ON MASSIVE MIMO CELLULAR SYSTEMS RESILIENCE TO RADAR INTERFERENCE

Stefano Buzzi, Carmen D'Andrea, and Marco Lops

University of Cassino and Southern Latium, Dept. of Electrical and Inform. Eng., Cassino – Italy

ABSTRACT

In this paper we consider a massive multiple-input multiple-output (MIMO) communication system using 5G New Radio-compliant multiple access, which is to co-exist with a radar system using the same frequency band. Building upon a recently proposed system model taking into account the reverberation (clutter) produced by the radar system at the massive MIMO receiver, we provide a theoretical analysis, in terms of a lower bound on the achievable uplink (UL) spectral efficiency (SE) and in terms of the mutual information of the cellular massive MIMO system, showing that for large number of antennas at the base station the radar clutter effects can be suppressed. Simulation results confirm the paper theoretical findings.

***Index Terms*—** Massive MIMO, radar signal processing, co-existence, 5G wireless networks.

1. INTRODUCTION

Radar-Communications co-existence is a large cloak under which a variety of architectures and strategies can be found, mainly aimed at allowing high-rate wireless services to share spectrum with sensing systems [1]. In particular, since the evolution of wireless communications has produced a progressive scaling up of the carrier frequencies, not only the $2-8$ GHz range, but also the 24 GHz and 60 GHz bandwidths, devoted to very high resolution mapping, scientific remote sensing and airport (short-range) surveillance, will be supposedly overcrowded: not surprisingly, the Defense Advanced Research Projects Agency (DARPA) has recently announced the Shared SPectrum Access for Radar and Communications (SSPARC) program [2], which has aroused an intense scientific interest in the subject. A consensus has now been reached on the fact that one of the most damaging effects of such co-existence is the clutter produced by a search radar onto the base-station of the wireless network, which ultimately may result in a dramatic reduction of the UL rates. The co-existence between a radar system and a cellular network was studied in [3], where the design of radar precoders mitigating the interference to the cellular system was proposed, and in [4], where a joint optimization of the performance of a downlink multiuser cellular system and of a radar system is presented. This paper aims at showing that a 5G wireless network, employing a standard Orthogonal Frequency Division Multiplexing (OFDM) modulation format and endowed with a *massive* MIMO array at the base station may successfully co-exist with a wide-beam search radar, taking huge advantage of the massive nature of the receive array. Building upon our recent paper [5], wherein a system model for the considered scenario and some practical clutter-resistant data detection strategies have been presented, this paper provides the following contribution: (a) the data detection strategies are extended and assessed for the case of incomplete channel state information; (b) a lower-bound for the achievable UL SE for the massive MIMO system is derived using the use-and-then-forget bounding technique; (c) an information-theoretic analysis of the system confirming the massive MIMO system robustness to clutter disturbance is also briefly outlined; and (d) numerical results are shown corroborating the validity of the paper theoretical findings.

2. SYSTEM MODEL

Consider a single-cell massive MIMO communication system using SC-FDMA multiple access in the UL, operating at a carrier frequency $f_c = 3$ GHz, and coexisting with a radar system using the same frequency band as in [5]. We denote by $N = 4096$ the number of subcarriers of the SC-FDMA system, with M the number of elements of the uniform linear array (ULA) at the BS, with K the number of single antenna mobile stations (MSs). A block fading channel is assumed with channel coherence bandwidth equal to $C\Delta f$, with $C = 16$ and $\Delta f = 30$ kHz the subcarrier spacing. The UL channel between the k-th single-antenna MS and the BS on the n-th carrier is represented by the M-dimensional vector $\mathbf{h}_k^{(\lceil n/C \rceil)} = \beta_k \mathbf{g}_k^{(\lceil n/C \rceil)}$, where β_k represents the path-loss and the log-normal shadowing, while $\mathbf{g}_k \sim \mathcal{CN}(0, \mathbf{I}_M)$ denotes the small-scale fading. Each packet is made of a cyclic-prefix (CP) and of a sequence of data symbols; the CP discrete length is $N_{\rm CP} = 288$, while the length of the data symbols is N. The timing is such that $N_{\rm pkt} = 14$ packets fit into a 0.5 ms timeslot, which leads to a symbol time $T_s = 8.146$ ns[1].

