## AN ALGORITHM UNROLLING APPROACH TO DEEP IMAGE DEBLURRING

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#### **ABSTRACT**

While neural networks have achieved vastly enhanced performance over traditional iterative methods in many cases, they are generally empirically designed and the underlying structures are difficult to interpret. The algorithm unrolling approach has helped connect iterative algorithms to neural network architectures. However, such connections have not been made yet for blind image deblurring. In this paper, we propose a neural network architecture that advances this idea. We first present an iterative algorithm that may be considered a generalization of the traditional total-variation regularization method on the gradient domain, and subsequently unroll the half-quadratic splitting algorithm to construct a neural network. Our proposed deep network achieves significant practical performance gains while enjoying interpretability at the same time. Experimental results show that our approach outperforms many state-of-the-art methods.

# 1. INTRODUCTION

Blind image deblurring refers to the process of recovering a sharp image from its blurred observation. Among various deblurring problems, motion deblurring is an important topic because camera shaking is common during photography. Assuming a planar scene and translational camera motion, the blurring process is typically modeled as [1]:  $\mathbf{y} = \mathbf{k} * \mathbf{x} + \mathbf{n}$  where  $\mathbf{y}$  is the observed blur image,  $\mathbf{x}$  is the latent sharp image,  $\mathbf{k}$  is the blur kernel, and  $\mathbf{n}$  is noise which is often modelled as Gaussian. When  $\mathbf{k}$  is unknown the corresponding estimation problem is commonly called blind deconvolution.

The majority of existing blind motion deblurring methods rely on iterative optimization. These methods usually hinge on sparsity-inducing regularizers, either in the gradient domain [2, 3, 4, 5, 6, 7, 8, 9] or more general sparsifying transformation domains [10, 11, 12, 13]. Variants of such methods may arise indirectly from a statistical estimation perspective, such as [14, 15, 16]. While these methods are typically physically interpretable, their performance depends heavily on appropriate selection of parameters and careful design of regularizers/priors, which are difficult to determine analytically. Furthermore, hundreds of iterations are usually required to achieve an acceptable performance level, and thus these algorithms can be computationally expensive.

Complementary to the aforementioned approaches, learning based methods for determining a non-linear mapping that deblurs the image while adapting parameter choices to an underlying training image set have been developed. Principally important in this class are techniques that employ deep neural networks, including [17, 18, 19, 20]. Although they offer practical promises in certain scenarios such as video deblurring and achieves substantial performance gains in some cases, these works commonly regard neural networks as abstract function approximators. The structures of the networks are typically empirically determined and the actual functionality of the neural networks is hard to interpret.

In the seminal work of Gregor *et al.* [21], a novel technique called algorithm unrolling was proposed that provides a neural network interpretation of iterative sparse coding algorithms. Passing through the network is equivalent to executing the iterative algorithm a finite number of times, and the trained network can be naturally interpreted as a parameter optimized algorithm. In blind deblurring, Schuler *et al.* [22] employ neural networks as feature extraction modules and integrate it into a trainable deblurring system. However, the network portions are still empirical and the whole system remains hard to interpret. The link between traditional iterative algorithms and neural networks remains largely unexplored for the problem of blind deblurring.

In this paper, we develop a neural network approach for blind motion deblurring in the spirit of algorithm unrolling, called Deblurring via Algorithm Unrolling (DAU). Parameters of the algorithm are optimized by training the network and performance gains are achieved without sacrificing interpretability. We experimentally verify its superior performance, both over best-known iterative algorithms and more recent neural network approaches.

