BEAMFORMING DESIGN FOR COEXISTENCE OF FULL-DUPLEX MULTI-CELL MU-MIMO CELLULAR NETWORK AND MIMO RADAR

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ABSTRACT

In this paper we investigate the co-existence between a multi-cell multi-user (MU) full-duplex (FD) multiple-input multiple-output (MIMO) cellular network and a MIMO radar. While a joint beamforming design technique at the cellular base-stations and users is proposed to maximize the detection probability of the MIMO radar subject to constraints of data rate per user in each cell and transmit power, null-space based waveform projection is used to mitigate the interference from the radar towards the cellular network. In particular, the proposed technique optimizes the performance of detection probability by maximizing its lower bound, which is obtained by exploiting the monotonically increasing relationship of detection probability and its non-centrality parameter. Numerical results show the feasibility of spectrum sharing between both systems.

1. INTRODUCTION

The dearth of available spectrum below 2GHz has empowered the vision of spectrum sharing between radar and commercial communication systems. Further, it has been identified that one of the many reasons of spectrum scarcity is the low-utilization of available spectrum due to fixed static spectrum allocation [1]. Among the various governing bodies around the world, the Federal Communications Commission and the National Telecommunication and Information Administration of the US are exploring the feasibility of using the huge underutilized spectrum held by other applications [1]–[4], such as radar (maritime, weather, etc.). Further, by 2030, the number of wirelessly connected devices is expected to jump to more than 500 billion. Hence, as a part of the global initiative to address the overwhelming demand for wireless broadband capacity, it is of paramount importance to develop innovative technologies for efficient spectrum sharing between radar and cellular networks, to fully deliver on the promise of future wireless communications systems such as 5G. As a result, recently spectrum sharing between radar and communication systems has captured the attention of both academia and industry [5]–[18]. However, spectrum sharing between radar and communication systems brings a new set of challenges into picture such as the *harmful interferences generated by cellular systems towards the radar* and vice-versa, which may greatly degrade the quality-of-service (QoS) of both systems. Besides spectrum sharing, full-duplex (FD) is another technology, which can potentially double the throughput of cellular systems [19]–[23]. However, the self-interference (SI) due to signal leakage from the transmitting antennas to its receiving antennas dominates the performance of FD systems. Due to the recent advances in interference cancellation techniques [20], SI can be combated to the extent that only residual SI is left behind. This residual SI can be mitigated through digital beamforming techniques [19]–[22], allowing us to truly explore the benefits of FD systems.

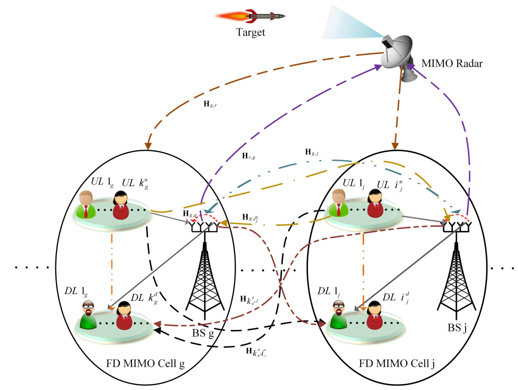

Fig. 1: Coexistence of FD multi-cell cellular systems with a MIMO radar.

Among the various challenges involved in implementation of the concerned model, the interference from cellular network towards the radar is of utmost concern owing to the level of seriousness involved in the operation of radars, particularly in defence scenarios. Hence, any new spectrum sharing design will need to ensure that the target detection capability of the radar is not affected due to the interference from the cellular network. Accordingly, in this paper, we adopt the following two approaches to enable efficient spectrum sharing between a MIMO radar and a cellular network: 1) null space based beamforming from the MIMO radar towards the multi-cell network to mitigate interference from radar towards cellular network and 2) joint design of beamforming matrices at the FD BSs and users to maximize the detection probability of MIMO radar.

