Low-Complexity Compressive Analysis in Sub-Eigenspace for ECG Telemonitoring System

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***Abstract*—Compressive sensing (CS) is attractive in long-term electrocardiography (ECG) telemonitoring to extend life-time for resource-limited wireless wearable sensors. Moreover, health monitoring has emphasized the need for edge computing to process real-time data without the bandwidth costs. However, the reconstructed analysis (RA) and the compressed learning (CL) frameworks have extremely high memory and computational overhead, cost-prohibitive for online usage at resource-constrained edge device. In this paper, to efficiently analyze the received CS measurements with different levels of compression, we propose a low-complexity framework of Compressive Analysis in Sub-Eigenspace (CA-SE) based on subspace-based representation. The dictionary is used for sifting the sub-eigen information from the CS measurements online, and it is built by eigenspace learning offline. The framework can reduce the memory overhead with a single light-weight machine learning model and multiple small filter matrices, and the computational complexity with sifting by matrix-vector product rather than sparse coding. CA-SE is implemented in ECG-based atrial fibrillation detection. The memory overhead of CA-SE is 13 and 39 times fewer compared with RA and CL, respectively, and the computational complexity of CA-SE is 42 and 10 times fewer compared with RA and CL, respectively.**

Keywords— Compressive sensing, edge computing, compressive analytics, sub-eigenspace, subspace-based representation

I. Introduction

Long-term patient monitoring, especially outside the hospital setting, offers the potential to substantially improve patient health, quality of life, and outcomes [1]. This potential depends on two aspects: 1) the ability to acquire signals that are informative with respect to patient stage; and 2) the ability to make relevant inference from such signals. Since the electrocardiography (ECG) signal recorded from the electrical activity of the heart over a period of time has been utilized for diagnosis for many diseases, the ECG telemonitoring [2] is recognized as a promising technique to realize telemedicine.

As wireless wearable biomedical sensor nodes are known to be resource-limited, it is a crucial problem to reduce the signal acquisition on these sensing system and enhance the energy efficiency of data transmission. Compressive sensing (CS) is an emerging technique combining both sampling and compression through random projection [3], which enables sub-Nyquist sampling and low-energy data reduction, resulting in life-time extension of the sensor node [4] and making the technique especially attractive in telemonitoring systems [5].

Moreover, health monitoring has emphasized the need for edge computing [6], leveraging the benefits of analyzing real-time data, without the bandwidth costs that come with sending the data offsite (*i.e.*, to the cloud or the data center). However, it remains a grand challenge to efficiently analyze the received CS measurements with different levels of compression online at resource-constrained edge. On the one hand, in reconstructed analysis (RA) framework as shown in Fig. 1(a), signal reconstruction and inference in high dimensional original space result in high computational complexity. On the other hand, in compressed learning (CL) framework [7], [8] as shown in Fig. 1(b), multiple machine learning (ML) models in measurement domain lead to large memory overhead.

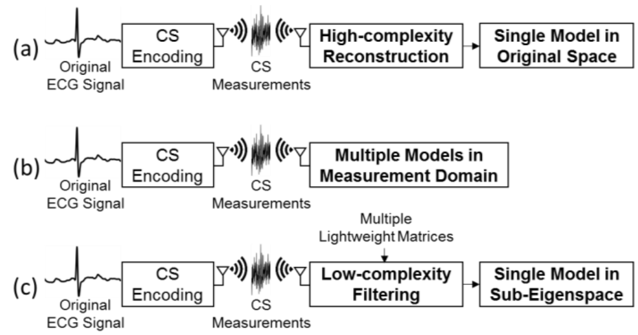

Fig. 1. (a) In RA, signal reconstruction and inference in original space result in high computational complexity. (b) In CL, multiple ML models in measurement domain lead to large memory overhead. (c) The proposed CA-SE takes care of both memory and computational efficiency.

In this paper, to efficiently analyze the received CS measurements, we propose a low-complexity framework of Compressive Analysis in Sub-Eigenspace (CA-SE), including two stages: I) In off-line stage, the dictionary is constructed through eigenspace learning and the ML model is trained in sub-eigenspace. II) In on-line stage as shown in Fig. 1(c), based on subspace-based representation, a low-complexity sifting algorithm is proposed using the pre-trained filter matrix. Our specific contributions are as follows: I) CA-SE can dramatically reduce the memory overhead with a single light-weight ML model and multiple small filter matrices. II) CA-SE can significantly reduce the computational cost with sifting by matrix-vector product rather than sparse coding. III) This framework is implemented in ECG-based atrial fibrillation (AF) detection. The total required number of parameters of CA-SE is reduced 13.3 and 39.3 times compared with RA and CL, respectively. On the other hand, the average required multiplications of CA-SE are reduced 42 and 10 times compared with RA and CL, respectively.