### 2. DEBLURRING VIA ALGORITHM UNROLLING

The total-variation regularization approach in the gradient domain [23] solves the following optimization problem:

$$\min_{\mathbf{k},\mathbf{g}_{1},\mathbf{g}_{2}} \frac{1}{2} \left( \|D_{x}\mathbf{y} - \mathbf{k} * \mathbf{g}_{1}\|_{2}^{2} + \|D_{y}\mathbf{y} - \mathbf{k} * \mathbf{g}_{2}\|_{2}^{2} \right)$$

$$+ \lambda_{1} \|\mathbf{g}_{1}\|_{1} + \lambda_{2} \|\mathbf{g}_{2}\|_{1} + \frac{\epsilon}{2} \|\mathbf{k}\|_{2}^{2},$$
subject to  $\mathbf{1}^{T}\mathbf{k} = 1, \quad \mathbf{k} \geq 0,$  (1)

where  $D_x \mathbf{y}, D_y \mathbf{y}$  are the partial derivates of  $\mathbf{y}$  in horizontal and vertical directions respectively, 1 is a vector whose entries are all ones, and  $\|\cdot\|_p$  denotes the  $\ell^p$  vector norm. The parameters  $\lambda_1, \lambda_2, \varepsilon$  are positive constants which balance the contributions of each term. The  $\geq$  sign acts elementwise.

In practice,  $D_x \mathbf{y}$  and  $D_y \mathbf{y}$  are usually computed using discrete filters, such as the Prewitt and Sobel filters. From this viewpoint, a straightforward generalization of (1) is to use more than two filters. We formulate the generalized optimization problem as the following:

$$\min_{\mathbf{k}, \{\mathbf{g}_i\}_i} \sum_{i=1}^{C} \left( \frac{1}{2} \| \mathbf{f}_i * \mathbf{y} - \mathbf{k} * \mathbf{g}_i \|_2^2 + \lambda_i \| \mathbf{g}_i \|_1 \right) + \frac{\epsilon}{2} \| \mathbf{k} \|_2^2,$$
subject to  $\| \mathbf{k} \|_1 = 1$ ,  $\mathbf{k} \ge 0$ , (2)

where  $\{\mathbf{f}_i\}_{i=1}^C$  is a collection of C filters that will be determined subsequently through learning.

## **Algorithm 1** Half-quadratic Splitting Algorithm

**Input:** Blurred image y, filter banks  $\{\mathbf{f}_i^l\}_{i,l}$ , positive constant parameters  $\{\zeta_i^l, \lambda_i^l\}_{i,l}, \varepsilon$ , number of iterations L.

**Output:** Estimated kernel  $\widetilde{\mathbf{k}}$ , feature maps  $\{\widetilde{\mathbf{g}}_i\}_{i=1}^C$ .

1: Initialize 
$$\mathbf{k} \leftarrow \delta; \mathbf{z}_i \leftarrow 0, i = 1, \dots, C$$
.

2: for 
$$l=1$$
 to  $L$  do

3: **for** 
$$i = 1$$
 **to**  $C$  **do**

4: 
$$\mathbf{v}_i^l \leftarrow \mathbf{f}_i^l * \mathbf{v}$$
,

4: 
$$\mathbf{y}_{i}^{l} \leftarrow \mathbf{f}_{i}^{l} * \mathbf{y},$$
5:  $\mathbf{g}_{i}^{l+1} \leftarrow \mathcal{F}^{-1} \left\{ \frac{\zeta_{i}^{l} \hat{\mathbf{k}}^{l} * \odot \hat{\mathbf{y}}_{i}^{l} + \mathbf{z}_{i}^{l}}{\zeta_{i}^{l} |\hat{\mathbf{k}}^{l}|^{2} + 1} \right\},$ 
6:  $\mathbf{z}_{i}^{l+1} \leftarrow \mathcal{S}_{\lambda_{i}^{l} \zeta_{i}^{l}} \left\{ \mathbf{g}_{i}^{l+1} \right\},$ 
7: **end for**

6. 
$$\mathbf{z}^{l+1} \leftarrow \mathcal{S}_{\mathcal{M}, \mathcal{S}_{l}} \left\{ \mathbf{g}^{l+1} \right\}$$