The radar system operates at the same carrier frequency as the wireless cellular system, and transmits a coded waveform,

[1]These numbers are inspired by the December 2017 3GPP first realease of the 5G New Radio standard.

of duration LT_s, whose baseband equivalent is expressed as $s_R(t) = \sqrt{P_T}\sum_{\ell=0}^{L-1} c_\ell \psi(t-\ell T_s)$, wherein P_T is the radar transmitted power, $[c_0, c_1, \dots, c_{L-1}]$ is the unit-energy radar code, and $\psi(\cdot)$ is the base pulse. The value $L = 32$ is assumed in this paper. The waveform $s_R(t)$ is transmitted periodically every $T_{\rm PRT} = 1$ ms, with $T_{\rm PRT}$ the Pulse Repetition Time (PRT).

Signal model during UL data transmission Consider the generic ℓ-th data packet; denote by $\mathbf{x}_k(\ell)$ an N-dimensional vector containing the data symbols from the k-th MS to be transmitted in the ℓ-th data packet; denote by $\mathbf{X}_k(\ell)$ the N-dimensional vector representing the isometric FFT of $\mathbf{x}_k(\ell)$. The observable corresponding to the n-th subcarrier after the FFT operation is the M-dimensional vector

$$\mathbf{y}(\ell)^{(n)} = \sum_{k=1}^{K} \sqrt{p_k}\mathbf{X}_k(\ell)^{(n)}\mathbf{h}_k^{(\lceil n/C \rceil)} + \mathbf{W}(\ell)^{(n)} + \mathbf{C}(\ell)^{(n)} \ , \tag{1}$$

for $n = 1, \dots, N$. In the above equation, p_k is the power transmitted by the k-th MS, $\mathbf{X}_k(\ell)^{(n)}$ is the n-th entry of the vector $\mathbf{X}_k(\ell)$, $\mathbf{W}(\ell)^{(n)}$ is a $\mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}_M)$ random vector representing the additive thermal noise, while $\mathbf{C}(\ell)^{(n)}$ is the clutter disturbance. Grouping together the observable corresponding to the N subcarriers we finally get the following $(M \times N)$-dimensional matrix for the ℓ-th data packet data:

$$\mathbf{Y}(\ell) = \sum_{k=1}^{K} \sqrt{p_k}\left(\left[\mathbf{h}_k^{(1)} \dots \mathbf{h}_k^{(Q)}\right] \otimes \mathbf{1}_{1\times C}\right) \mathrm{diag}(\mathbf{X}_k(\ell)) + \mathbf{W}(\ell) + \mathbf{C}(\ell) \ , \tag{2}$$

where $Q = N/C$, $\otimes$ denotes Kronecker product and $\mathbf{1}_{1\times C}$ denotes a C-dimensional row vector with unit entries.

Signal model during UL training Consider now the case in which the MSs transmit known pilot sequences to enable channel estimation (CE) at the BS. Let T denote the number of consecutive packets devoted to training, and let $\mathbf{p}_k(1), \dots, \mathbf{p}_k(T)$ denote N-dimensional vectors containing the k-th MS pilots to be used in the T packets used for CE. Focusing on the ℓ-th packet (with now $\ell = 1, \dots, T$), and following the same steps as in Eq. (2), we can obtain a compact expression for the $(M \times N)$-dimensional matrix $\mathbf{Y}(\ell)$ at the output of the FFT block at the BS receiver. Assuming that the M-dimensional channel vectors $\mathbf{h}_k^{(q)}, \forall k = 0, \dots, K-1$, are to be estimated, the columns from the $[(q-1)C+1]$-th to the $[qC]$-th of the matrices $\mathbf{Y}(1), \dots, \mathbf{Y}(T)$ are to be picked; they form the following observable:

$$\mathcal{Y}_q = \sum_{k=1}^{K} \sqrt{p_{\rm p,k}}\mathbf{h}_k^{(q)}\mathbf{P}_k^{(q)\,T} + \mathcal{W}_q + \mathcal{C}_q \ , \tag{3}$$

where $p_{\rm p,k}$ is the power transmitted by the $k-$th MS during the UL training phase, $\mathcal{W}_q$ and $\mathcal{C}_q$ are suitably defined matrices, and $\mathbf{P_k}^{(q)}$ is a (TC)-dimensional vector containing FFT samples of the k-th MS pilots (see [5] for further details).

Clutter modeling The clutter disturbance is generated by a large set of discrete scatterers in the surrounding environment. Given the BS array dimension, it is reasonable to assume that these scatterers are seen by the BS as "colocated" and, thus, the radar-to-BS channel can be modeled as a LTI channel with the following vector-valued impulse response:

$$\mathbf{h}(t) = \sum_{q=0}^{N_s-1}\sum_{m=0}^{Q-1} \beta_{q,m}\mathbf{b}(\theta_q)\delta(t-\tau_q - m/W) \ . \tag{4}$$

In the above equation, N_s denotes the number of scatterers in the surrounding environment; θ_q and τ_q are the direction of arrival and the propagation delay of the clutter contribution from the q-th scatterer, and $\mathbf{b}(\cdot)$ is the BS ULA array response[2]. Moreover, since the signal bandwidth W exceeds the channel coherence time, we also assume that each physical scatterer generates Q clutter echoes spaced apart by integer multiples of $1/W$; accordingly, $\beta_{q,m}$ is the reflection coefficient associated to the m-th replica from the q-th scatterer. Accordingly, after A/D conversion, the baseband equivalent of the clutter disturbance at the BS is the following vector-valued discrete-time (at rate $1/T_s$) signal:

$$\widetilde{\mathbf{s}}_R(\eta) = \sum_{q=0}^{N_s-1}\sum_{m=0}^{Q-1}\sum_{p=0}^{L-1} \sqrt{P_T}\beta_{q,m}c_p\mathbf{b}(\theta_q) \, r_\psi((\eta-p)T_s - m/W - \tau_q) \ , \tag{5}$$

with $r_\psi(\cdot)$ the autocorrelation function of the base pulse. Let now $\mathcal{S}(\ell)$ denote the set of the scatterers corrupting the reception of the ℓ-th data packet; the clutter $(M \times N)$-dimensional matrix appearing in (2) can be shown to be expressed as

$$\mathbf{C}(\ell) = \sum_{q \in \mathcal{S}(\ell)}\sum_{m=0}^{Q-1}\sum_{p=0}^{L-1} \sqrt{P_T}\beta_{q,m}c_p\mathbf{b}(\theta_q)\mathbf{r}_{q,p,m}^T(\ell)\mathbf{W}_{N,FFT} \ , \tag{6}$$

wherein $\mathbf{W}_{N,FFT}$ is the isometric FFT matrix,

$$\mathbf{r}_{q,p,m}(\ell) = \left[r_\psi\left(\ell T_{\rm pkt} + T_{\rm CP} + T_s - pT_s - \tfrac{m}{W} - \tau_q\right), \dots, r_\psi\left((\ell+1)T_{\rm pkt} - pT_s - \tfrac{m}{W} - \tau_q\right)\right]^T .$$

and we define $\widetilde{\mathbf{R}}_{q,\ell,m}^T = \sum_{p=0}^{L-1} c_p \mathbf{r}_{q,p,m}^T(\ell)\mathbf{W}_{N,FFT}$.