II. BACKGROUND

A. *Compressive Sensing* [3]

CS is a novel technique that can be used to acquire signals with fewer measurements than Nyquist rate to estimate sparse signals, which can be modeled in matrix form as

$$\hat{\mathbf{x}} = \mathbf{\Phi x}, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^N$ is N-sample signal; $\hat{\mathbf{x}} \in \mathbb{R}^M$ is M-compressed measurements; and $\mathbf{\Phi} \in \mathbb{R}_{M,N}$ is the CS measurement matrix whose entries are independent identically distributed (i.i.d) samples. Although sensing can incur very little energy, the reconstruction of $\mathbf{x}$ from $\hat{\mathbf{x}}$ can be costly.

B. *Related Works*

In reconstructed analysis (RA), original signal is reconstructed before processing. The challenge is that both sparse signal reconstruction and ML inference in high dimensional original space can be extremely energy-intensive and time-consuming [9].

In order to prevent signal reconstruction, in compressed learning (CL) [7], [8], the classification is directly performed in the measurement domain, and the learnability is remained because the distances between the points are preserved by the Johnson-Lindenstrauss lemma (JLL) [10]. To support different levels of compression, multiple ML models are required and thus leads to large memory overhead.

III. PROPOSED COMPRESSIVE ANALYSIS IN SUB-EIGENSPACE (CA-SE) FRAMEWORK

To take care of both memory and computational efficiency, the CA-SE framework is proposed as shown in Fig. 2. In off-line stage, we aim to construct the dictionary and train a single ML model in sub-eigenspace. In on-line stage, based on subspace-based representation for signal, we aim to develop a low-complexity sifting algorithm using the pre-trained filter matrix.

A. *Learning in Sub-Eigenspace*

Consider a dataset of n vectors $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \cdots \mathbf{x}_n]$, $\mathbf{x}_i \in \mathbb{R}^N$. We aim to find a reduced space such that the projected vectors have 1) maximum variance (minimum projection error equivalently), and 2) learnable features (classification capability). Hence, we find the basis preserving the information of the signal for reconstruction and classification through learning the eigenspace of the dataset.

The eigenspace can be derived by computing the covariance matrix $\mathbf{S} \in \mathbb{R}_{N,N}$ and solving the eigenvalue decomposition problem as follows,

$$\mathbf{S} = \frac{1}{n}(\mathbf{X} - \bar{\mathbf{x}}\mathbf{h})(\mathbf{X} - \bar{\mathbf{x}}\mathbf{h})^T, \tag{2}$$

where $\bar{\mathbf{x}} \in \mathbb{R}^N$ is the mean vector from each row of $\mathbf{X}$ and $\mathbf{h}$ is a $1 \times n$ vector of all 1s.

$$\mathbf{SW} = \mathbf{W\Lambda}, \tag{3}$$

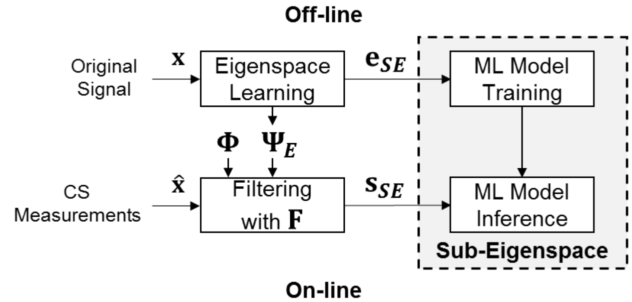

Fig. 2. The proposed Compressive Analysis in Sub-Eigenspace (CA-SE) framework.

where $\mathbf{\Lambda} \in \mathbb{R}_{N,N}$ is a diagonal matrix with eigenvalue in descending order, and $\mathbf{W} \in \mathbb{R}_{N,N}$ is the corresponding eigenvector matrix of $\mathbf{S}$.

