Communications under the Constraint of De-Chirp Channel Distortion

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ABSTRACT

We analyze the effects of radar stretch processing on a system that jointly receives communications waveforms and radar waveforms. This effort is motivated by the growing interest in RF convergence, for which spectrum is cooperatively used for multiple functions simultaneously. In this paper, we first introduce and motivate the use of chirp radar waveforms and review stretch processing for radar applications. We then consider communications signaling under the constraint of a stretch processing radar receiver that de-chirps pulses at the first stage of the processing chain. Since it may not be possible in some systems to perform communications reception prior to the de-chirp processing block, we analyze the effects of de-chirp processing as part of the communications channel.

1. INTRODUCTION

RF convergence, which removes the traditional system spectral isolation requirement for spectrum sharing, enables numerous potential advantages. Communications, sensing, radar, and location estimation tasks can all be performed with the same devices. Furthermore, all of these operations can be done within the same spectrum simultaneously [1, 2, 3]. These capabilities are of interest to traditional military applications. However, the much larger market is likely to be commercial as the prevalence of many modern applications dramatically increase. A wide range of these emerging applications, which are categorized in greater detail in [4], include vehicle positioning and vehicle-to-vehicle communications systems [5, 6], automated flight control and collision avoidance [7, 8], high frequency imaging [9], and health monitoring [10]. As radar-on-a-chip devices become potentially cheaper than low-cost cameras, many other applications will be invented.

Stretch processing has a number of practical advantages for a radar system [11, 12]. Consequently, for many joint radar-communications systems (particularly legacy systems), the communications receiver will need to operate after the process of de-chirping. Theoretically, this operation is invertible; however, in practice, limitations on bandwidth and observation periods disables invertibility. This observation motivates our interest in understanding the effects of the de-chirping channel on communications links.

In this paper, we consider a scenario as described in Figure 1. In this scenario, we consider the joint radar-communications receiver to be a radar transmitter/receiver that can also act as a communications receiver. The joint receiver can simultaneously estimate the radar target parameters from the radar return and decode a received communications signal. We investigate communications performance of the joint receiver under the channel constraint that the system performs de-chirping prior to communications reception. We also provide a brief discussion on designing communications waveforms that are more tolerant to the distortion that de-chirping inflicts.

The paper is organized as follows. In Section 2, we explain stretch processing from a radar system's perspective in detail. In Section 3, we define the effective de-chirped communications channel by considering the effects of de-chirping (or stretch processing) on a narrowband signal. We also provide a brief discussion on how to design communications signals that are robust to de-chirping. Finally, we summarize our results in Section 4.

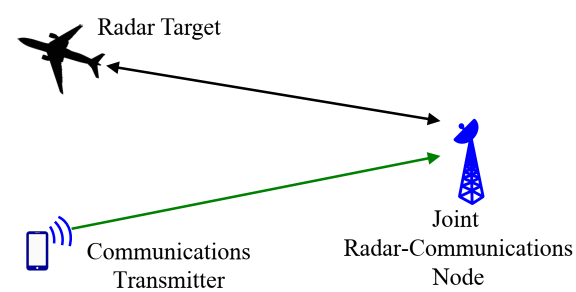

Fig. 1. Notional example of simultaneous joint communications and radar return reception.

2. STRETCH PROCESSING

In this section, we talk about stretch processing and the advantages the technique provides to a radar system. By as-

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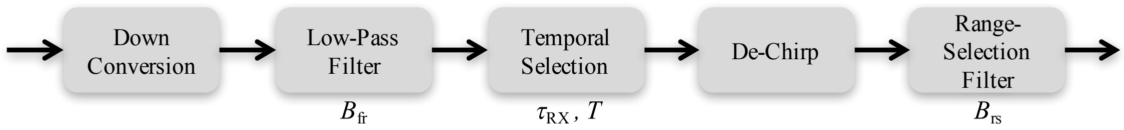