8: 
$$\mathbf{k}^{l+\frac{1}{3}} \leftarrow \mathcal{F}^{-1} \left\{ \frac{\sum_{i=1}^{C} \widehat{\mathbf{z}_{i}^{l+1}}^{*} \odot \widehat{\mathbf{y}_{i}^{l}}}{\sum_{i=1}^{C} \left| \widehat{\mathbf{z}_{i}^{l+1}} \right|^{2} + \epsilon} \right\},$$

9: 
$$\mathbf{k}^{l+\frac{2}{3}} \leftarrow \left[\mathbf{k}^{l+\frac{1}{3}}\right]_{+}, \mathbf{k}^{l+1} \leftarrow \frac{\mathbf{k}^{l+\frac{2}{3}}}{\|\mathbf{k}^{l+\frac{2}{3}}\|}$$

10: **end for** 

## 2.1. Efficient Minimization via Half-quadratic Splitting

A common approach to solve (1) and more generally (2) is the half-quadratic splitting algorithm [24]. The basic idea is to perform variable-splitting and then alternating minimization on the penalty function. To this end, we first cast (2) into the following approximation model:

$$\begin{aligned} \min_{\mathbf{k}, \{\mathbf{g}_{i}, \mathbf{z}_{i}\}_{i}} & \sum_{i=1}^{C} \left(\frac{1}{2} \|\mathbf{f}_{i} * \mathbf{y} - \mathbf{k} * \mathbf{g}_{i}\|_{2}^{2} \right. \\ &+ \left. \lambda_{i} \|\mathbf{z}_{i}\|_{1} + \frac{1}{2\zeta_{i}} \|\mathbf{g}_{i} - \mathbf{z}_{i}\|_{2}^{2} \right) + \frac{\epsilon}{2} \|\mathbf{k}\|_{2}^{2}, \\ \text{subject to } \|\mathbf{k}\|_{1} &= 1, \quad \mathbf{k} \geq 0, \end{aligned} \tag{3}$$

by introducing auxiliary variables  $\{\mathbf{z}_i\}_{i=1}^C$  and constant parameters  $\zeta_i$ ,  $i = 1, \dots, C$ . We then alternately minimize over  $\{\mathbf{x}_i\}_i, \{\mathbf{z}_i\}_i$  and **k** and iterate until convergence.

In practice, a common strategy is to alter the parameters per iteration [24, 7, 23, 9]. In numerical analysis and optimization, this strategy is formally called continuation method. By adopting this strategy, we choose different parameters  $\{\zeta_i^l, \lambda_i^l\}_{i.l}$  across the iterations l. We take this idea one step further by varying the filters  $\{\mathbf{f}_i\}_i$  as well. The complete algorithm is summarized in Algorithm 1. We let  $\hat{\cdot}$  denote the Discrete Fourier Transform (DFT) and  $\mathcal{F}^{-1}$  be the inverse DFT. We define  $[x]_{+} = \max\{x, 0\}, \delta$  is the unit impulse function, \* is the complex conjugation and ⊙ is the Hadamard product operator. Finally,  $S_{\lambda}(\cdot)$  is the softthresholding operator:  $S_{\lambda}(x) = \operatorname{sgn}(x) \cdot \max\{|x| - \lambda, 0\}.$ Operations matrices and vectors act elementwise.

After algorithm 1 converges, we obtain the estimated feature maps  $\{\widetilde{\mathbf{g}}_i\}_i$  and the estimated kernel k. When k approximates  $\mathbf{k}$ ,  $\widetilde{\mathbf{g}_i}$  should approximate  $\mathbf{f}_i * \mathbf{x}$ . Therefore, we retrieve the image x by solving the following optimization problem:

$$\widetilde{\mathbf{x}} \leftarrow \arg\min_{\mathbf{x}} \frac{1}{2} \left\| \mathbf{y} - \widetilde{\mathbf{k}} * \mathbf{x} \right\|_{2}^{2} + \sum_{i=1}^{C} \frac{\eta_{i}}{2} \left\| \mathbf{f}_{i} * \mathbf{x} - \widetilde{\mathbf{g}}_{i} \right\|_{2}^{2}$$