3. RECEIVER PROCESSING

We now briefly review the signal processing algorithms at the BS to estimate the UL channels and decode the MSs data symbols. In the following, we assume knowledge of the delays τ_q and directions of arrival θ_q of the clutter echoes. Regarding channel estimation, given the data model (3), we detail two different CE strategies, exploiting knowkedge of the normalized pilot sequences $\widetilde{\mathbf{P}}_k^{(q)} \triangleq \frac{\mathbf{P}_k^{(q)}}{\left\|\mathbf{P}_k^{(q)}\right\|^2} \, \forall \, k, q$.

[2]Model (4) permits considering also a possible direct path from the radar to the BS antenna array.

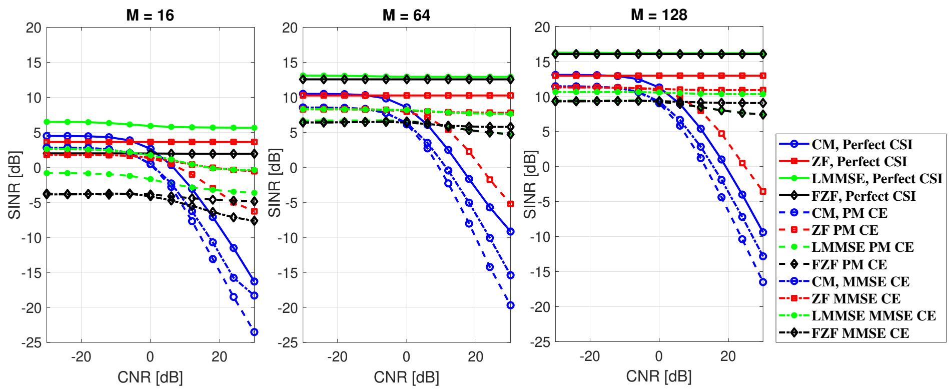

Fig. 1. SINR versus CNR of four detection strategies in the cases of perfect CSI, PM CE and MMSE CE, with $K = 10$ and different values of M.

Pilot-matched CE (PM CE) A simple estimator for the channel vector $\mathbf{h}_k^{(q)}$, $\forall k, q$, is obtained through the PM processing $\widehat{\mathbf{h}}_k^{(q)} = p_{\mathrm{p,k}}{}^{-1/2}\mathcal{Y}_q\widetilde{\mathbf{P}}_k^{(q)\,*}$.

Minimum-mean-square-error CE (MMSE CE) A better performing estimator can be obtained by resorting to the linear MMSE criterion. Given the observable in Eq. (3), the BS forms the following $M-$ dimensional vector $\mathbf{r}_{q,k} = \mathcal{Y}_q\widetilde{\mathbf{P}}_k^{(q)\,*}$. The linear MMSE estimate [6] of the $M-$dimensional channel vector $\mathbf{h}_k^{(q)}$ can be computed as $\widehat{\mathbf{h}}_k^{(q)} = \mathbf{D}_{q,k}\mathbf{r}_{q,k}$, with $\mathbf{D}_{q,k} = \mathbb{E}\left[\mathbf{h}_k^{(q)}\mathbf{r}_{q,k}^H\right]\left(\mathbb{E}\left[\mathbf{r}_{q,k}\mathbf{r}_{q,k}^H\right]\right)^{-1}$.

UL data detection Regarding data detection, in this paper performance results for the following linear detectors are presented: channel-matched beamforming (CM), zero-forced clutter (ZF), linear minimum mean square (LMMSE), and full zero-forcing (FZF) (see [5] for further details).

4. THEORETICAL ANALYSIS

UL SE lower-bound computation We now provide closed-form formulas for a lower bound to the UL SE under the hypothesis of CM detection. To do so, the key idea is to utilize the channel estimates only for computing the receive combining vectors, while not exploiting this side-information for signal detection[3], whereby the bounding technique is known as the use-and-then-forget (UatF) bound [7, 8]. Focusing on the k-th MS and $n-$th subcarrier, the achievable UL SE can be shown to be lower bounded by

$$\mathrm{SE}_k^{(n)} \geq \frac{N_{\mathrm{pkt}} - T}{N_{\mathrm{pkt}}}\log_2\left(1 + \mathrm{SINR}_k^{(n)}\right)\ [\text{bit/s/Hz}]\,, \quad (7)$$

where $\mathrm{SINR}_k^{(n)}$ can be shown to be expressed as in Eqs. (8) and (9) at the top of next page, for PM and MMSE CE, respectively, with $q = \lceil n/C \rceil$.

