LOW POWER PILOT AIDED SUB-SAMPLE BASED CHANNEL ESTIMATION FOR MMWAVE CELLULAR SYSTEMS

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ABSTRACT

The fast temporal changes of a millimeter wave channel necessitate frequent estimation of the channel. Power reduction techniques for the channel estimation process for ultra-wideband 5G systems are highly desirable. High speed analog-to-digital converters for the wideband data conversion and high speed baseband processing of the Nyquist rate digital samples are the main contributors to high power consumption. This work utilizes the subsequence properties of Zadoff Chu sequences and presents a training based channel estimation algorithm that can operate at a fraction of the symbol rate and thus save power. The algorithm also provides a framework for tradeoff between channel estimation performance and computational complexity. This can allow a receiver to go into power saving mode during high signal to noise ratio channel estimation. Our analysis and simulation results show that our sub-Nyquist based approach achieves maximum likelihood performance at full rate sequence for a single path channel model.

Index Terms— Channel estimation, mmWave, pilot design

1. INTRODUCTION

Millimeter wave (mmWave) communications can fulfill the ultra-high speed data needs of 5G [1]. Estimation of ultra-wide bandwidth mmWaves channels is challenged by the inherent hardware implementation complexities of mmWave devices [2][3] and the high-speed signal processing and data conversion requirements of 5G systems. Signal transmissions in such channel environments suffer huge propagation and reflection losses, and are highly susceptible to blockage. Frequent channel estimation is necessary to establish and maintain a reliable communication between a base-station (BS) unit and a user-equipment (UE). These unfavorable channel characteristics mandate narrow beam transmission in mmWaves channel to save power and to reduce interference. The small wavelengths of mmWave make narrow beam-forming (BF) possible with large-scale antenna arrays [1]–[3]. Typical transceiver architectures with large-scale antenna arrays utilize a low-power power amplifier (PA)/low noise amplifier (LNA) per antenna and a pair of high-speed digital-to-analog (DAC)/analog-to-digital (ADC) circuits per RF chain [4]. Power consumption and cost of the DAC/ADC circuits dominate the ultra-wideband mmWave transceiver design challenges. The baseband implementation of high-speed data sampling can also be difficult to balance in terms of power, complexity, performance, and cost. The high speed processing of the conventional channel estimator adds higher power consumption at the baseband hardware implementation.

Power-saving solutions using one-bit or low resolution A/D-converters [5] received a lot of attention in the research community. Various channel estimation algorithms based on 1-bit or sub-resolution ADC's have been researched [6]–[9]. These techniques operate on high-speed (above Nyquist rate), low resolution data samples. There has also been research utilizing compressed sensing based algorithms on sparse channel models [10]–[13]. These methods reduce the number of pilot signals and optimally place them in multi-carrier transmissions in order to improve spectral efficiency. There has been limited research on sub-Nyquist based channel estimation algorithms. In [14], a sub-Nyquist based channel estimation solution is described for IEEE 802.11ad links. In this approach, subsampling is achieved using sampler designs in hardware. In [15], estimation of delay and doppler shifts with sub-Nyquist sampling is described. In this work sub-sampling is achieved by jointly optimizing analog transmit and receive filters. Prior research attempts to reduce power consumptions using low-resolution or sub-Nyquist sampler designs.

In this paper, we address the channel estimation problem for ultra wide-band mmWave communication channels by developing a novel approach that reduces power consumption via subsampling of the received signal itself. Sub-Nyquist sampling without additional hardware modification is made possible with the design of a pilot signal. The channel is assumed to have a single path between the transmitter and the receiver.

Our algorithm utilizes a pilot signal's subsequence properties and develops an algorithm that is capable of processing the subsequences of the received pilot signal in parallel at a much lower clock speed than the Nyquist rate. Processing a smaller block of decimated data at a time also reduces the complexity of the algorithm. Lastly, the proposed algorithm can estimate the channel from a sub-Nyquist sampled signal by trading off performance with ADC sampling speed. This mode of operation can allow the receiving device to go in a power saving mode when the received signal to noise ratios are high and acceptable channel estimations can still be achieved when the ADC samples the pilot signal at a fraction of the Nyquist rate.

In the rest of the paper we develop a maximum likelihood (ML) estimation of the given channel model in section 2, the proposed algorithm is presented in section 3 followed by simulation results in section 4 and conclusions in section 5. This paper uses lower and upper case bold letters for vector and matrix notations. Transpose, Hermitian, and modulo n are represented by $(.)^T$, $(.)^H$, and $((.))_n$ respectively.

