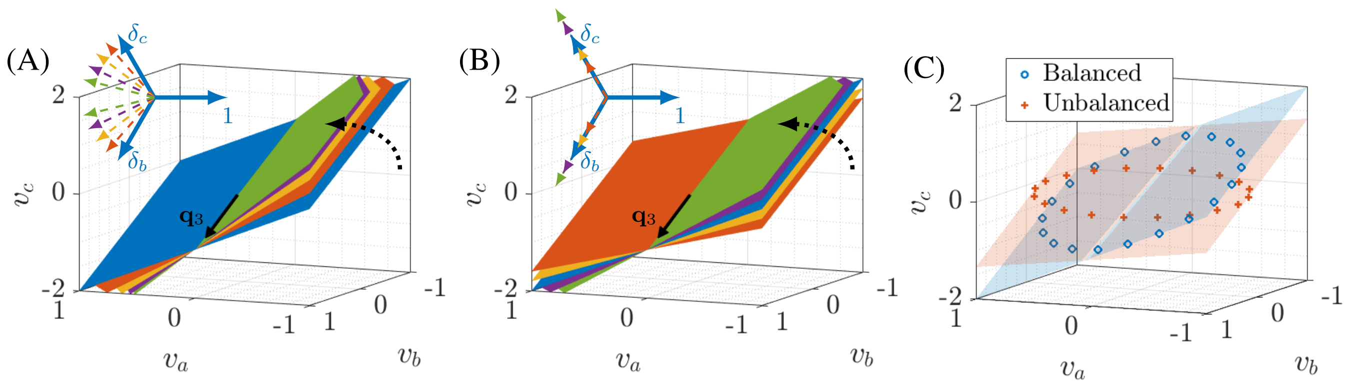

Fig. 1. Visualisation of characteristics of three-phase voltage signals, $\boldsymbol{s}_k$, in a 3D space. (A) The effect of conjugate off-nominal phase angles on the data subspace where $\boldsymbol{s}_k$ resides in. (B) The effect of conjugate voltage sags on the data subspace where $\boldsymbol{s}_k$ resides in. (C) The effect of general system imbalances on the trajectory of $\boldsymbol{s}_k$ in 3D space.

phase angles and voltage sags for a symmetric conjugate type of imbalance.

4.2. Trajectory of data

We shall now examine the problem from a Data Analytics perspective. Fig 1 (C) shows that with an imbalance, the trajectory of the data (blue dots) changes from circular to elliptical (red dots). This can be confirmed by noticing that for unbalanced data, the non-zero eigenvalues of the covariance matrix are no longer equal. In other words, the variation along the two principal axes are different.

The above phenomenon can be mathematically described using complex algebra. Upon combining the complex Clarke transform in (7) with the phasor representation of the data in (2), we arrive at a physically meaningful representation through the counter–clockwise rotating *positive–sequence voltage*, $\bar{V}_+$, and the clockwise rotating *negative–sequence voltage*, $\bar{V}_-$, both rotating at the system frequency, ω, to assume the form

$$\bar{s}_k = \frac{1}{\sqrt{2}}\left(\bar{V}_+ e^{j\omega k} + \bar{V}_-^* e^{-j\omega k}\right), \tag{16}$$

where

$$\bar{V}_+ = \frac{1}{\sqrt{3}}\left[V_a e^{j\phi_a} + V_b e^{j\phi_b} + V_c e^{j\phi_c}\right] \tag{17}$$
$$\bar{V}_-^* = \frac{1}{\sqrt{3}}\left[V_a e^{-j\phi_a} + V_b e^{-j\left(\phi_b + \frac{2\pi}{3}\right)} + V_c e^{-j\left(\phi_c - \frac{2\pi}{3}\right)}\right].$$

Remark 2: *For balanced three–phase power systems, with $V_a = V_b = V_c$ and $\phi_a = \phi_b = \phi_c$, the negative sequence voltage component within the Clarke voltage, $\bar{V}_-^*$ in* (16)*, vanishes and the Clarke voltage attains a single degree of freedom. For unbalanced systems, characterised by unequal phase voltage amplitudes and/or phases, $\bar{V}_-^* \neq 0$, and thus the Clarke voltage in* (16) *exhibits two degrees of freedom - a signature of system imbalance.*

5. SMART DSP FOR A SMARTER POWER GRID

We have so far established that two key characteristics of the data in off-nominal conditions are: (i) the location of the 2D subspace where the data resides changes (ii) the trajectory of the data changes from circular to elliptical. This motivates the use of an adaptive algorithm with sufficient degrees of freedom and naturally points towards using adaptive complex widely linear modelling.