2. SYSTEM MODEL

We consider the coexistence between a MIMO radar and a cellular network consisting of G-cells as shown in Fig. 1. Each cell consists of a FD BS, K_g^d DL users, and K_g^u UL users and operates in the spectrum shared by the radar. Each BS is equipped with $2M_B$ antennas (M_B antennas are used for receiving the signals and M_B antennas are used for transmitting the signals) and DL and UL users are equipped with M_d and M_u HD antennas, respectively. Additionally, the MIMO radar is equipped with $M_{T_x}^R$ transmit and $M_{R_x}^R$ receive antennas for detecting/estimating a point-like target in the far-field. The detection probability performance and the accuracy of Direction of Arrival (DoA) estimation of MIMO radar are effected by the interferences generated by the multi-cell BSs and all UL users. Accordingly, first we define $\mathbf{H}_{k_g^d,j} \in \mathbb{C}^{M_d \times M_B}$ as the channel between the j-th BS and DL user k_g^d, $\mathbf{H}_{k_g^d,i_j^u} \in \mathbb{C}^{M_d \times M_u}$ as the channel between UL user i_j^u and DL user k_g^d, $\mathbf{H}_{g,j} \in \mathbb{C}^{M_B \times M_B}$ as the interference channel from the j-th BS to the g-th BS, $\mathbf{H}_{g,i_j^u} \in \mathbb{C}^{M_B \times M_u}$ as the channel between UL user i_j^u and BS g, $\mathbf{H}_{k_g^d,r} \in \mathbb{C}^{M_d \times M_{T_x}^R}$ as the channel between the MIMO radar and DL user k_g^d, and $\mathbf{H}_{r,g} \in \mathbb{C}^{M_{R_x}^R \times M_B}$ as the channel from BS g to MIMO radar. Further, we assume that global channel state information (CSI) is available at all the nodes [22]. Since the cellular network uses the radar's spectrum, the cellular system will induce interference on the MIMO radar and vice-versa. Further, to model the residual SI, due to FD operation at the BSs the limited DR model (a.k.a., hardware impairment model) in [20, 24] is adopted. Accordingly, the signal received by the DL user k_g^d and that received by the BS g at time index l, $l = 1, \ldots, L$, where L is total number of time samples for cellular communication, can be given, respectively as

$$\begin{aligned}\mathbf{z}_{k_g^d}(l) = &\sum_{j=1}^{G} \sum_{i=1}^{K_j^d} \mathbf{H}_{k_g^d,j} \left(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)\right) \\ &+ \sum_{j=1}^{G} \sum_{i=1}^{K_j^u} \mathbf{H}_{k_g^d,i_j^u} \left(\mathbf{V}_{i_j^u}\mathbf{s}_{i_j^u}(l) + \mathbf{c}_{i_j^u}(l)\right) \\ &+ \sqrt{P_R}\mathbf{H}_{k_g^d,r}\mathbf{s}_R(l) + \mathbf{e}_{k_g^d}(l) + \mathbf{n}_{k_g^d}(l), \end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{z}_{g}(l) = &\sum_{j=1}^{G} \sum_{i=1}^{K_j^u} \mathbf{H}_{g,i_j^u} \left(\mathbf{V}_{i_j^u}\mathbf{s}_{i_j^u}(l) + \mathbf{c}_{i_j^u}(l)\right) \\ &+ \sum_{j=1}^{G} \sum_{i=1}^{K_j^d} \mathbf{H}_{g,j} \left(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)\right) \\ &+ \sqrt{P_R}\mathbf{H}_{g,r}\mathbf{s}_R(l) + \mathbf{e}_{g}(l) + \mathbf{n}_{g}(l). \end{aligned} \tag{2}$$

Here, $\mathbf{s}_{i_j^u}(l) \in \mathbb{C}^{d_{i_j^u} \times 1}$ denotes the data transmitted by UL user i in cell j with length $d_{i_j^u}$ and $\mathbb{E}\left[\mathbf{s}_{i_j^u}(l)\mathbf{s}_{i_j^u}^H(l)\right] = \mathbf{I}_{d_{i_j^u}}$, and $\mathbf{s}_{i_j^d}(l) \in \mathbb{C}^{d_{i_j^d} \times 1}$ denotes the data transmitted by j-th BS to i-th DL user in cell j with length $d_{i_j^d}$ and $\mathbb{E}\left[\mathbf{s}_{i_j^d}(l)\mathbf{s}_{i_j^d}^H(l)\right] = \mathbf{I}_{d_{i_j^d}}$. Further, $\mathbf{V}_{i_j^d} \in \mathbb{C}^{M_B \times d_{i_j^d}}$ and $\mathbf{V}_{i_j^u} \in \mathbb{C}^{M_u \times d_{i_j^u}}$ are the transmit beamforming matrices for $\mathbf{s}_{i_j^u}(l)$ and $\mathbf{s}_{i_j^d}(l)$, respectively. The symbol $\mathbf{s}_R(l) \in \mathbb{C}^{M_{T_x}^R \times 1}$ is the waveform transmitted by the MIMO radar at l-th time slot with $\mathbb{E}\left[\mathbf{s}_R(l)\mathbf{s}_R^H(l)\right] \simeq \frac{1}{L_R}\sum_{l=1}^{L_R}\left[\mathbf{s}_R(l)\left(\mathbf{s}_R(l)\right)^H\right] = \mathbf{I}_{M_{T_x}^R}$, where L_R expresses the total number of time samples for the radar communication. Further, P_R is the power of the MIMO radar signal. For the ease of derivation, we assume that the time duration of the radar waveform is the same as the communication signals and $L_R = L$. The terms $\mathbf{n}_{k_g^d}(l) \in \mathbb{C}^{M_d \times 1}$ and $\mathbf{n}_g(l) \in \mathbb{C}^{M_B \times 1}$ in (1) and (2) represent the additive white Gaussian noise (AWGN) vector with zero mean and variance σ_U^2 and σ_B^2 at the DL user k_g^d and BS g, respectively. Furthermore, $\mathbf{c}_{i_j^d}(l) \in \mathbb{C}^{M_B \times 1}$ and $\mathbf{c}_{i_j^u}(l) \in \mathbb{C}^{M_u \times 1}$ denote transmit distortion at the BS j and UL user i_j^u, respectively, while $\mathbf{e}_{k_j^d}(l) \in \mathbb{C}^{M_d \times 1}$ and $\mathbf{e}_g(l) \in \mathbb{C}^{M_B \times 1}$ are receiver distortions at the DL user k_g^d and BS g, respectively, which are modeled as [24]

