OPTIMIZATION OF A MOVING COLORED CODED APERTURE IN COMPRESSIVE SPECTRAL IMAGING

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ABSTRACT

Coded aperture compressive spectral imagers allow sensing a three-dimensional (3D) data cube by using two-dimensional (2D) projections of the coded and spectrally dispersed source. The traditional block-unblock coded apertures have been recently replaced by patterned optical filter arrays, allowing to modulate the spatial and spectral information. The real implementation of these patterned or "colored" coded apertures in terms of cost and complexity, directly depends on the number of filters to be used as well as the number of snapshots to be captured. This paper introduces a coded aperture optimization having in consideration these restrictions, the final design obtained is a moving colored coded aperture, which improves the reconstruction quality of the data cube and is physically implementable. Simulations show the accuracy and performance achieved with the proposed approach yielding up to 3 dB gain in PSNR over the current literature designs.

***Index Terms*—** Spectral imaging, coded aperture optimization, patterned optical filter arrays.

1. INTRODUCTION

Coded aperture compressive spectral imagers capture three-dimensional scenes by two-dimensional projections, multiplexing the spatio-spectral information of the scene through a coded aperture (CA), and a dispersive element. Many compressive spectral imaging (CSI) systems have been developed following this framework, however all of them share the attempt to perform a direct 2D measurement, achieving a mapping of each point of the scene to a single point in the optical sensor [1].

The CA can be implemented by a photomask with a permeability to block or let pass the light for certain spectral bands. The CA patterns have been usually modeled as matrices, whose entries are realizations of a Bernoulli random variable, Hadamard matrices, S-matrices and cyclic S matrices obtained by cyclic permutations of a codeword, and these distributions have shown to obtain good reconstructions, and have been widely used [2, 3, 4]. In addition to the use of these distributions, different approaches have been reported in which the authors attempt to find an optimal structure for the CA, not only to increase the reconstruction quality, but to take fewer measurements, revealing the benefits of optimal sampling applied in conjunction with compressive sensing [5, 6, 7, 8, 9, 10, 11]. Approaches using singular value decomposition [5, 12], genetic algorithms [6], adaptive schemes [13, 14], shrinkage methods [5], computational-based [15, 16], among others approaches [17, 18, 19] have been proposed. Although gradient-based methods has been successfully

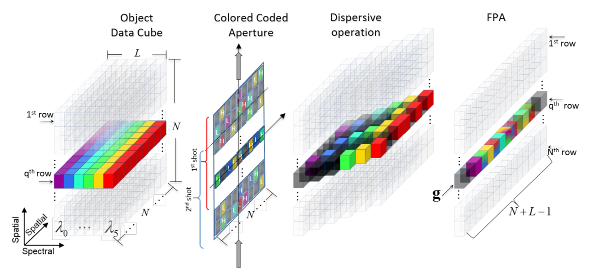

Fig. 1. Physical sensing phenomena in colored CASSI. L spectral bands of the data cube $\mathbf{F}$ are coded spatially and spectrally by a moving colored coded aperture, and dispersed by the prism. The detector captures the intensity $\mathbf{g}$ by integrating the coded and dispersed light.

used, there is no report of its use for the design of colored CA, having into account the spatial-spectral modulation as well as its simultaneous design for different number of snapshots, which is the approach presented in this work. This work specifically develops a colored coded aperture optimization, which includes variability and uniformity constraints as well as hardware restrictions such as the number of filters and a novel moving strategy, which can be implemented as a moving colored lithographic mask using a micro-piezo electric device. Figure 1 depicts the physical sensing phenomenon in a CSI system. Notice the colored coded aperture, which can be moved vertically to acquire two different snapshots.