Based on $\mathbf{W}$, the columns in dictionary $\mathbf{\Psi}_E \in \mathbb{R}_{N,k_E}$ are formed by the eigenvectors corresponding to the k_E largest eigenvalues for signal reconstruction as follows,

$$\mathbf{\Psi}_E = \mathbf{W}(:,1{:}\,k_E), \tag{4}$$

where k_E represents the intrinsic dimension of the signal, also the dimension of the eigenspace, decided by scree-plot. After that, the sub-eigenspace sub-dictionary $\mathbf{\Psi}_{SE} \in \mathbb{R}_{N,k_{SE}}$ is further formed with the first k_{SE} columns in $\mathbf{\Psi}_E$ for classification as follows,

$$\mathbf{\Psi}_{SE} = \mathbf{\Psi}_E(:,1{:}\,k_{SE}), \tag{5}$$

where k_{SE} represents the dimension of sub-eigenspace, decided by classification performance. Next, we apply the matrix $\mathbf{\Psi}_{SE}$ to project the original raw data $\mathbf{x} \in \mathbb{R}^N$ to the sub-eigen information $\mathbf{e}_{SE} \in \mathbb{R}^{k_{SE}}$, and we call the projected space the sub-eigenspace.

$$\mathbf{e}_{SE} = \mathbf{\Psi}_{SE}{}^T(\mathbf{x} - \bar{\mathbf{x}}). \tag{6}$$

The sub-eigen information $\mathbf{e}_{SE}$ is first extracted with the sub-eigenspace sub-dictionary $\mathbf{\Psi}_{SE}$ in off-line stage, and we further implement ML model training in sub-eigenspace.

B. *Subspace-based Signal Representation with Dictionary* $\mathbf{\Psi}_E$

After eigenspace learning, subspace-based representation is proposed. The reconstruction error is small enough if the dimension k_E chose is large enough. On the other hand, $\mathbf{\Psi}_E$ is orthonormal. Therefore, signal can be represented by the information in low dimensional eigenspace with the basis $\mathbf{\Psi}_E$. Furthermore, the data in eigenspace can be represented by the information from two subspaces, the sub-eigenspace and the complementary eigenspace corresponding to the remaining k_{CE} eigenvalues ($k_{CE} = k_E - k_{SE}$). Therefore, the signal can be represented as follows,

$$\tilde{\mathbf{x}} \cong \mathbf{\Psi}_E\boldsymbol{\alpha} = [\mathbf{\Psi}_{SE} \quad \mathbf{\Psi}_{CE}]\begin{bmatrix}\mathbf{s}_{SE}\\ \mathbf{s}_{CE}\end{bmatrix}, \tag{7}$$

where $\tilde{\mathbf{x}} = \mathbf{x} - \bar{\mathbf{x}}$ is the centered data. Equation (7) is called subspace-based signal representation. $\mathbf{\Psi}_E$ is composed of $\mathbf{\Psi}_{SE}$ and complementary eigenspace sub-dictionary $\mathbf{\Psi}_{CE} \in \mathbb{R}_{N,k_{CE}}$.

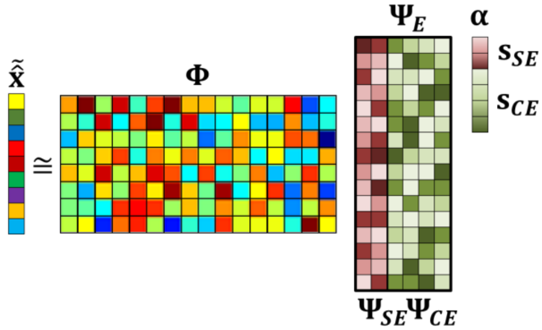

Fig. 3. Centered CS measurements $\tilde{\hat{\mathbf{x}}}$ can be represented with the measurement matrix $\boldsymbol{\Phi}$ and the dictionary $\boldsymbol{\Psi}_E$.

$\mathbf{s}_{SE} \in \mathbb{R}^{k_{SE}}$ and $\mathbf{s}_{CE} \in \mathbb{R}^{k_{CE}}$ are the coefficients corresponding to $\boldsymbol{\Psi}_{SE}$ and $\boldsymbol{\Psi}_{CE}$, representing the recovered sub-eigen information and the recovered complementary information.