$$= \mathcal{F}^{-1} \left\{ \frac{\widehat{\widetilde{\mathbf{k}}}^{*} \odot \widehat{\mathbf{y}} + \sum_{i=1}^{C} \eta_{i} \widehat{\mathbf{f}}_{i}^{*} \odot \widehat{\widetilde{\mathbf{g}}}_{i}}{\left| \widehat{\widetilde{\mathbf{k}}} \right|^{2} + \sum_{i=1}^{C} \eta_{i} \left| \widehat{\mathbf{f}}_{i} \right|^{2}} \right\}, \tag{4}$$

where  $\eta_i$ 's are positive constant parameters.

### 2.2. Network Construction via Algorithm Unrolling

Each step of Algorithm 1 is in analytic form and can be implemented using a series of basic functional operations. Therefore, each iteration of Algorithm 1 admits a layered representation, and repeating it L times yields an L-layer neural network (assuming L iterations). For notational brevity, we concatenate the parameters in each layer and let  $\mathbf{f}^l$  $(\mathbf{f}_{i}^{l})_{i=1}^{C}, \zeta^{l} = (\zeta_{i}^{l})_{i=1}^{C}, \lambda^{l} = (\lambda_{i}^{l})_{i=1}^{C} \text{ and } \eta = (\eta_{i})_{i=1}^{C}.$  We also concatenate  $\mathbf{y}_i^l$ 's,  $\mathbf{z}_i^l$ 's and  $\mathbf{g}_i^l$ 's by letting  $\mathbf{y}^l = (\mathbf{y}_i^l)_{i=1}^C$ ,  $\mathbf{z}^l = (\mathbf{z}_i^l)_{i=1}^C$  and  $\mathbf{g}^l = (\mathbf{g}_i^l)_{i=1}^C$ , respectively.

To handle large blur kernels, we alter the size of the filter banks  $\{\mathbf{f}_i\}_i$  in different layers in the following way:

size of 
$$\mathbf{f}_i^1 > \text{ size of } \mathbf{f}_i^2 > \text{ size of } \mathbf{f}_i^3 > \dots$$

so that high-level representations features are captured first, and fine details emerge in later iterations. To facilitate training, we produce large filters by cascading small  $3 \times 3$  filters, following the same principle as [25]. Formally speaking, we set  $\mathbf{f}_i^L = \mathbf{w}_{i1}^L$  where  $\{\mathbf{w}_{i1}^L\}_{i=1}^C$  is a collection of  $3 \times 3$  filters, and recursively obtain  $\mathbf{f}_i^l$  by:  $\mathbf{f}_i^l \leftarrow \sum_{j=1}^C \mathbf{w}_{ij}^l * \mathbf{f}_j^{l+1}$ . Using this representation, we obtain the network structure in Fig. 1. The parameters  $\{\mathbf{w}^l, b^l, \lambda^l\}_{l=1}^L$  will be learned from the training data, as explained in the next Section.

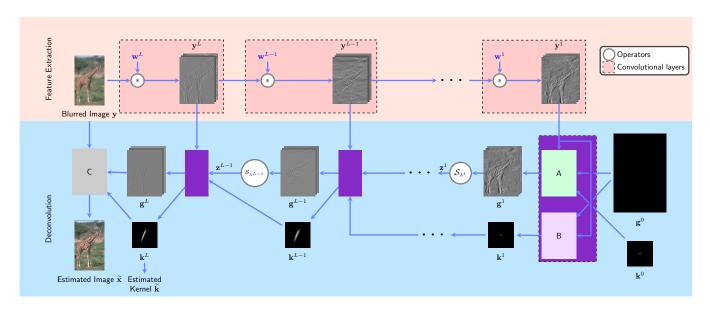


Fig. 1. Structure of the deep network constructed by algorithm unrolling and cascaded filtering. Block A and B implements Step 5 and Step 8 in Algorithm 1 respectively, while block C implements (4). A diagram representation can be found at http://signal.ee.psu.edu/diagram.pdf. Intermediate data (hidden layers) on the trained network are also shown. It can be observed that, as l increases, more details are extracted in  $\mathbf{g}^l$  and finer kernel coefficients are recovered. The parameters that will be learned from real datasets are colored in blue.