$$\mathbf{c}_{i_j^d}(l) \sim \mathcal{CN}\left(\mathbf{0}, \alpha_B \operatorname{diag}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H\right)\right), \mathbf{c}_{i_j^d}(l) \perp \mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l), \tag{3}$$

$$\mathbf{c}_{i_j^u}(l) \sim \mathcal{CN}\left(\mathbf{0}, \alpha_U \operatorname{diag}\left(\mathbf{V}_{i_j^u}\mathbf{V}_{i_j^u}^H\right)\right), \mathbf{c}_{i_j^u}(l) \perp \mathbf{V}_{i_j^u}\mathbf{s}_{i_j^u}(l), \tag{4}$$

with $\alpha_B, \alpha_U \ll 1$, and

$$\mathbf{e}_{k_j^d}(l) \sim \mathcal{CN}\left(\mathbf{0}, \beta_U \operatorname{diag}\left(\mathbf{\Phi}_{k_j^d}\right)\right), \ \mathbf{e}_{k_j^d}(l) \perp \mathbf{u}_{k_j^d}(l), \tag{5}$$

$$\mathbf{e}_{g}(l) \sim \mathcal{CN}\left(\mathbf{0}, \beta_B \operatorname{diag}\left(\mathbf{\Phi}_{g}\right)\right), \ \mathbf{e}_{g}(l) \perp \mathbf{u}_{g}(l), \tag{6}$$

with $\mathbf{\Phi}_{k_j^d} = \mathrm{Cov}\{\mathbf{u}_{k_j^d}(l)\}$, $\mathbf{\Phi}_g = \mathrm{Cov}\{\mathbf{u}_g(l)\}$ and $\beta_B, \beta_U \ll 1$. Here, $\mathbf{u}_{k_j^d}(l)$ and $\mathbf{u}_g(l)$ are the undistorted received signal vector at the DL user k_g^d and BS g, respectively. Note that the BS g knows its own transmitted signal $\sum_{i=1}^{K_g^d}\mathbf{H}_{g,g}\mathbf{V}_{i_g^d}\mathbf{s}_{i_g^d}(l)$, and thus, this term can be cancelled out in (2), yielding

$$\begin{aligned}\hat{\mathbf{z}}_{g}(l) = &\sum_{j=1}^{G} \sum_{i=1}^{K_j^u} \mathbf{H}_{g,i_j^u} \left(\mathbf{V}_{i_j^u}\mathbf{s}_{i_j^u}(l) + \mathbf{c}_{i_j^u}(l)\right) \\ &+ \sum_{j=1, j\neq g}^{G} \sum_{i=1}^{K_j^d} \mathbf{H}_{g,j} \left(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)\right) \\ &+ \sqrt{P_R}\mathbf{H}_{g,r}\mathbf{s}_R(l) + \mathbf{e}_{g}(l) + \mathbf{n}_{g}(l). \end{aligned} \tag{7}$$

For the radar system, an echo wave in a single range-Doppler bin of the MIMO radar is considered. The signal vector received by the MIMO radar at time index l and at angle θ can be given as[1]

$$\begin{aligned}\mathbf{y}_R(l) = &\kappa\sqrt{P}_R\mathbf{A}(\theta)\,\mathbf{s}_R(l) \\ &+ \sum_{j=1}^{G} \sum_{i=1}^{K_j^d} \mathbf{H}_{r,j}(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)) + \mathbf{n}_{R_x}(l), \end{aligned} \tag{8}$$

where κ denotes the complex path loss of the radar-target-radar path including the propagation loss and the coefficient of reflection. $\mathbf{n}_{R_x}(l) \in \mathbb{C}^{M_{R_x}^R \times 1}$ denotes the AWGN at radar receiver with variance σ_R^2 and $\mathbf{A}(\theta)$ is the transmit-receive steering matrix written as