2. CASSI SYSTEM WITH COLORED CODED APERTURES

The coded aperture snapshot spectral imager (CASSI), which is an example of a compressive spectral imager (CSI) architecture is composed by a coded aperture, a dispersive element, and a focal plane array (FPA). Figure 1 shows the main components of the colored CASSI. The micro-lithography and coating technology boost the fabrication of the colored CA with different optical filters, allowing not only the spatial but the spectral modulation as well. Therefore, and taking advantage of the recent advances in mask fabrication, a colored coded aperture defined as $\mathbf{T}^{\ell}_{mnk}$ modulates the source, a spatio-spectral image defined as $\mathbf{F}_{mnk}$, where m and n index the spatial coordinates, k determines the k^{th} spectral band, and the ℓ index the number of snapshots to be captured, each one using a different colored CA. The ℓ^{th} FPA measurement, using this notation is defined as,

$$\mathbf{G}^{\ell}_{mn} = \sum_{k=0}^{L-1} \mathbf{T}^{\ell}_{m(n-k)k}\mathbf{F}_{m(n-k)k} + \boldsymbol{\omega}_{mn}, \quad (1)$$

where $m, n = 0, 1, \ldots, N-1$, $k = 0, 1, \ldots, L-1$, and $\boldsymbol{\omega}$ represents the noise of the sensing system. Notice that $\mathbf{F} \in \mathbb{R}^{N^2L}$, $\mathbf{T}^{\ell} \in \mathbb{R}^{N^2L}$, and $\mathbf{G}^{\ell} \in \mathbb{R}^{N^2}$.

2.1. Binary representation of the colored coded aperture

A colored CA $\mathbf{T}_{mnk}$ can be represented as an arrangement of binary CAs $\mathbf{T}_{mn}$ per each of the L bands, as it is presented in fig. 2. Other representation using the binary CA can be used instead, with the aim of reducing the complexity of the coded aperture design proposed in this work. This matrix arrangement is defined as the horizontal concatenation of the L binary $\mathbf{T}_{mn}$ CAs or the respective binary representation of the colored coded aperture such that $\mathbf{X}^{i} = [\mathbf{T}^{\intercal}_{mn1} \ldots \mathbf{T}^{\intercal}_{mnL}]^{\intercal}$, and the subsequent vertical concatenation of the previous codes for the $\mathbf{K}$ shots. Then, the colored CA binary matrix for K snapshots is defined as $\mathbf{X} = \left[(\mathbf{X}^1)^{\intercal}, \ldots, (\mathbf{X}^i)^{\intercal}, \ldots (\mathbf{X}^K)^{\intercal}\right]^{\intercal}$, such that $\mathbf{X} \in \mathbb{R}^{KN \times NL}$, and $\mathbf{X}^{i} \in \mathbb{R}^{N \times N \cdot L}$. A cartoon representation of the arrangement is displayed in fig. 3, this colored CA modulates a spatiospectral data cube of dimensions $16 \times 16 \times 3$, and for 2 shots. The pattern distribution used in the sketch is a random pattern.

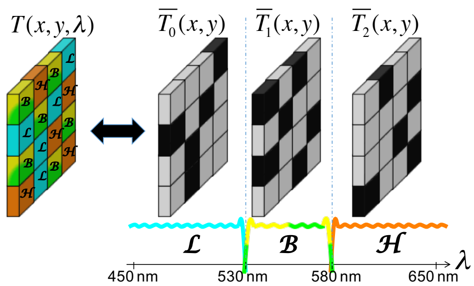

Fig. 2. Color coded aperture with low, band and high pass filters and its equivalent set of binary coded apertures.

2.2. Reconstruction Algorithm

The multispectral signal $\mathbf{F} \in \mathbb{R}^{N \times N \times L}$, or its vector representation $\mathbf{f} \in \mathbb{R}^{N \cdot N \cdot L}$ is S-sparse on some basis $\boldsymbol{\Psi}$. Hence, the signal can be approximated by a linear combination of S vectors from $\boldsymbol{\Psi}$ with $S \ll (N \cdot N \cdot L)$ as $\mathbf{f} = \boldsymbol{\Psi}\boldsymbol{\theta}$. An alternative representation of the projections in CASSI is given by $\mathbf{g} = \mathbf{H}\boldsymbol{\Psi}\boldsymbol{\theta} = \mathbf{A}\boldsymbol{\theta}$, where $\mathbf{H}$ is a matrix structure, determined by the coded aperture entries and the dispersive element effect, and the matrix $\mathbf{A} = \mathbf{H}\boldsymbol{\Psi}$ is the sensing matrix. An estimate of the spatio-spectral data cube from the measurements $\mathbf{G}$ can be attained by solving the optimization problem,