[3] This simplification is reasonable when there is substantial channel hardening.

Information Theoretic Analysis Consider now a single packet transmission in a single-user scenario, and neglect clutter cross-packet effects. In order to simplify the notation, we omit the MS and the packet indexes k and ℓ. Let us thus consider C consecutive sub-carriers, extending from $(n-1)C+1$ to nC, which experience the same channel fading; Eq. (2) simplifies to:

$$\begin{aligned}\boldsymbol{y}_{(n-1)C+1:nC} &= \sqrt{p}\boldsymbol{X}^{(n-1)} \otimes \boldsymbol{h}^{(n-1)} \\ &+ \widetilde{\boldsymbol{w}}_{(n-1)C+1:nC} + \widetilde{\boldsymbol{c}}_{(n-1)C+1:nC} \in \mathbb{C}^{CM}\,,\end{aligned} \quad (10)$$

where $\boldsymbol{X}^{(n-1)} = \left[X^{(n-1)C}, \ldots, X^{nC-1}\right]^T$, $\boldsymbol{y} = \mathrm{vec}\left(\boldsymbol{Y}\right)$, $\widetilde{\boldsymbol{w}} = \mathrm{vec}\left(\boldsymbol{W}\right)$, $\widetilde{\boldsymbol{c}} = \mathrm{vec}\left(\boldsymbol{C}\right)$, and $\boldsymbol{h}^{(n-1)} \in \mathbb{C}^M$. We can thus form the $QCM = NM-$dimensional vector

$$\boldsymbol{y} = \left[\boldsymbol{y}_{1:C}^T, \boldsymbol{y}_{C+1:2C}^T, \ldots, \boldsymbol{y}_{(Q-1)C+1:QC}^T\right]^T \quad (11)$$

We assume to have $\widetilde{N}_s$ scatterers in the packet under test, each contributing Q replicas of the radar signal. Under these circumstances, the $(MN \times MN)-$dimensional clutter covariance matrix from $\widetilde{N}_s$ scatterers can be written as

$$\boldsymbol{K}_{\mathrm{c}} = \mathbb{E}\left[\widetilde{\boldsymbol{c}}\widetilde{\boldsymbol{c}}^H\right] = \sum_{q=1}^{\widetilde{N}_s}\sum_{m=0}^{Q-1} P_T\sigma_{q,m}^2\widetilde{\mathbf{A}}_{q,m} \otimes \boldsymbol{b}(\theta_q)\boldsymbol{b}^H(\theta_q)\,, \quad (12)$$

where $\widetilde{\mathbf{A}}_{q,m} = \widetilde{\mathbf{R}}_{q,m}\widetilde{\mathbf{R}}_{q,m}^H$. For the transmission phase, the single-user mutual information can be expressed as [9]:

$$\begin{aligned}&I\left(\boldsymbol{y}; \boldsymbol{X}^{(0)}, \boldsymbol{X}^{(1)}, \ldots, \boldsymbol{X}^{(Q-1)} \middle| \boldsymbol{h}^{(0)}, \ldots, \boldsymbol{h}^{(Q-1)}\right) \\ &= \log\det\left[\boldsymbol{I}_{NM} + \boldsymbol{K}'\left(N_0\boldsymbol{I}_{NM} + \boldsymbol{K}_{\mathrm{c}}\right)^{-1}\right]\,,\end{aligned}$$

where

$$\boldsymbol{K}' = p\,\mathrm{blkdiag}\Big(\boldsymbol{I}_C \otimes \boldsymbol{h}^{(0)}\boldsymbol{h}^{(0),H}, \ldots, \boldsymbol{I}_C \otimes \boldsymbol{h}^{(Q-1)}\boldsymbol{h}^{(Q-1),H}\Big) \quad (13)$$