5.1. Widely Linear Modelling

Recall that the widely linear estimator for complex-valued data is given by

$$\hat{y}_k = E\{y_k|\mathbf{x}_k, \mathbf{x}_k^*\} = \mathbf{h}^{\mathsf{H}}\mathbf{x}_k + \mathbf{g}^{\mathsf{H}}\mathbf{x}_k^*. \tag{18}$$

A comparison with the expression for the unbalanced Clarke voltage in (16), reveals that it is governed by a widely linear autoregressive (WLAR) model, in the form [10]

$$\bar{s}_k = \frac{1}{\sqrt{2}}\left(e^{j\omega}\bar{V}_+ e^{j\omega(k-1)} + e^{-j\omega}\bar{V}_-^* e^{-j\omega(k-1)}\right) \tag{19}$$
$$= h^*\bar{s}_{k-1} + g^*\bar{s}_{k-1}^*, \tag{20}$$

from which the system frequency, ω, can be estimated as

$$\hat{\omega} = \tan^{-1}\left[\frac{\sqrt{\mathsf{Im}\{h\}^2 - |g|^2}}{\mathsf{Re}\{h\}}\right]. \tag{21}$$

For a general case of both unbalanced system voltages and time-varying frequencies, the coefficients h and g will also assume a time-varying form, h_k and g_k, and can be estimated using the augmented complex least square (ACLMS) algorithm [11, 10], to yield an instantaneous frequency estimate

$$\hat{\omega}_k = \tan^{-1}\left[\frac{\sqrt{\mathsf{Im}\{h_k\}^2 - |g_k|^2}}{\mathsf{Re}\{h_k\}}\right]. \tag{22}$$

5.2. Smart Clarke and Park transforms

Having demonstrated that accurate estimation in unbalanced systems requires an additional degree of freedom, achieved through widely linear modelling, it is natural to ask whether we can use Signal Processing to "equalise" the noncircular Clarke trajectories associated with unbalanced systems, so as to make them amenable to standard (strictly linear) single–degree–of–freedom system analysis tools. To this end, consider the voltage unbalance factor (VUF) in a power system, defined as $\kappa \stackrel{\text{def}}{=} \bar{V}_-/\bar{V}_+$ [12]. The system frequency and VUF can now be expressed through the WLAR coefficients, h and g, as

$$e^{j\omega} = h^* + g^*\kappa \quad \text{and} \quad e^{-j\omega} = h^* + \frac{g^*}{\kappa^*}. \tag{23}$$