The CS can be modeled in matrix form in (1). Combing (1) with (7), we can derive:

$$\tilde{\hat{\mathbf{x}}} \cong \boldsymbol{\Phi}\boldsymbol{\Psi}_E\boldsymbol{\alpha} = \boldsymbol{\Theta}_E\boldsymbol{\alpha}, \\ \boldsymbol{\Theta}_E = [\boldsymbol{\Theta}_{SE} \quad \boldsymbol{\Theta}_{CE}], \boldsymbol{\alpha} = [\mathbf{s}_{SE} \quad \mathbf{s}_{CE}]^T, \tag{8}$$

where $\tilde{\hat{\mathbf{x}}} = \hat{\mathbf{x}} - \boldsymbol{\Phi}\bar{\mathbf{x}}$ is the centered CS measurements, and $\boldsymbol{\Theta}_E \in \mathbb{R}_{M,k_E}$ is composed of $\boldsymbol{\Theta}_{SE} \in \mathbb{R}_{M,k_{SE}}$ ($\boldsymbol{\Theta}_{SE} = \boldsymbol{\Phi}\boldsymbol{\Psi}_{SE}$) and $\boldsymbol{\Theta}_{CE} \in \mathbb{R}_{M,k_{CE}}$ ($\boldsymbol{\Theta}_{CE} = \boldsymbol{\Phi}\boldsymbol{\Psi}_{CE}$). Therefore, the centered CS measurements $\tilde{\hat{\mathbf{x}}}$ can be represented with the measurement matrix $\boldsymbol{\Phi}$ and the dictionary $\boldsymbol{\Psi}_E$, as shown in Fig. 3.

C. Filtering Matrix $\mathbf{F}$ for Recovered Sub-Eigen Information

Based on subspace-based representation, we aim to develop a low-complexity sifting algorithm, which sifts the recovered sub-eigen information from the centered CS measurements with different C_r, using the pre-trained filter matrix. To solve for $\boldsymbol{\alpha}$ in (8), we have to solve the following optimization problem:

$$\min_{\boldsymbol{\alpha}} \left\|\tilde{\hat{\mathbf{x}}} - \boldsymbol{\Theta}_E\boldsymbol{\alpha}\right\|_2^2. \tag{9}$$

The problem can be solved with a least-square (LS) approach. We thus have the following solution:

$$\boldsymbol{\alpha} = {\boldsymbol{\Theta}_E}^{\dagger}\tilde{\hat{\mathbf{x}}}, \tag{10}$$

where ${\boldsymbol{\Theta}_E}^{\dagger}$ is the Moore-Penrose inverse of $\boldsymbol{\Theta}_E$. Furthermore, according to [11], the particular formulate for the Moore-Penrose inverse of a columnwise partitioned matrix is:

$${\boldsymbol{\Theta}_E}^{\dagger} = [\boldsymbol{\Theta}_{SE} \quad \boldsymbol{\Theta}_{CE}]^{\dagger} = \left[(\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{CE}}\boldsymbol{\Theta}_{SE})^{\dagger} \quad (\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{SE}}\boldsymbol{\Theta}_{CE})^{\dagger}\right], \tag{11}$$

where the orthogonal projectors specified as:

$$\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{CE}} = \mathbf{I}_M - \boldsymbol{\Theta}_{CE}{\boldsymbol{\Theta}_{CE}}^{\dagger}, \\ \text{and } \mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{SE}} = \mathbf{I}_M - \boldsymbol{\Theta}_{SE}{\boldsymbol{\Theta}_{SE}}^{\dagger}. \tag{12}$$

Combing (8), (10), and (11), by comparing the coefficients in $\boldsymbol{\alpha}$,

$$\boldsymbol{\alpha} = \left[(\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{CE}}\boldsymbol{\Theta}_{SE})^{\dagger}\tilde{\hat{\mathbf{x}}} \quad (\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{SE}}\boldsymbol{\Theta}_{CE})^{\dagger}\tilde{\hat{\mathbf{x}}}\right] = [\mathbf{s}_{SE} \quad \mathbf{s}_{CE}]^T, \tag{13}$$

TABLE I
EXPERIMENTAL SETTINGS.

Data Parameters	
Source	NTUH ICU ECG data
Sampling Frequency	512 Hz
Input Dimension (*N*)	512 (1 sec. ECG)
Number of Classes	2 (Normal/AF)
Number of training / inference data	2500 / 1000 each class
CS Sampling Matrix	
Type	Random Bernoulli
Compression Ratio (C_r)	[0.7 to 0.2]
Machine Learning Model	
Type	RBF Kernel SVM by LIBLVM
Cost (C) Search Range	[1, 10, 100]
Gamma (γ) Search Range	[10^{-3} to 10^{2}]
Cross-validation	5-fold
Simulation	
Simulator	PYTHON
Trails	100