$$\text{SINR}_k^{(q)} = \frac{p_k p_{\text{p,k}} \beta_k^4 M^2}{\sum\limits_{\substack{j=1 \\ j\neq k}}^{K} p_j p_{\text{p,j}} \beta_j^4 M^2 \left| \mathbf{P}_j^{(q)\,T} \widetilde{\mathbf{P}}_k^{(q)\,*} \right|^2 + \sum\limits_{j=1}^{K} p_j \beta_j^2 \text{tr}\left(\mathbf{R}_{q,k}\right) + \sigma_w^2 \text{tr}\left(\mathbf{R}_{q,k}\right) + \text{tr}\left(\mathbf{R}_{q,k} \mathbf{K}_{\mathbf{C}^{(n)}}\right)}, \tag{8}$$

$$\text{SINR}_k^{(q)} = \frac{p_k p_{\text{p,k}} \beta_k^4 \text{tr}\left(\mathbf{D}_{q,k}\right)^2}{\sum\limits_{\substack{j=1 \\ j\neq k}}^{K} p_j p_{\text{p,j}} \beta_j^4 \text{tr}\left(\mathbf{D}_{q,k}\right)^2 \left| \mathbf{P}_j^{(q)\,T} \widetilde{\mathbf{P}}_k^{(q)\,*} \right|^2 + \sqrt{p_{\text{p,k}}} \left[\sum\limits_{j=1}^{K} p_j \beta_j^2 \beta_k^2 \text{tr}\left(\mathbf{D}_{q,k}\right) + \beta_k^2 \left[\sigma_w^2 \text{tr}\left(\mathbf{D}_{q,k}\right) + \text{tr}\left(\mathbf{D}_{q,k} \mathbf{K}_{\mathbf{C}^{(n)}}\right) \right] \right]}. \tag{9}$$

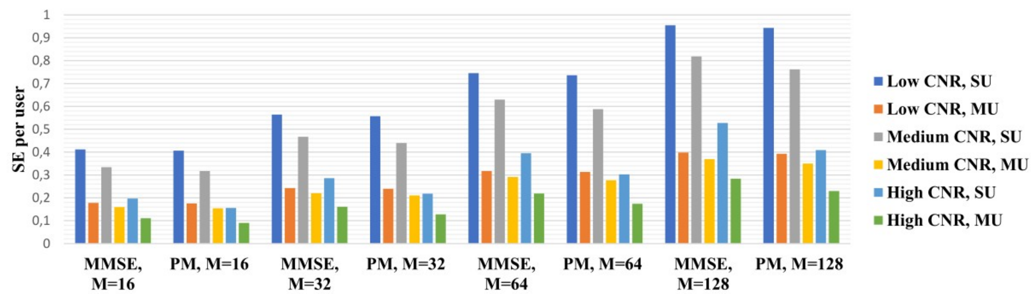

Fig. 2. SE per user lower bounds, in SU and MU scenarios, for increasing values of M, for CNR $= -20$ dB (Low CNR), for CNR $= 0$ dB (Medium CNR), for CNR $= 10$ dB (High CNR), and with PM and MMSE CE techniques.

Notice that the massive MIMO structure allows defining the M-dimensional unitary matrix

$$\boldsymbol{U} = \left[\boldsymbol{h}^{(0)}/\|\boldsymbol{h}^{(0)}\|, \ldots, \boldsymbol{h}^{(Q-1)}/\|\boldsymbol{h}^{(Q-1)}\|, \underbrace{\boldsymbol{u}^{(Q)}, \ldots, \boldsymbol{u}^{(M-1)}}_{\text{arbitrary}} \right]$$