$$\hat{\mathbf{f}} = \boldsymbol{\Psi}\left\{ \underset{\boldsymbol{\theta}}{\arg\min} \|\mathbf{g} - \mathbf{H}\boldsymbol{\Psi}\boldsymbol{\theta}\|_2 + \tau\|\boldsymbol{\theta}\|_1 \right\}, \quad (2)$$

where τ is a regularization constant. The basis representation $\boldsymbol{\Psi}$ is set as the Kronecker product between a 2D-Wavelet Symmlet 8 basis and the 1D-Discrete Cosine Transform. Different algorithms have been proposed to solve the optimization problem in Eq. (2), including the two-step iterative shrinkage/thresholding (TwIST) [20], the gradient projection for sparse reconstruction (GPSR) [21], and the Gaussian mixture models (GMM) [22]. In this work, the GPSR algorithm was used, although any of the other algorithms could be used as well.

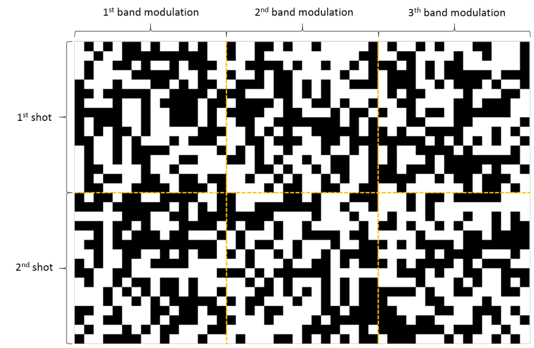

Fig. 3. Proposed $\mathbf{X}$ matrix arrangement of a colored coded aperture in binary representation to modulate a spatio-spectral data cube of dimensions $16 \times 16 \times 3$, and for 2 snapshots.

3. COLORED CODED APERTURE OPTIMIZATION

The optimization of the colored CA proposed in this work is based on the promotion of the variability and uniformity. The variability regards the reduction of the correlation between the rows and columns of the $\mathbf{X}$ matrix arrangement. The uniformity is referred to the regularity of the sensing process through the spatial dimensions, the spectral bands, and the number of snapshots. In addition, other two considerations are used to guide the design process, the cost and fabrication complexity of the CA masks.

3.1. Variability constraint

The sampling process is directly affected by the CA from Eq. (1), therefore by $\mathbf{X}$. The Gram matrix of $\mathbf{X}$ is used in the design optimization problem as the constraint inducing the low correlation between the rows and columns of $\mathbf{X}$. The row-wise and column-wise correlations define the variability constraint as,

$$\underset{\mathbf{X}}{\arg\min} \|\mathbf{I}_1 - \mathbf{X}\mathbf{X}^{\intercal}\|_F^2, \quad (3)$$

$$\underset{\mathbf{X}}{\arg\min} \|\mathbf{I}_2 - \mathbf{X}^{\intercal}\mathbf{X}\|_F^2, \quad (4)$$

where $\mathbf{I}_1$ and $\mathbf{I}_2$ are identity matrices of size $KN \times KN$, and $LN \times LN$, respectively, and $\mathbf{X}$ is the optimization variable matrix.

3.2. Uniformity constraint

On the other hand, the uniformity constraint promotes the reduction of the spatial, spectral and snapshot correlation of the voxels in

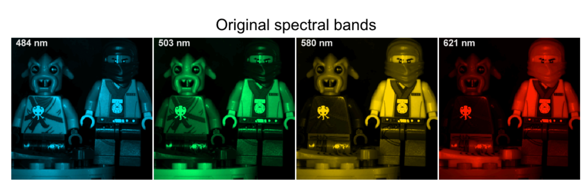

Fig. 4. Four of the eight original spectral bands of the data cube used in simulations.