**Table 1**. Quantitative comparison averaged over 200 images from the BSDS500 [27] set and 4 linear kernels. The RMSE values are computed over kernels. Best scores are in bold.

Metrics	DAU	[23]	[26]
PSNR (dB)	27.21	22.23	25.23
ISNR (dB)	4.36	2.06	1.88
SSIM	0.88	0.76	0.81
$RMSE (\times 10^{-3})$	2.21	5.21	_

#### 3. EXPERIMENTS

**Training**: We use the training and validation portions from the Berkeley Segmentation Dataset (BSDS500) [27] as training images. The linear motion kernels are generated by uniformly sampling 16 angles in  $[0,\pi]$  and 16 lengths in [5,20]. The images are convolved with each kernel and white Gaussian noise with standard deviation 0.01 (suppose the image intensity is in [0,1]) is added. For each blurred image  $\mathbf{y}_t^{\text{train}}(t=1,\ldots,T)$ , we let the corresponding sharp image and kernel be  $\mathbf{x}_t^{\text{train}}$  and  $\mathbf{k}_t^{\text{train}}$ , respectively. We re-parametrize  $\lambda_i^l$  in step 6 of Algorithm 1 by letting  $b_i^l = \lambda_i^l \zeta_i^l$  and let  $b^l = (b_i^l)_{i=1}^C$ ,  $l = 1,\ldots,L$ . The network outputs  $\widetilde{\mathbf{x}}_t$ ,  $\widetilde{\mathbf{k}}_t$  corresponding to  $\mathbf{y}_t^{\text{train}}$  depend on the network parameters  $\mathbf{w}^l$ ,  $b^l$ ,  $\lambda^l$ ,  $l = 1,2,\ldots,L$ , and  $\widetilde{\mathbf{x}}_t$  further depends on  $\eta$ . We train the network to determine those parameters by minimizing:

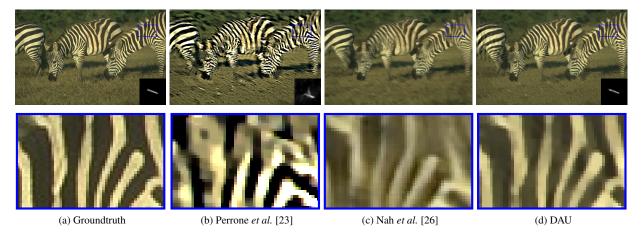
**Table 2.** Quantitative comparison on nonlinear motion (average over 4 images and 8 kernels from [28]). The RMSE values are computed over kernels. Best scores are in bold.

	DAU	[23]	[26]	[19]	[20]
PSNR (dB)	27.15	26.79	24.51	23.18	26.75
ISNR (dB)	3.79	3.63	1.35	0.02	3.59
SSIM	0.88	0.89	0.81	0.81	0.89
$\overline{\text{RMSE} (\times 10^{-3})}$	3.87	3.83	-	_	3.98