$$\mathbf{A}(\theta) \triangleq \mathbf{a}_{R_x}(\theta)\,\mathbf{a}_{T_x}^T(\theta)\,, \tag{9}$$

where $\mathbf{a}_{T_x} \in \mathbb{C}^{M_{T_x}^R \times 1}$ and $\mathbf{a}_{R_x} \in \mathbb{C}^{M_{R_x}^R \times 1}$ indicate the transmit and receive steering vectors of radar antenna array. With assumptions of $M_{T_x}^R = M_{R_x}^R = M$ as $\mathbf{a}_{R_x}(\theta) = \mathbf{a}_{T_x}(\theta) = \mathbf{a}(\theta)$, and $\mathbf{A}_{ir}(\theta) = \mathbf{a}_i(\theta)\,\mathbf{a}_r(\theta) = \exp\left(-j\omega\tau_{ir}(\theta)\right)$, let us define $\mathbf{A}_{ir}(\theta)$ as [10]

$$\mathbf{A}_{ir}(\theta) = \exp\left(-j\frac{2\pi}{\lambda}\left[\sin(\theta);\cos(\theta)\right]^T(\mathbf{z}_i + \mathbf{z}_r)\right). \tag{10}$$

Here, $\mathbf{A}_{ir}(\theta)$ represents the ith element at the rth column of the matrix $\mathbf{A}$ and $\mathbf{z}_i = \left[z_i^1; z_i^2\right]$ denotes the location of the ith element of the antenna array. The symbols ω and λ indicate the frequency and the wavelength of the carrier.

3. DETECTION PROBABILITY OF MIMO RADAR

By utilizing a binary hypothesis test, and ignoring the interference by clutter and false targets, we select between two hypothesis for the target detection and estimation. From (8), the hypothesis testing problem can be constructed as

$$\mathbf{y}_R(l) = \begin{cases} \mathcal{H}_1: \kappa\sqrt{P}_R\mathbf{A}(\theta)\,\mathbf{s}_R(l) + \mathbf{n}_{R_x}(l) \\ +\sum_{j=1}^{G}\sum_{i=1}^{K_j^d}\mathbf{H}_{r,j}\left(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)\right), \ \forall l\,, \\ \mathcal{H}_0: \sum_{j=1}^{G}\sum_{i=1}^{K_j^d}\mathbf{H}_{r,j}\left(\mathbf{V}_{i_j^d}\mathbf{s}_{i_j^d}(l) + \mathbf{c}_{i_j^d}(l)\right) \\ +\mathbf{n}_{R_x}(l), \ \forall l\,, \end{cases} \tag{11}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ present the null and alternate hypotheses on the MIMO radar detection. Specifically, $\mathcal{H}_0$ represents the case with no target but active cellular network. The hypothesis $\mathcal{H}_1$ is used to indicate the case when both the target and the cellular network are active. Due to the unknown nature of the deterministic parameters κ and θ, the generalized likelihood ratio test (GLRT) [25] is adopted for determining the detection probability P_D, which is given as

$$P_D = 1 - \mathfrak{F}_{\mathcal{J}_2^2(\rho)}\left(\mathfrak{F}_{\mathcal{J}_2^2}^{-1}(1 - P_{FA})\right). \tag{12}$$

Here, P_{FA} denotes the probability of false alarm, and $\mathfrak{F}_{\mathcal{J}_2^2(\rho)}$ and $\mathfrak{F}_{\mathcal{J}_2^2}^{-1}$ represent the non-central and inverse central chi-squared distribution functions with two degrees of freedom (DoFs), respectively. Further, ρ denotes the non-centrality parameter, given as

[1]The interference from the UL users towards the MIMO radar is ignored due to the low transmit power of UL users as compared to FD BSs.

$$\rho = \Gamma_R \sigma_R^2 \text{tr}\left(\mathbf{A}(\theta)\mathbf{A}^H(\theta)\hat{\mathbf{\Xi}}^{-1}\right), \quad (13)$$

where $\Gamma_R = |\kappa|^2 LP_R/\sigma_R^2$. The terms $\mathbf{\Xi} \in \mathbb{C}^{M^2\times M^2}$ and $\hat{\mathbf{\Xi}}$ are defined as

$$\mathbf{\Xi} = \begin{pmatrix} \overbrace{\tilde{\mathbf{\Xi}} + \sigma_R^2 \mathbf{I}_M}^{\hat{\mathbf{\Xi}}} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\mathbf{\Xi}} + \sigma_R^2 \mathbf{I}_M \end{pmatrix}, \quad (14)$$

where $\tilde{\mathbf{\Xi}} = \sum_{j=1}^{G}\sum_{i=1}^{K_j^d} \mathbf{H}_{r,j}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H + \alpha_B \text{ diag}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H\right)\right)\mathbf{H}_{r,j}^H$. Since the GLRT in [26] was applied in the presence of white noise only, we convert the covariance matrix in (14) into white by applying a whitening filter.