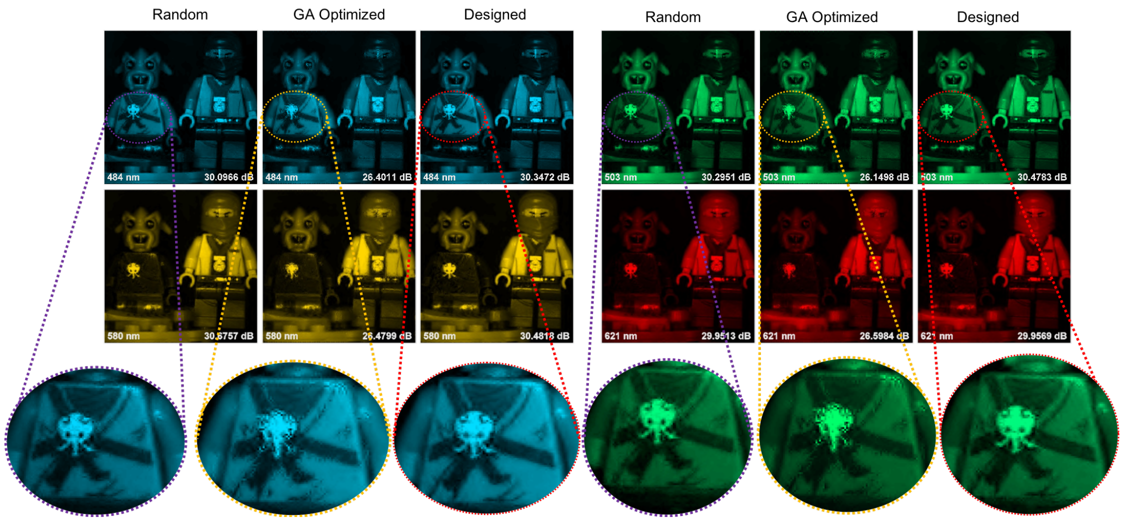

Fig. 5. Reconstruction of four spectral bands using the CASSI with color coded apertures. For each spectral band, three reconstructions from 2 measurements using the random, the genetic algorithm optimization, and the designed coded apertures are shown.

the acquisition process. In the case of the snapshots, when multiple snapshots are acquired, the number of times a voxel is sensed across shots given a CA arrangement $\mathbf{X}$, can be calculated as the product $\mathbf{RX}$, where $\mathbf{R} = [\mathbf{I}_1, \cdots, \mathbf{I}_k]^\intercal$, and $\mathbf{I}_i$ is an identity matrix of size $N \times N$. Then, the shots uniformity can be minimized by solving,

$$\underset{\mathbf{X}}{\arg\min} \, \|\mathbf{U} - \mathbf{RX}\|_F^2, \tag{5}$$

where $\mathbf{U}$ is a matrix with constant values.

Regarding the spectral sensing, the uniformity is guaranteed if the number of times a spectral voxel is sensed is as uniform as possible, and it is calculated as $\mathbf{XD}$, with $\mathbf{D} = [\mathbf{0}^\intercal_{N\times L-L}, \mathbf{I}^\intercal_N, \mathbf{0}^\intercal_{N\times L-1}, \ldots, \mathbf{0}^\intercal_{N\times L-1}, \mathbf{I}^\intercal_N, \mathbf{0}^\intercal_{N\times L-L}]^\intercal$, where $\mathbf{0}_{N\times L-1}$ is a $0-$valued $N \times L-1$ matrix, and $\mathbf{I}_N$ is an identity $N \times N$ matrix. The uniformity is then given by the constraint,

$$\underset{\mathbf{X}}{\arg\min} \, \|\mathbf{V} - \mathbf{XD}\|_F^2, \tag{6}$$

where $\mathbf{V}$ is a matrix with constant values, keeping the sensing proportions.

With respect to the spatial uniformity, the purpose, is to avoid the clusters of one-valued entries both in the columns, and in the rows of the CA pattern. These constraints are defined as,

$$\underset{\mathbf{X}}{\arg\min} \, \|\mathbf{B} - \mathbf{XW}\|_F^2, \tag{7}$$

$$\underset{\mathbf{X}}{\arg\min} \, \|\mathbf{C} - \mathbf{ZX}\|_F^2, \tag{8}$$

where $\mathbf{B}$ and $\mathbf{C}$ are matrices with constant values, $\mathbf{W}$ and $\mathbf{Z}$ are positive definite Toeplitz matrices of size $LN \times LN$ and $KN \times KN$ respectively. The selection of the number of 1-value diagonals in the Toeplitz matrices determine the number of neighbor pixels to analize of a row/column. The expected behavior of the resulting matrices as in the previous constraints is to be as constant as possible and therefore promote a more uniform sensing.