we can derive $\mathbf{s}_{SE}$ as follows:

$$\mathbf{s}_{SE} = (\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{CE}}\boldsymbol{\Theta}_{SE})^{\dagger}\tilde{\hat{\mathbf{x}}} = \mathbf{F}\tilde{\hat{\mathbf{x}}}, \tag{14}$$

where $\mathbf{F} \in \mathbb{R}_{k_{SE},M}$ ($\mathbf{F} = (\mathbf{P}^{\perp}_{\boldsymbol{\Theta}_{CE}}\boldsymbol{\Theta}_{SE})^{\dagger}$) is the filter matrix which sifts the recovered sub-eigen information $\mathbf{s}_{SE}$ from the centered CS measurement $\tilde{\hat{\mathbf{x}}}$. The recovered sub-eigen information $\mathbf{s}_{SE}$ is first sifted with the filter matrix $\mathbf{F}$ in on-line stage, and we further implement ML model inference in sub-eigenspace.

IV. Numerical Experiments and Complexity Analysis

The learning performance, memory and computational overhead in on-line stage are compared between this work (CA-SE), RA, and CL. We use a case study of AF [12] detection to validate the benefits of our proposed algorithm. The simulation setup is in TABLE I. The data were recorded from the intensive care unit (ICU) of stroke in National Taiwan University Hospital (NTUH), and visually checked and labeled by doctors. ECG samples are applied random projection to obtain the CS measurements with compression ratio $C_r = 0.7 - 0.2$. Then, the CS measurements are analyzed in different frameworks. Model selection for SVM (by LIBSVM [13]) is performed by cross-validated grid-search.

A. Size Determination of Dictionary $\boldsymbol{\Psi}_E$

The dictionary is learnt offline by eigenspace learning. To ensure the dictionary spans the vector space the signals lie within, the dimension of eigenspace k_E is set to fulfill the following criteria:

$$\frac{\sum_{i=1}^{i=k_E}\lambda_i}{\sum_{i=1}^{i=N}\lambda_i} > \beta, \tag{15}$$

where λ_i is the eigenvalue of the ith principal component, which represents the data variance. The percentage of the accumulated eigenvalue needs to be greater than $\beta = 0.995$ as shown in Fig. 4(a). Therefore, we choose $k_E = 83$ for $\boldsymbol{\Psi}_E$. The dimension of sub-eigenspace k_{SE} is decided by the classification performance as depicted in Fig. 4(b), and is set 35 for highest learning accuracy. Therefore, the size of dictionary $\boldsymbol{\Psi}_E$ is: $k_E = 83$, $k_{SE} = 35$, $k_{CE} = 48$.

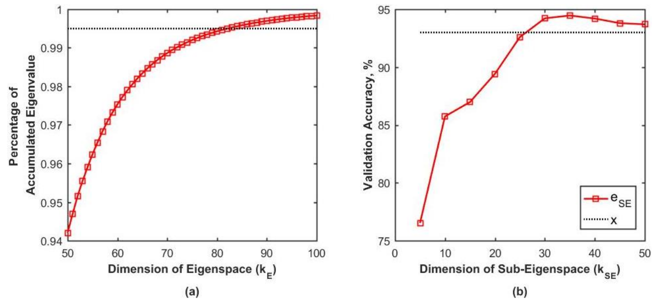

Fig. 4. ECG signal from ICU dataset: (a) Accumulated eigenvalues for $k_E \in [50,100]$. (b) Classification accuracy for $k_{SE} \in [5,50]$.

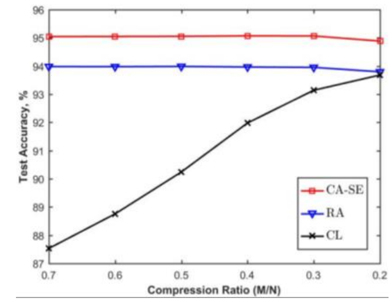

Fig. 5. Classification accuracy for compression ratio $\in [0.7,0.2]$.

B. Learning Performance Evaluation

Fig. 5 shows the average classification accuracy with different levels of compression ($C_r = 0.7 - 0.2$). The proposed CA-SE framework has the highest accuracy compared with both RA and CL. This improvement benefits from the noise mitigation in sub-eigenspace.