Fig. 2. Block diagram of a stretch processing receiver.

suming that the illumination waveform is a chirp, stretch processing converts delay associated with the time of flight to a tone of finite duration [11]. The delay associated with the radar return arrival is proportional to the ratio of the length of channel to the speed of propagation. By multiplying the received chirp by the complex conjugate of the illumination chirp and integrating over its duration, delay is translated into frequency which is interpreted by employing spectral analysis techniques for ranging estimation. From a radar perspective, one of the advantages of this approach is that the chirp can have a wide bandwidth that provides good distance estimation resolution, but if the range of distances that are of interest is relatively small, then only a narrow bandwidth signal needs to be analyzed reducing hardware complexity.

We depict the block diagram of receiver processing in Figure 2. While the details of the stretch processing may be different for each system, these systems have a processing chain similar to that in the figure. First, down conversion shifts the signal to complex baseband. Extraneous spectral energy is removed by a front-end low-pass filter that is idealized by having a double-sided bandwidth of B_{fr}. A period of duration in time associated with the ranges of interest is extracted that starts at delay τ_{RX} and duration T. The de-chirp transformation is applied, and finally the ranges of interest are selected with a final post-chirp filter with idealized bandwidth B_{pc}.

We begin by defining the characteristics of a chirp. For convenience, we define an unnormalized tophat $\Pi(x)$ as

$$\Pi(x) = \begin{cases} 1 & ; \quad \|x\| \leq 1/2 \\ 0 & ; \quad \text{otherwise}\,. \end{cases} \tag{1}$$

Represented at complex baseband, the chirp waveform $c(t)$ is given by

$$c(t) = e^{i\,\pi\,t^2\,B/T}\,\Pi(t/T)\,. \tag{2}$$

The chirp has a duration of T and an approximate bandwidth of B. The ratio B/T describes the chirp rate. The energy spectral density of the chirp waveform is given by

$$\begin{aligned} C(f) &= \int_{-\infty}^{\infty} dt\, e^{-i\,2\pi\,f\,t}\,c(t) \\ &= \int_{-T/2}^{T/2} dt\, e^{-i\,2\pi\,f\,t}\,e^{i\,\pi\,t^2\,B/T} \qquad (3) \\ &= -\frac{e^{i\pi/4}\,e^{-\frac{i\pi f^2 T}{B}}}{2}\sqrt{\frac{T}{B}} \\ &\quad \cdot \left[\operatorname{erf}\left(\frac{e^{i\,3\pi/4}}{2}\sqrt{\frac{\pi T}{B}}(B-2f)\right)\right. \\ &\qquad \left. + \operatorname{erf}\left(\frac{e^{i\,3\pi/4}}{2}\sqrt{\frac{\pi T}{B}}(B+2f)\right)\right]\,, \qquad (4) \end{aligned}$$

where the Gauss error function $\operatorname{erf}(\cdot)$ is given by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x dy\, e^{-y^2}\,. \tag{5}$$

We depict a representation of the energy spectral density of a chirp at complex baseband in Figure 3.

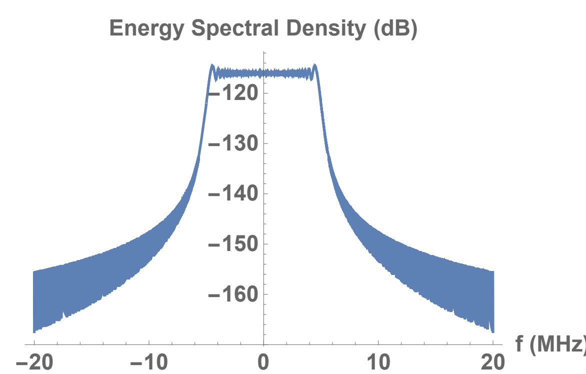

Fig. 3. Example of the energy spectral density of a chirp at complex baseband with an approximate bandwidth of 10 MHz and a duration of 25 μs.