Considering the variability constraints in Eqs. (3) and (4), and the uniformity constraints in Eqs. (5), (6), (7), and (8), the cost function to solve is defined as,

$$\begin{aligned}\underset{\mathbf{X}}{\arg\min} \;\; c(\mathbf{X}) = {} & \phi_1\|\mathbf{I}_1 - \mathbf{XX}^\intercal\|_F^2 + \phi_2\|\mathbf{I}_2 - \mathbf{X}^\intercal\mathbf{X}\|_F^2 \\ & + \phi_3\|\mathbf{U} - \mathbf{RX}\|_F^2 + \phi_4\|\mathbf{V} - \mathbf{XD}\|_F^2 \\ & + \phi_5\|\mathbf{B} - \mathbf{XW}\|_F^2 + \phi_6\|\mathbf{C} - \mathbf{ZX}\|_F^2,\end{aligned} \tag{9}$$

where $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5$, and ϕ_6 are step control variables. The minimization problem is solved with a gradient descent algorithm, which iteratively minimizes Eq. (9), starting with a realization of a random CA, and with the aim to find an optimized coded aperture arrangement $\mathbf{X}^*$.

3.3. Cost and fabrication complexity constraint

The cost limitations are given by the type and number of filters used in the fabrication, and the number of masks required for multishot systems. In order to reduce the cost and fabrication complexity, the spectral response of the filters in the CA is limited to be either low or high pass filters, and the cut-off wavelengths of the filters are assumed to be selected from the subset $\lambda_0, \ldots, \lambda_{L-1}$. Thus, only $2\lambda_L$ colored filters can be selected for each coded aperture pixel. A thresholding operator is applied at each iteration of the gradient descent algorithm, to reduce the resultinf filters of the coded aperture, to those belonging to the set $\mathbf{\Lambda} \in \left\{\mathbf{\Lambda}^{Low} \cup \mathbf{\Lambda}^{High}\right\}$, where the set of low pass filters is $\mathbf{\Lambda}^{Low} = \left\{\lambda_0^{Low}, \ldots, \lambda_L^{Low}\right\}$, and the set of high pass filters is $\mathbf{\Lambda}^{High} = \left\{\lambda_0^{High}, \ldots, \lambda_L^{High}\right\}$. On the other hand, the design of the colored CA patterns for a multishot system is proposed such that only one moving mask is required. The strategy consists in the concatenation of vertical complementary colored coded aperture patches of size $S \times N$, where S is the number of pixels the mask should be moved between shots. The final spatial dimension of the moving colored coded aperture is $N + (S * (K-1))$.

4. SIMULATIONS RESULTS

In order to verify the proposed moving coloring CA design, three sets of compressive measurements are calculated using the model

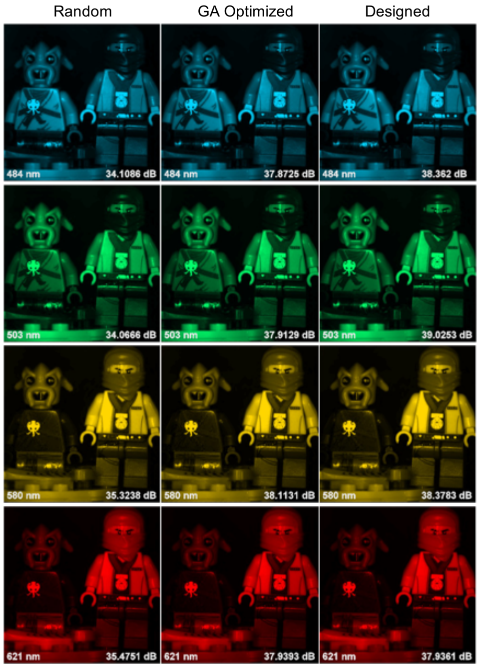

Fig. 6. Reconstruction of four spectral bands using the CASSI with color coded apertures. For each spectral band, three reconstructions from 4 measurements using the random, the genetic algorithm optimization, and the designed coded apertures are displayed.