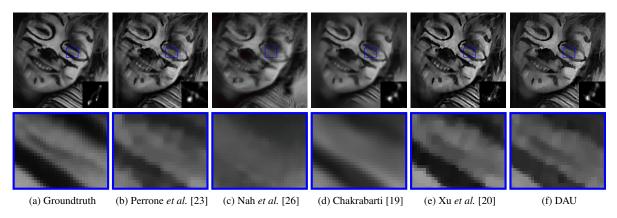
$$\begin{split} \min_{\left\{\mathbf{w}^{l},b^{l},\lambda^{l}\right\}_{l=1}^{L},\eta} \sum_{t=1}^{T} \mathrm{MSE}\left(\mathbf{x}_{t}^{\mathrm{train}} - \widetilde{\mathbf{x}}_{t}\left(\left\{\mathbf{w}^{l},b^{l},\lambda^{l}\right\}_{l=1}^{L},\eta\right)\right) \\ + \kappa \, \mathrm{MSE}\left(\mathbf{k}_{t}^{\mathrm{train}} - \widetilde{\mathbf{k}}_{t}\left(\left\{\mathbf{w}^{l},b^{l},\lambda^{l}\right\}_{l=1}^{L}\right)\right), \\ \mathrm{subject} \ \mathrm{to} \ b_{i}^{l} \geq 0, \ \lambda_{i}^{l} \geq 0, \quad l=1,\ldots,L, i=1,\ldots,C, \end{split}$$

where  $\kappa>0$  is a constant parameter which is fixed to  $10^5$  and MSE is the Mean Squared Error. We choose L=10 and C=16 by cross-validation. The minimization is performed by stochastic gradient descent, followed by a gradient projection step to enforce the non-negative constraints. We use the Adam [29] solver for faster training. The learning rate is set to  $1\times 10^{-3}$  initially and decayed by a factor of 0.9 per epoch. We terminate training after 20 epochs. The parameters  $\{\lambda_i^l\}_{i,l}$  are initialized to zeros,  $\{b_i^l\}_{i,l}$  to 1, and  $\{\eta_i\}_i$  to 20, respectively. The weights are initialized according to [30]. **Evaluation**: We use 200 images from the test portion from

**Evaluation**: We use 200 images from the test portion from the BSDS500 dataset [27] as test images. We randomly



**Fig. 2**. Qualitative comparisons on the BSDS500 dataset [27]. The blur kernels are placed at the right below corner. DAU recovers the kernel at higher accuracy and therefore the estimated images are more faithful to the groundtruth.



**Fig. 3**. Qualitative comparisons on the dataset from [28]. The blur kernels are placed at the right below corner. DAU generates fewer artifacts and preserves more details than competing state of the art methods.

choose angles from  $[0,\pi]$  and lengths from [5,20] to generate 4 test kernels. We compare with state-of-the art algorithms, Perrone et~al.~[23] and Nah et~al.~[26], which are representatives of iterative algorithms and deep-learning approaches. We assess the performance using four commonly used evaluation metrics: Peak Signal-to-Noise-Ratio (PSNR), Improvement in Signal-to-Noise-Ratio (ISNR), Structural Similarity Index (SSIM) [31], and Root-Mean-Square Error (RMSE) between the estimated kernel and the groundtruth kernel. The average scores are in Table 1. Clearly, DAU outperforms state-of-the art algorithms by a significant margin.

Fig. 2 shows example images and kernels for a qualitative comparison. Although Perrone *et al.*'s method can roughly infer the directions of the blur kernels, the recovered coefficients are unsatisfactory. As a result, the recovered image contains clearly visible artifacts. Nah *et al.*'s method effectively removes most of the blurs, but blurring artifacts still remain locally and the details are not faithfully preserved. In contrast, the kernel recovered by DAU is closer to the ground truth and hence leads to a more accurate estimated image.

Additionally, we compare the performance of various methods on deblurring under non-linear motion kernels, which is a more realistic scenario as discussed in [28]. We collect training kernels by interpolating the paths provided by [32] and created by ourselves in the same manner. We further augment these kernels by scaling and rotations. We use the standard image set from [28] (comprising 4 images and 8 kernels) as the test set. The average scores are presented in Table 2. Again DAU outperforms state-of-the-art methods. A visual example is shown in Fig. 3,

# 4. CONCLUSION

We propose a neural network deblurring architecture built by unrolling an iterative algorithm. We show how a generalized TV-regularized algorithm can be recast into a neural network, and train it to optimize the parameters. Unlike most existing deblurring networks, our work has the benefit of interpretability, while exhibiting performance benefits that are shared with modern deep-nets and exceed state of the art performance.

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