Now, the received radar return signal is given by

$$\begin{aligned} z(t) &= a\,c(t-\tau) + n(t) \qquad (6) \\ &= e^{i\,\pi\,(t-\tau)^2\,B/T}\,\Pi([t-\tau]/T)\,, \qquad (7) \end{aligned}$$

where τ is the target delay, a is the complex gain due to channel propagation, and $n(t)$ is the additive channel noise (commonly modelled as additive white Gaussian noise (AWGN)).

The de-chirp process is done by

$$\tilde{z}(t) = (a\, c(t-\tau) + n(t))\, c^*(t-\tau_{\mathrm{RX}})\,, \tag{8}$$

where

$$\begin{aligned}
&c(t-\tau)\, c^*(t-\tau_{\mathrm{RX}}) \\
&\quad = e^{i\,\pi\,(t-\tau)^2\,B/T}\,\Pi([t-\tau]/T) \\
&\qquad \cdot e^{-i\,\pi\,(t-\tau_{\mathrm{RX}})^2\,B/T}\,\Pi([t-\tau_{\mathrm{RX}}]/T) \\
&\quad = e^{i\,2\pi\,t\,[\tau_{\mathrm{RX}}-\tau]\,B/T} e^{i\,\pi\,[\tau^2-\tau_{\mathrm{RX}}^2]\,B/T} \\
&\qquad\quad \cdot \Pi([t-\tau]/T)\,\Pi([t-\tau_{\mathrm{RX}}]/T) \\
&\quad = e^{i\,2\pi\,t\,[\tau_{\mathrm{RX}}-\tau]\,B/T}\, e^{i\,\pi\,[\tau^2-\tau_{\mathrm{RX}}^2]\,B/T} \\
&\qquad\quad \cdot \Pi\left(\frac{t-\frac{\tau+\tau_{\mathrm{RX}}}{2}}{T-\|\tau_{\mathrm{RX}}-\tau\|}\right) \qquad (9) \\
&\text{if} \quad T > \|\tau_{\mathrm{RX}}-\tau\|\,. \qquad (10)
\end{aligned}$$

Thus, for a duration defined by $\Pi(\cdot)$

$$\tilde{z}(t) \propto e^{i\,2\pi\,t\,[\tau_{\mathrm{RX}}-\tau]\,B/T} + \tilde{n}(t)\,, \tag{11}$$

where $\tilde{n}(t)$ is the de-chirp transformed receiver noise. The result is that the scatterer range which is encoded in the time delay is transformed to a frequency

$$f_\tau = [\tau_{\mathrm{RX}} - \tau]\,B/T\,. \tag{12}$$

In some cases, only a subset of delays (which translates to frequencies) is considered.

3. EFFECT OF DE-CHIRP DISTORTION ON A NARROWBAND SIGNAL

In this section, we analyze the effects of de-chirping on a communications system. We also provide a brief discussion on designing a communications signal that is robust to de-chirping. We notionally represent the effects of the de-chirped channel on a narrowband communications signal in Figure 4. For the sake of a simplified representation in the figure, we employ the admittedly poorly defined concept of instantaneous frequency. The channel approximately selects a region in time and frequency and modulates the narrowband communications signal with the de-chirp transformation. This selection process destroys the invertibility of the de-chirp transform.

If a source is sending a complex tone represented at baseband, the signal is represented by

$$s(t) = e^{i\,2\pi\,f_0\,t}\,, \tag{13}$$

where f_0 is the frequency of the tone at complex baseband. The received signal is then given by

$$\tilde{z}(t) = [a\,s(t) + n(t)]\,c^*(t-\tau_{\mathrm{RX}}) \tag{14}$$

$$s(t)\,c^*(t-\tau_{\mathrm{RX}}) = e^{i\,2\pi\,f_0\,t}\,e^{-i\,\pi\,(t-\tau_{\mathrm{RX}})^2\,B/T} \cdot \Pi([t-\tau_{\mathrm{RX}}]/T)\,, \tag{15}$$