As stated before, to mitigate the interference from radar towards cellular system, null space based waveform projection is used. Let us consider that the radar shares $\mathcal{I}$ interference channels, denoted as $\mathbf{W}_i \in \mathbb{C}^{G\times R_T}$ with the cellular system, where $i = 1, \ldots, \mathcal{I}$. Now, singular value decomposition (SVD) can be utilized to find the null space of $\mathbf{W}_i$, which can then be used to create a projector matrix.

4. JOINT BEAMFORMING DESIGN FOR FD MIMO MULTI-CELL SYSTEM

To mitigate the interference from cellular network towards MIMO radar, in this section we formulate the joint beamforming design problem for maximizing P_D of the MIMO radar, subject to the constraints of minimum data rate for each UL and DL users in each cell and transmit powers. For notational simplicity, hereinafter we ignore the time index l unless otherwise stated.

After eliminating the interference from radar towards cellular system, through null-space projection, the rate of the DL user k_g^d and UL user k_g^u can be given from (1) and (7) as

$$R_{k_g^d} = \log_2\left|\mathbf{I}_{M_d} + \mathbf{\Sigma}_{k_g^d}^{-1}\mathbf{H}_{k_g^d,g}\mathbf{V}_{k_g^d}\mathbf{V}_{k_g^d}^H\mathbf{H}_{k_g^d,g}^H\right|, \quad (15)$$

$$R_{k_g^u} = \log_2\left|\mathbf{I}_{M_B} + \mathbf{\Sigma}_{k_g^u}^{-1}\mathbf{H}_{g,k_g^u}\mathbf{V}_{k_g^u}\mathbf{V}_{k_g^u}^H\mathbf{H}_{g,k_g^u}^H\right|, \quad (16)$$

where $\mathbf{\Sigma}_{k_g^d}$ and $\mathbf{\Sigma}_{k_g^u}$ denote the interference-plus-noise covariance matrices for the DL user k_g^d and UL user k_g^u. The details expression of $\mathbf{\Sigma}_{k_g^d}$ and $\mathbf{\Sigma}_{k_g^u}$ is omitted due to space limitations.

The P_D maximization problem can now be formulated as

$$\begin{aligned} \textbf{P0:}\ \max_{\mathbf{V}} \quad & P_D \\ \text{s.t.}\ (C.1)\quad & R_{k_g^u} \geq R_{k_g^u,min}, \quad k = 1,\ldots,K_g^u,\ \forall g, \\ (C.2)\quad & R_{k_g^d} \geq R_{k_g^d,min}, \quad k = 1,\ldots,K_g^d,\ \forall g, \\ (C.3)\quad & \text{tr}\left(\mathbf{V}_{k_g^u}\mathbf{V}_{k_g^u}^H\right) \leq P_{k_g^u},\ k = 1,\ldots,K_g^u,\ \forall g, \\ (C.4)\quad & \sum_{k=1}^{K_g^d}\text{tr}\left(\mathbf{V}_{k_g^d}\mathbf{V}_{k_g^d}^H\right) \leq P_g, \quad \forall g, \end{aligned} \quad (17)$$

where the constraints $(C.1)$ and $(C.2)$ ensure the minimum QoS requirements for the UL user k_g^u and DL user k_g^d, respectively in terms of data rate. The constraints $(C.3)$ and $(C.4)$ represent the transmit power constraints for each UL user and BS, respectively. In the above, $\mathbf{V} = \{\mathbf{V}_{k_g^u}, \mathbf{V}_{k_g^d}\}$ denotes the set of all transmit beamforming matrices. According to [27], P_D is a monotonically increasing function with respect to non-centrality parameter (ρ_{NSP} in this case). Thus, the problem **(P0)** can be equivalently cast as

$$\begin{aligned} \textbf{P1:}\ \max_{\mathbf{V}}\ & \text{tr}\left(\mathbf{A}(\theta)\hat{\mathbf{P}}\hat{\mathbf{P}}^H\mathbf{A}^H(\theta)\hat{\mathbf{\Xi}}^{-1}\right) \\ \text{s.t.}\ & (C.1) - (C.4). \end{aligned} \quad (18)$$

Since the objective function in (18) is non-concave [28], we cannot solve the problem **(P1)** in its current form. Thus, a lower bound on the objective function is considered for tractability of the problem.