so that, letting $\boldsymbol{\Lambda}_i = \text{diag}\left(\mathbf{0}_{i-1}, \|\boldsymbol{h}^{(i)}\|^2, \mathbf{0}_{M-i}\right)$ with $\mathbf{0}_p$ a p-dimensional row vector with zero entries, $\boldsymbol{h}^{(i)}\boldsymbol{h}^{(i),H} = \boldsymbol{U}\boldsymbol{\Lambda}_i\boldsymbol{U}^H$. Notice also that, given the properties of the Kronecker product, $\boldsymbol{K}'$ has rank $CQ = N$, while $\boldsymbol{K}_\text{c}$ has maximum rank $\widetilde{N}_s Q$. As a consequence, a clutter-free direction exists only if $\widetilde{N}_s < C$. Assume now that the massive MIMO has been optimized with no concern on the presence of clutter, and let us consider the term $\boldsymbol{K}'^{\dagger}\boldsymbol{K}_\text{c}$. Using Eqs. (12) and (13), and after some algebraic manipulations, the $(M \times M)-$dimensional (i, j) block of the product reads

$$\left[\boldsymbol{K}'^{\dagger}\boldsymbol{K}_\text{c}\right](i+1, j) = \boldsymbol{U}\boldsymbol{\Lambda}^{\dagger}_{\lfloor \frac{i}{C} \rfloor}\boldsymbol{U}^H\boldsymbol{K}_\text{c}(i+1, j). \tag{14}$$

Let us examine the terms $\boldsymbol{U}\boldsymbol{\Lambda}_0^{\dagger}\boldsymbol{U}^H\boldsymbol{K}_\text{c}(i, j)$, $i = 1, \ldots, C$, $j = 1, \ldots N$. Since $\boldsymbol{\Lambda}_0^{\dagger} = \text{diag}\left[\left\|\boldsymbol{h}^{(0)}\right\|^{-2}, 0, \ldots, 0\right]$, the matrices $\boldsymbol{\Lambda}_0^{\dagger}\boldsymbol{U}^H\boldsymbol{K}_\text{c}(i, j)$ can be shown to have only the first row non-zero $\forall i, j$. This row is written as

$$\boldsymbol{z}_{0,(i,j)} = \boldsymbol{h}^{(0),H}/\|\boldsymbol{h}^{(0)}\|^3\boldsymbol{K}_\text{c}(i, j) \, . \tag{15}$$

Using some algebraic manipulations, and applying the strong law of large numbers [10] and the continous mapping theorem in [11, 12], we can conclude that $\boldsymbol{z}_{0,(i,j)} \to \mathbf{0}_{1\times M}$, with $M \to \infty$ and $\forall\, i, j$ almost surely. This result thus proves that, in the large number of antennas regime, the clutter contribution has no effect on the single-user mutual information.

5. NUMERICAL RESULTS AND CONCLUSIONS

We will use the system parameters detailed in Section 2. Moreover, the MSs-BS distance is uniform in the range $[20, 500]$ m, the additive noise has a power spectral density of -174 dBm/Hz, and the front-end receiver noise figure is 3 dB. The MSs transmit power is: i.e. $p_k = p_{\text{p},k} = 100$ mW, $\forall k = 1, \ldots, K$. The presence of the direct path from the radar to the BS is also considered in our results.

Fig. 1 reports the SINR versus the Clutter-to-Noise Ratio (CNR) for the four detection stategies mentioned in Section 3, considering the case of perfect CSI, PM CE and MMSE CE (with $T = 7$), for different numbers of BS antennas M, and assuming $K = 10$ active MSs. The results show that for perfect CSI the only receiver sensitive to clutter is the CM beamformer. With CE, instead, all the receivers are sensitive to the clutter, but it is clearly seen that increasing the number of antennas M the clutter effect is attenuated. Fig. 2 reports the lower bound of the average SE per user for the case of CM detection, with PM and MMSE CE, and for several number of BS antennas M, both in the single user (SU) and in the multiple users (MU) scenarios. Results clearly show that the performance of the system increases with the number M of antennas at the BS. The above results confirm the theoretical findings that increasing the number of antennas at the BS provides increased robustness against the clutter disturbance originating from the radar system.

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