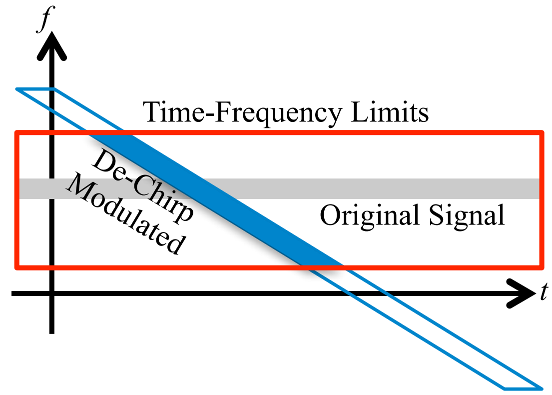

Fig. 4. Notional time-frequency representation of the effects of the de-chirp channel.

which, in the frequency domain, produces a frequency-shifted version of the original chirp spectrum. This frequency-shift may cause much of the spectral energy of the received joint waveform to not be within the spectral window of the receiver processing chain. If the signal remains essentially intact, an

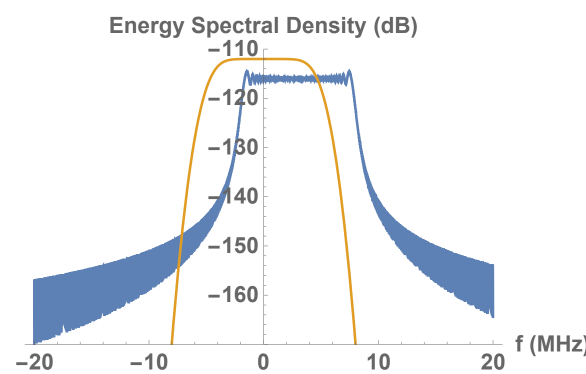

Fig. 5. Example of the energy spectral density of a de-chirp transformed complex tone at complex baseband with an approximate bandwidth of 10 MHz and a duration of 25 μs. A notional lowpass filter with spectral width of 10 MHz.

estimate of the original transmitted tone, $\hat{s}(t)$, can be recovered by applying a chirp transform

$$\begin{aligned}
\tilde{z}(t) &= [a\,s(t) + n(t)]\,c^*(t-\tau_{\mathrm{RX}}) && (16) \\
\hat{s}(t) &= \frac{1}{\hat{a}}\tilde{z}(t)\,c(t-\tau_{\mathrm{RX}}) \\
&= \left[\frac{a}{\hat{a}}s(t) + \frac{n(t)}{\hat{a}}\right]\Pi\left(\frac{t-\tau_{\mathrm{RX}}}{T}\right) && (17) \\
&\approx \left[s(t) + \frac{n(t)}{\hat{a}}\right]\Pi\left(\frac{t-\tau_{\mathrm{RX}}}{T}\right)\,, && (18)
\end{aligned}$$

where $\hat{a}$ is an estimate of a and

$$\begin{aligned} c^*(t-\tau_{\mathrm{RX}})\, c(t-\tau_{\mathrm{RX}}) \\ &= e^{-i\,\pi\,(t-\tau_{\mathrm{RX}})^2\,B/T}\,\Pi\left(\frac{t-\tau_{\mathrm{RX}}}{T}\right) \\ &\quad \cdot e^{i\,\pi\,(t-\tau_{\mathrm{RX}})^2\,B/T}\,\Pi\left(\frac{t-\tau_{\mathrm{RX}}}{T}\right) \\ &= \Pi([t-\tau_{\mathrm{RX}}]/T)\,. \end{aligned} \tag{19}$$

The effect of receiver filtering and de-chirping can be thought of as filtering with some spectral filter, $G(f)$, with corresponding temporal representation $g(t) = c(t-\tau_{\mathrm{RX}})$. Thus, the spectrally filtered de-chirp distorted received signal is given by

$$\breve{z}(t) = [g * \tilde{z}](t)\,. \tag{20}$$

This filter would effectively truncate our finite tone in time.