Lemma 1 *Let $\varphi = tr(\hat{\boldsymbol{P}}\hat{\boldsymbol{P}}^H)$. For $M_{T_x}^R = M_{R_x}^R = M$, the lower bound on $tr\left(\mathbf{A}(\theta)\hat{\boldsymbol{P}}\hat{\boldsymbol{P}}^H\mathbf{A}^H(\theta)\hat{\mathbf{\Xi}}^{-1}\right)$ can be expressed as*

$$tr\left(\mathbf{A}(\theta)\hat{\boldsymbol{P}}\hat{\boldsymbol{P}}^H\mathbf{A}^H(\theta)\hat{\mathbf{\Xi}}^{-1}\right) \geq \quad (19)$$
$$\frac{\varphi M^2}{\sum_{j=1}^{G}\sum_{i=1}^{K_j^d} tr\{\mathbf{H}_{r,j}(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H + \alpha_B diag(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H))\mathbf{H}_{r,j}^H\} + M\sigma_R^2}.$$

Using Lemma 1, the problem **(P1)** can be transformed as

$$\begin{aligned} \textbf{P2:}\ \min_{\mathbf{V}}\ & \sum_{j=1}^{G}\sum_{i=1}^{K_j^d}\text{tr}\left\{\mathbf{H}_{r,j}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H + \alpha_B \text{ diag}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H\right)\right)\mathbf{H}_{r,j}^H\right\} \\ \text{s.t.}\ & (C.1) - (C.4). \end{aligned} \quad (20)$$

The optimization problem **(P2)** in (20) is still intractable. To transform this problem into a tractable one, we convert the rate constraints $(C.1)$ and $(C.2)$ into equivalent mean squared error (MSE) constraints as [22]

$$R_{k_g^u} \simeq -\text{tr}\{\mathbf{W}_{k_g^u}\mathbf{E}_{k_g^u}\} + \log_2|\ln 2\mathbf{W}_{k_g^u}| + d_{k_g^u}/\ln 2, \quad (21)$$

$$R_{k_g^d} \simeq -\text{tr}\{\mathbf{W}_{k_g^d}\mathbf{E}_{k_g^d}\} + \log_2|\ln 2\mathbf{W}_{k_g^d}| + d_{k_g^d}/\ln 2, \quad (22)$$

where $\mathbf{W}_{k_g^u}$ and $\mathbf{W}_{k_g^d}$ represent the weight matrix for the UL user k_g^u and DL user k_g^d.

Now, using the decompositions $\mathbf{W}_{k_g^u} = \mathbf{B}_{k_g^u}^H\mathbf{B}_{k_g^u}$ and $\mathbf{W}_{k_g^d} = \mathbf{B}_{k_g^d}^H\mathbf{B}_{k_g^d}$, the terms $\text{tr}\{\mathbf{W}_{k_g^u}\mathbf{E}_{k_g^u}\}$ in (21) and $\text{tr}\{\mathbf{W}_{k_g^d}\mathbf{E}_{k_g^d}\}$ in (22) are explicitly expressed as in (23) and (24) shown on the top of the next page, where $\mathbf{U}_{k_g^u} \in \mathbb{C}^{\mathbf{I}_{d_{k_g^u}}\times M_B}$ and $\mathbf{U}_{k_g^d} \in \mathbb{C}^{\mathbf{I}_{d_{k_g^d}}\times M_d}$ denote linear receivers at BS g and at DL user k_g^d, respectively.

Using (21), (22) and writing the constraints $(C.3)$ and $(C.4)$ in vector forms, the problem **(P2)** can be reformulated as

$$\begin{aligned} \textbf{P3:}\ \min_{\mathbf{U},\mathbf{V},\mathbf{W}}\ & \sum_{j=1}^{G}\sum_{i=1}^{K_j^d}\text{tr}\left\{\mathbf{H}_{r,j}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H + \alpha_B \text{ diag}\left(\mathbf{V}_{i_j^d}\mathbf{V}_{i_j^d}^H\right)\right)\mathbf{H}_{r,j}^H\right\} \\ \text{s.t.}\ (C.1)\ & \text{tr}\{\mathbf{W}_{k_g^u}\mathbf{E}_{k_g^u}\} - \log_2|\ln 2\mathbf{W}_{k_g^u}| - \frac{d_{k_g^u}}{\ln 2} \\ & \leq -R_{k_g^u,min},\ k = 1,\ldots,K_g^u,\ \forall g, \\ (C.2)\ & \text{tr}\{\mathbf{W}_{k_g^d}\mathbf{E}_{k_g^d}\} - \log_2|\ln 2\mathbf{W}_{k_g^d}| - \frac{d_{k_g^d}}{\ln 2} \\ & \leq -R_{k_g^d,min},\ k = 1,\ldots,K_g^d,\ \forall g, \\ (C.3)\ & \left\|\text{vec}\left(\mathbf{V}_{k_g^u}\right)\right\|_2^2 \leq P_{k_g^u}, \quad k = 1,\ldots,K_g^u,\ \forall g, \\ (C.4)\ & \sum_{k=1}^{K_g^d}\left\|\text{vec}\left(\mathbf{V}_{k_g^d}\right)\right\|_2^2 \leq P_g, \quad \forall g. \end{aligned} \quad (25)$$