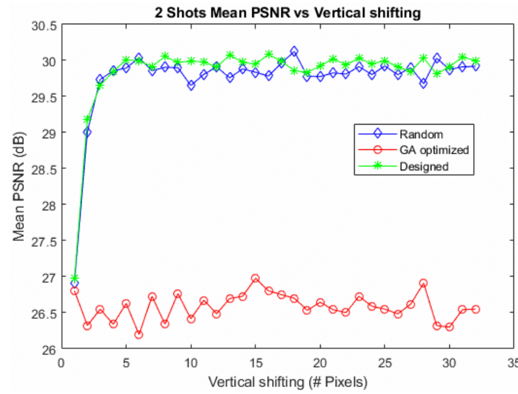

Fig. 7. Mean PSNR achieved with $K = 2$ measurements snapshots for different vertical shifting value S from 1 to 32 pixels.

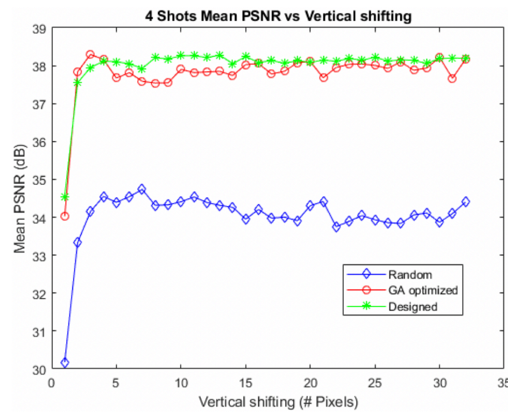

Fig. 8. Mean PSNR achieved with $K = 4$ measurements snapshots for different vertical shifting value S from 1 to 32 pixels.

in Eq. (1). The difference between each set of measurements is the modulation pattern. The first set is modulated by a random *LH*-colored CA, the second set is modulated by a genetic algorithm (GA) optimal *LH*-colored CA in literature [6], and the third set is modulated by the proposed moving colored CA pattern. In order to make a fair comparison, the three CAs are designed to be moving patterns. A test data cube $\mathbf{F}$ with 256×256 pixels of spatial resolution and $L = 8$ spectral bands is used. To construct these measurements, the spectral data cube F was acquired by a monochromator in the spectral range between $450\,nm$ and $650\,nm$. A CCD camera AVT Marlin F0033B, with 656×492 pixels and a pixel pitch size of $9.9\,\mu m$ is used. The resolution of all the three CAs is 256×256 pixels, they have the same set of 16 filters, corresponding to the design for $L = 8$ spectral bands. The transmittance, defined as the amount of light the CA let pass, depends on the number of shots acquired, using the relation $T = 1/K$. The simulations were performed for $K = 2, 4$ shots.

The GPSR algorithm is used to obtain the reconstructions of the data cube [21]. Figure 4 shows four of the eight spectral bands of the original data cube used for the simulations. Figures 5 and 6 present the respective reconstructions for two and four measurements snapshots, and for a shifting value of $S = 8$ pixels. For each spectral band, the reconstructions from the measurements acquired using the random, the GA optimized, and the designed moving color CA are presented. The spatial quality is improved when the designed coded aperture is used, as can be seen in the zoom sections of fig. 5, and it can be also noticed with the PSNR values for 2 and 4 snapshots.

In order to analyse the influence of the shifting paremeter S, a set of simulations were performed for $K = 2$ and $K = 4$ snapshots, and for the three colored CAs. The overall performance achieved by the designed colored CA is superior for 2 and 4 snapshots. Figures 7 and 8 report these results. The results correspond with results in literature [6], where random colored CAs are shown to behave closely as the optimized designs for $K = 2$. For greater number of snapshots $K = 4$, the designed colored CA from literature and the ones proposed get better reconstruction performance than random codes for all the shifting values. In general, the terms in the optimization function cover the constraints imposed by works in the state of the art, which results in the superior performance of the proposed codes in comparison with other approaches.

5. CONCLUSIONS

An optimization of a moving colored CA in compressive spectral imaging is proposed. The optimization promotes the variability and uniformity of the patterns, as well as the consideration of hardware limitations to reduce the cost of fabrication of the masks. The moving colored CA designs where simulated and the achieved improvement for the reconstruction PSNR is up to 3 dB, obtained in comparison with random and optimal LH-colored CAs in literature.

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