To analyze the effects of de-chirping on communications performance, we define the error magnitude as

$$err = \frac{\|s(t)-\hat{s}(t)\|_2}{N_{dur}}\,, \tag{21}$$

where N_{dur} is the total number of samples of the tone. Through simulation, we analyze the error magnitude performance vs. signal-to-noise ratio (SNR) for a communications system that performs de-chirping and a communications system that does not. Both communications systems transmitted a complex tone at 3 MHz at complex baseband. A de-chirping transformation with a bandwidth of 10 MHz and duration of 25 μs was employed by one system. The effects of de-chirping are shown in Figure 6. From the figure, it is evident that de-chirping drastically increases the error magnitude of a communications system (by approximately 25dB) and is independent of the SNR. Thus, de-chirp processing significantly degrades communications performance and designing communications waveforms that are robust to de-chirp processing is of significance for joint radar-communications systems. In the following subsections, we briefly discuss how to build such waveforms.

3.1. Synchronized Waveform

If the transmitter is completely synchronized with the radar receiver and has knowledge of the processing chain and timing, then we can construct a waveform that maximizes the received signal SNR. Under the constraint that the transmitter cannot significantly change its carrier frequency, it is of no value to transmit during a region of time for which the signal is admitted. From Figure 4, we see the region of time during which transmission would occur. The duration of this transmission is limited by $B_{\mathrm{pc}}/(B/T)$.

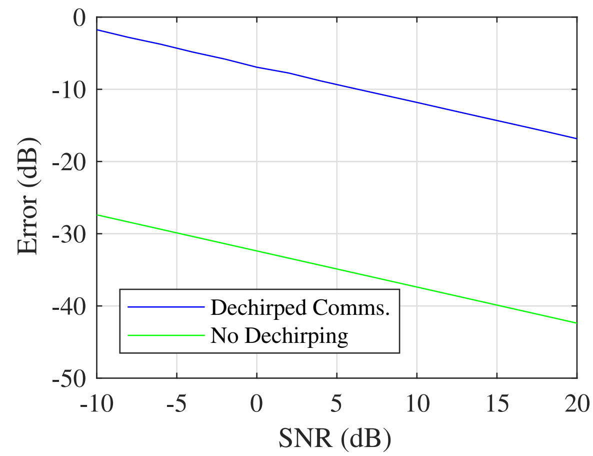

Fig. 6. Simulated error magnitude plot vs. SNR for a communications system that has to perform stretch processing (de-chirping) and a communications system that does not. It is evident that the effect of de-chirping has a significant effect on the performance of a communications system. Both communications systems transmitted a complex tone at 3 MHz at complex baseband. A de-chirping transformation with a bandwidth of 10 MHz and duration of 25 μs was applied.

3.2. Unsynchronized Waveform

If the transmitter does not have knowledge of the timing or selected range of the radar receiver, then to assure reception, repeating the transmission sequences with a duration of less than $B_{\mathrm{pc}}/(B/T)$ can be used to ensure that the complete sequence is received. Here, we do assume that the post-chirp baseband bandwidth and the chirp rate are known. Because there is a potential cyclic permutation at the receiver, the communications decoding must take this into account. The typical solution would be to include a known training sequence to enable channel compensation.

4. CONCLUSION

We investigated a joint radar-communications system under the channel constraint that the system performs de-chirping prior to communications reception. We briefly discussed stretch processing from a radar system's perspective. We also analytically studied the effects of de-chirping on communications reception. Through simulation, we saw that de-chirping can significantly increase the error magnitude of a communications system, degrading it's performance. We also provided a brief discussion on designing communications waveforms that are more tolerant to the distortion that de-chirping inflicts.

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