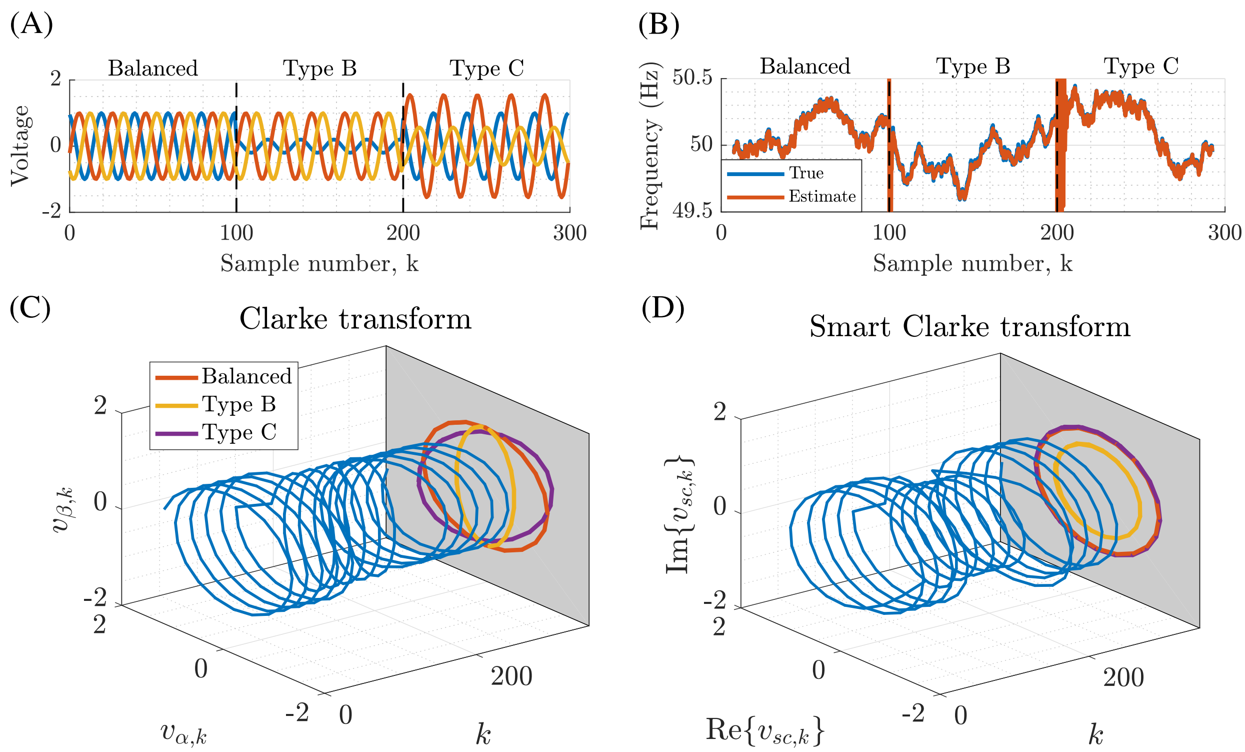

Fig. 2. Illustrative examples of the Smart Clarke transform (SCT) and Smart Park transform (SPT). (A) An example of a three-phase time waveform which evolves from a balanced state into type B and C faults. (B) Frequency estimation using the Smart Park transform. Observe the self-stabilising effect when faults occur. (C) The elliptical trajectory of the Clarke voltage in unbalanced conditions. (D) The self-balancing nature of the Smart Clarke transform.

Upon solving for the system frequency, ω, and VUF, κ, we have [10]

$$e^{j\omega} = \mathsf{Re}\{h\} + j\sqrt{\mathsf{Im}^2\{h\} - |g|^2}, \tag{24}$$

$$\kappa = \frac{\bar{V}_-}{\bar{V}_+} = \frac{j}{g^*}\left(\mathsf{Im}\{h\} + \sqrt{\mathsf{Im}^2\{h\} - |g|^2}\right). \tag{25}$$

The knowledge of the VUF, κ in (25), allows us to eliminate the negative sequence phasor, $\bar{V}_-$, from the Clarke voltage, $\bar{s}_k$ in (16). To this end, consider the expression

$$m_k \stackrel{\text{def}}{=} \sqrt{2}\,(\bar{s}_k - \kappa^* \bar{s}_k^*) = \bar{V}_+ \left(1 - |\kappa|^2\right) e^{j\omega k}, \tag{26}$$

whereby the value of κ is readily available from the WLAR coefficients in (25). This makes it possible to eliminate the "noncircular" effects of voltage imbalance on the Clarke $\alpha\beta$ voltage, based on

$$\bar{m}_k = m_k/(1 - |\kappa|^2) = \bar{V}_+ e^{j\omega k}. \tag{27}$$

Finally, from (24) and (25), the self–balancing adaptive Smart Clarke transform (SCT) and Smart Park transform (SPT) [13] can be summarised as

$$\text{SCT}: \quad v_{sc,k} = \sqrt{2}(\bar{s}_k - \kappa_k^* \bar{s}_k^*)/(1 - |\kappa_k|^2) \tag{28a}$$

$$\text{SPT}: \quad v_{sp,k} = e^{-j\omega_k k} v_{sc,k}. \tag{28b}$$

For real–time adaptive mode of operation, the SCT and SPT can be implemented using adaptive learning algorithms (e.g. ACLMS or Kalman filter) to track the VUF, κ_k, and system frequency, ω_k.

6. SIMULATIONS

In order to illustrate the power behind the derived Smart transforms, consider a three-phase time waveform that evolves from a balanced state into a type B fault, followed by a type C fault, as shown in Fig 2 (A). Real-time frequency estimation can be achieved using the SPT based on the augmented complex least mean square (ACLMS) as illustrated in Fig 2 (B) [11]. Observe the self-stabilising effect of the SPT at the time where faults occur. Fig 2 (C) shows the elliptical trajectory of the standard Clarke voltage for the type B and type C faults (a signature of imbalance), as opposed to the circular trajectory for a balanced Clarke voltage. The self-balancing nature of the SCT is self-evident, as it equalises the elliptical trajectories of the type B and type C faults into the "balanced" circular ones, Fig 2 (D).

7. CONCLUSION

The future Smart Grid will be permanently dynamically unbalanced and, therefore, its successful operation requires close cooperation between the Power Systems and Data Analytics communities, especially those working in Signal Processing and Machine Learning. However, a lack of common language has been identified as a major prohibitive factor in this endeavour. Also, the fact that the fundamental power system analysis tools, e.g. Clarke and Park transforms, have been designed from a Circuit Theory perspective makes it awkward for linking up with Data Analytics communities. To help bridge this gap, we have provided a modern interpretation of the three-phase voltage transforms through principal component analysis, and in this way have established a cross-community link. In addition, the flexibility and rigour of our DSP interpretation has made it possible to simultaneously develop advanced solutions for unbalanced power systems, such as the Smart transforms for frequency estimation, which are not accessible using the power community approaches. For the electronic supplement consisting of teaching material and code, please see `http://www.commsp.ee.ic.ac.uk/~mandic/DSP_ML_for_Power.htm`.

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