C. Analysis of Memory and Computational Overhead

In this subsection, we seek to compare the overhead in analyzing the CS measurements with different C_r in on-line stage. TABLE II presents the memory requirement and the computational complexity of different frameworks. Under the experimental settings, TABLE III shows the total required number of parameters and the average required multiplications in different C_r. The parameter N is the original dimension; M is the measurement length; k_{SE} is the dimension of sub-eigenspace; n_{SV} is the number of support vectors in SVM; and K is the signal sparsity in noiseless scenario.

In RA, a single big reconstruction matrix ($M_{max}N = 358 \times 512 \cong 0.2\text{M}$) multiplied by the measurement matrix and the sparsifying matrix is required, where M_{max} is the measurement length in maximum $C_r = 0.7$. Also, a single heavyweight ML model in original domain ($n_{SV}N = 3515 \times 512 \cong 1.8\text{M}$) is required. In CL, multiple ML models in measurement domain ($\sum_{C_r}(n_{SV}M) \cong 5.9\text{M}$) are required. Furthermore, n_{SV} increases (*i.e.*, $n_{SV} = 4300$ in $C_r = 0.5$) because of learnability degradation in measurement domain. In the proposed CA-SE, multiple small filter matrices $\mathbf{F}$ ($\sum_{C_r}(Mk_{SE}) \cong 0.05\text{M}$) are required. Also, a single lightweight ML model in sub-eigenspace ($n_{SV}k_{SE} = 2812 \times 35 \cong 0.1\text{M}$) is required. In addition, n_{SV} decreases due to learnability improvement in sub-eigenspace with noise reduction. Therefore, the total required number of parameters of CA-SE (0.15M) is 13.3 and 39.3 times fewer compared with RA (2M) and CL (5.9M), respectively.

Most works in RA adopted orthogonal matching pursuit (OMP) to realize the CS reconstruction engine [14] because OMP has less complexity and is more feasible for implementation compared with convex optimization based sparse coding algorithm. The complexity of OMP is at least $\mathcal{O}(MNK)$ [15]; therefore, the average required multiplications are $\sum_{C_r}\mathcal{O}(MNK)/6 = \sum_{C_r} M \times 512 \times 20/6 \cong 2.4\text{M}$. On the contrary, the sifting algorithm in the proposed CA-SE only involves matrix-vector product with the filter matrix $\mathbf{F}$; therefore, the average required multiplications are $\sum_{C_r}\mathcal{O}(Mk_{SE})/6 = \sum_{C_r} M \times 35/6 \cong 8\text{K}$. Note that the filter matrix $\mathbf{F}$ can be precomputed offline referring to (14). On the other hand, the average required multiplications for SVM in RA, CL, and CA-SE are $\mathcal{O}(n_{SV}N) = 3515 \times 512 \cong 1.8\text{M}$, $\sum_{C_r}\mathcal{O}(n_{SV}M)/6 \cong 1\text{M}$, and $\mathcal{O}(n_{SV}k_{SE}) = 2812 \times 35 \cong 0.1\text{M}$, repectively. Therefore, the average required multiplications of CA-SE (0.1M) is 42 and 10 times fewer compared with RA (4.2M) and CL (1M), respectively.

TABLE II
MEMORY AND COMPUTATIONAL OVERHEAD OF DIFFERENT FRAMEWORKS.

Framework	Memory Requirement	Computational Complexity
RA	$M_{max}N + n_{SV}N$	$MNK + n_{SV}N$
CL	$\sum_{C_r}(n_{SV}M)$	$n_{SV}M$
Proposed CA-SE	$\sum_{C_r}(Mk_{SE}) + n_{SV}k_{SE}$	$Mk_{SE} + n_{SV}k_{SE}$

TABLE III
REQUIRED # PARAMETERS AND MULTIPLICATIONS.

Framework	Total Required # Parameters	Average Required Multiplications
RA	2M	4.2M
CL	5.9M	1M
CA-SE	0.15M	0.1M

V. CONCLUSIONS

In this work, we propose a novel compressive analytics in sub-eigenspace framework based on CS. The proposed CA-SE outperforms RA, CL in both memory overhead and computational complexity. The framework is implemented in ECG-based AF detection. The total required number of parameters of CA-SE is reduced 13.3 and 39.3 times compared with RA and CL, respectively. On the other hand, the average required multiplications of CA-SE are reduced 42 and 10 times compared with RA and CL, respectively.

VI. ACKNOWLEDGEMENT

This research was supported in part by the Ministry of Science and Technology of Taiwan (MOST 107-2633-E-002-001), National Taiwan University (NTU-107L104039), and Intel Corporations.

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