Note that the problem **(P3)** is not jointly convex in $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{W}$. An iterative algorithm is proposed to find the optimal transmit and receive beamforming matrices and the optimal weight matrices.

$$\mathrm{tr}\{\mathbf{W}_{k_g^u}\mathbf{E}_{k_g^u}\} = \mathrm{tr}\Big\{\mathbf{B}_{k_g^u}\left(\mathbf{U}_{k_g^u}\mathbf{H}_{g,k_g^u}\mathbf{V}_{k_g^u} - \mathbf{I}_{d_{k_g^u}}\right)\left(\mathbf{U}_{k_g^u}\mathbf{H}_{g,k_g^u}\mathbf{V}_{k_g^u} - \mathbf{I}_{d_{k_g^u}}\right)^H \mathbf{B}_{k_g^u}^H\Big\} + \mathrm{tr}\left\{\mathbf{B}_{k_g^u}\mathbf{U}_{k_g^u}\mathbf{\Sigma}_{k_g^u}\mathbf{U}_{k_g^u}^H\mathbf{B}_{k_g^u}^H\right\}, \tag{23}$$

$$\mathrm{tr}\{\mathbf{W}_{k_g^d}\mathbf{E}_{k_g^d}\} = \mathrm{tr}\Big\{\mathbf{B}_{k_g^d}\left(\mathbf{U}_{k_g^d}\mathbf{H}_{k_g^d,g}\mathbf{V}_{k_g^d} - \mathbf{I}_{d_{k_g^d}}\right)\left(\mathbf{U}_{k_g^d}\mathbf{H}_{k_g^d,g}\mathbf{V}_{k_g^d} - \mathbf{I}_{d_{k_g^d}}\right)^H \mathbf{B}_{k_g^d}^H\Big\} + \mathrm{tr}\left\{\mathbf{B}_{k_g^d}\mathbf{U}_{k_g^d}\mathbf{\Sigma}_{k_g^d}\mathbf{U}_{k_g^d}^H\mathbf{B}_{k_g^d}^H\right\}, \tag{24}$$

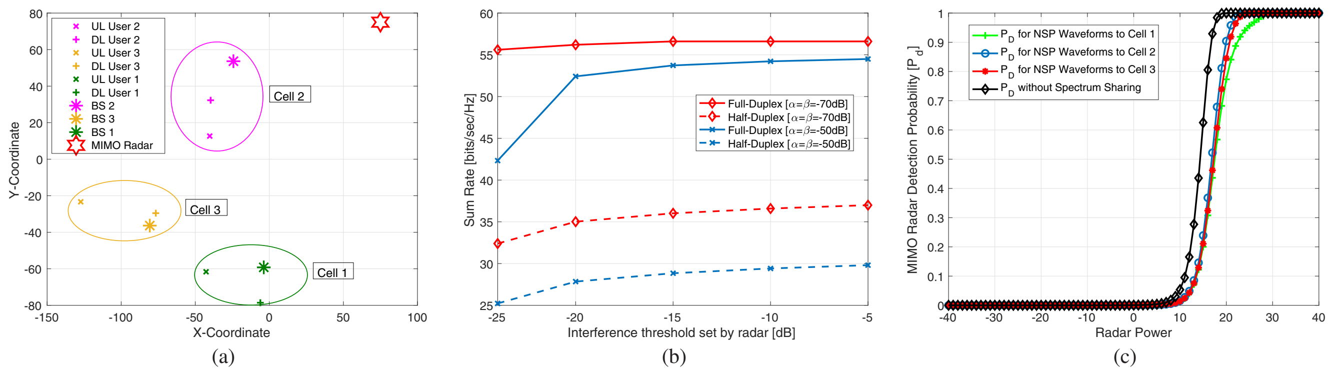

Fig. 2: (a) Simulation Setup. The MIMO radar (red hexagram) shares its spectrum with the cellular network consisting of 3 cells, (b) Sum rate of cellular network vs Interference threshold of radar; and (c) Detection probability (P_D) vs. radar transmit power (P_R).

5. NUMERICAL RESULTS

The maximum number of iterations of the iterative algorithm is set at 350. In conjunction with radar spectrums available for sharing [4], we perform the analyses for the carrier frequency of 3.6 GHz and a bandwidth of 50 MHz. An area of 150m is considered for the examined scenario. Fig. 2(a) illustrates the simulation setup, where the MIMO radar is located at the edge of the area and shares its spectrum with a cellular network involving three small cells, deployed under the 3GPP LTE specifications [29]. The small cells are randomly distributed within the area between 50−150m from the radar. Each cell has a radius of 50m and consists of one BS serving one UL and one DL user. While each BS is equipped with $2M_B$ antennas, each UL and DL user is equipped with M_u and M_d antennas, respectively. Note that when BS operates in HD mode, it uses only M_B antennas for transmission/reception. For the sake of simplicity, we consider $M_B = M_u = M_d = \hat{M} = 2$ and $M_{T_x}^R = M_{R_x}^R = \tilde{M} = 4$. Next, to model the path loss in our system, we consider the close-in (CI) free space reference distance path loss models as depicted in [30]. The CI model is a generic model that describes the large-scale propagation path loss at all relevant frequencies ($>$ 2 GHz) and can be easily implemented in existing 3GPP models. The thermal noise density is set at -174 dBm/Hz and the noise figures at BS and UEs are set at 13dB and 9dB respectively. The estimated channel gains in the network are multiplied by $\sqrt{\wp}$, where $\wp = 10^{(-A/10)}$, $A \in \{\mathrm{LOS}, \mathrm{NLOS}\}$[2] denotes the large scale fading consisting of path loss and shadowing. The path loss exponent for LOS and NLOS are set as 2.0 and 3.1, respectively, while the value of shadow fading standard deviation for LOS and NLOS are set at 2.9 dB and 8.1 dB, respectively [31]. To model the SI channel, the Rician model in [20] is adopted, wherein the SI channel is distributed as $\mathbf{H}_{g,g} \sim \mathcal{CN}\left(\sqrt{\frac{K_R}{1+K_R}}\hat{\mathbf{H}}_{g,g}, \frac{1}{1+K_R}\mathbf{I}_{M_B} \otimes \mathbf{I}_{M_B}\right)$, where the Rician factor is denoted by K_R, while the matrix $\hat{\mathbf{H}}_{g,g}$ is a deterministic matrix. Here, for simplicity, we take $K_R = 10$ and $\hat{\mathbf{H}}_{g,g}$ as the matrix of all ones for all simulations [32]. Unless otherwise stated, the values of $R_{k_g^u}$, $\forall k, g$, and $R_{k_g^d}$, $\forall k, g$, are set at 10 and 15 bits/sec/Hz, respectively, $\alpha_B(\beta_B) = \alpha_U(\beta_U) = \alpha(\beta) = -70$dB, $P_{k_g^u} = P_g = 0$dB, $\forall k, g$, and $P_{FA} = 10^{-5}$.

Next we quantify the performance of the cellular network operating in the spectrum shared by the MIMO radar. Fig. 2(b) shows the achievable sum rate of the cellular network as a function of interference threshold of the radar for different RSI cancellation levels, reflected here by transmitter/receiver (α/β) distortion values. It can be seen from the figure that as the RSI cancellation capability of the system increases, sum rate of the FD cellular system increases. Also, increasing the threshold of the interference temperature towards radar allows for better sum rate at the cellular system. However, in order to protect the radar, the proposed algorithm ensures that after a certain limit the sum rate becomes constant. It can be observed that an interference threshold of around -15dB is enough to guarantee considerable sum rate for the cellular network. Additionally, it can be seen that operating at FD mode improves the performance of the network by at least 60% over conventional HD operation.

Finally, we show the performance of the spectrum sharing MIMO radar in Fig. 2(c). In particular, we evaluate the detection probability of the MIMO radar w.r.t. radar transmit power. It can be observed that for a fixed P_{FA}, the MIMO radar needs to spend some extra power to achieve a particular P_D than for the scenario without spectrum sharing. However, the extra power is not significant, when compared to the gains achieved by the cellular network and can be compounded to around $2-3$dB only to achieve a P_D of 0.9.

6. CONCLUSION

Null-space based beamformers at radar and optimal digital beamformers at cellular systems in a multi-cell FD cellular network suffering from transmit/receive distortions was formulated to facilitate the coexistence between a cellular network and a MIMO radar under the same spectrum. Numerical results show the feasibility of spectrum sharing, albeit with certain tradeoffs in radar transmit power, detection probability and sum rate of the cellular system. In a nutshell, the designed framework provides a cornerstone and most importantly, the essential understanding for successful development of future cellular systems in-conjunction with MIMO radar that can operate under same spectrum resources.

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[2]LOS=Line-of-sight and NLOS=Non-line-of-sight.

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