2. MMWAVE CHANNEL MODEL AND ML ESTIMATION

We assume that the communication between two devices in mmWave channels happens in a single narrow-beam path between the transmitter and the receiver as shown below:

$$y(t) = \alpha p(t-\tau) + w(t)$$

Here, $y(t)$, $p(t)$, and $w(t)$ are the received signal, pilot signal, and additive noise at time t respectively. The propagation gain is α. Our understanding of the mmWave propagation channel model is based on measurements and studies done on mmWave bands [16]–[18]. The delay spread in mmWave for a typical indoor line-of-sight (LOS) scenario ranges from 4 to 11ns. For a GHz bandwidth system, this is equivalent to a duration of less than 12 symbols. Applying beamforming can further contain the delay spread to 0.5ns for LOS scenarios and 5ns for non-line-of-sight (NLOS) scenarios, which corresponds to 1 and 5 symbol-durations in a GHz bandwidth system respectively. Delay spread numbers for mmWave are much smaller than conventional cellular communication systems in the lower frequency bands. These smaller delay spreads found in mmWave propagation together with beamforming technology makes a mmWave channel an ideal candidate for being modeled mathematically as a sparse channel. A single path communication may appear simplistic, but it is applicable to many 5G scenarios as described below:

1) During the initial acquisition (IA) process, the UE receiver, most likely limited by analog beamforming, scans all directions to search for downlink (DL) pilot transmission. The reference pilot signal to assist in the BF process is also transmitted on a beam to cover sufficient range of transmission. Analog BF at the receiver end has the added benefit of rejecting all signals coming outside the steered direction. The rejected signals can be from other interferers or other multipath components from the same BS. The DL pilot signal received during this scan process, using analog BF at UE can be modeled as a single path channel model. The IA process is a complex and lengthy process adding overhead and draining power from the device. Latency and power reduction during this process is a crucial problem to solve for 5G mmWave communications.
2) After beamforming i.e., the aligning of angle-of-arrival (AoA) and angle-of-departure (AoD), a single narrow beam signal path survives at the UE receiver. This is a valid and quite common scenario for DL transmission in 5G mmWaves communication. Further, power conservation at the UE device is more critical compared to that at the BS ends. Our approach of reducing power consumption using algorithms aided by pilot design is very much applicable for DL narrow beam transmissions.
3) For higher frequency band communications, such as 90GHz bands, the channel is mostly LOS on a single narrow-beam path. For this scenario our channel model is valid for both UL and DL transmissions regardless of analog, digital or hybrid BF capabilities at the receivers.

2.1. Maximum-likelihood estimator for the given channel model:

In this subsection we formulate an ML channel estimation for the above channel model. The received signal is sampled at symbol timing and the delay in the channel can be resolved by symbol delays. The received signal at symbol timing can be expressed as follows:

$$y[n] = \alpha p[n-\tau] + w[n], \qquad n = 0, L-1.$$

Here, $y[n]$, $p[n]$, and $w[n]$ are the received signal, pilot signal and additive noise samples at the n^{th} symbol time respectively. The noise is zero mean additive white Gaussian noise (AWGN). The integer symbol delay is τ, the complex gain of the transmitted pilot at the receiver is α, and the symbol duration of the pilot signal is L.

For the ML method to estimate the dominant path, the problem becomes much simpler if the different delays of the pilot signal are uncorrelated to each other over the delay spread of the channel. Therefore, we assume that the transmitted pilot sequence is designed to have perfect circular autocorrelation properties. A cyclic prefix of duration greater than the maximum channel delay is appended to the pilot sequence so that reduced complexity frequency domain circular correlation can be applied. Due to this zero circular correlation property all of the circular shifted sequences of the pilot sequences form an orthogonal basis for the L dimensional subspace. Let us define the L dimensional τ circular shifted pilot sequence $\mathbf{p}_\tau$, received signal samples $\mathbf{y}$, and the additive noise $\mathbf{w}$, in vector notations as:

$$\begin{aligned}
\mathbf{p}_\tau &= \begin{bmatrix} p[\tau] & \cdots & p[L-1] & p[0] & \cdots & p[\tau-1] \end{bmatrix}^T \\
\mathbf{y} &= \begin{bmatrix} y[0] & \cdots & y[L-1] \end{bmatrix}^T \\
\mathbf{w} &= \begin{bmatrix} w[0] & \cdots & w[L-1] \end{bmatrix}^T.
\end{aligned}$$

Assuming unit norm for the pilot vectors, $\mathbf{p}_\tau$, the following unitary matrix can form an ortho-normal basis for the L dimensional subspace:

$$\mathbf{U} = \begin{bmatrix} \mathbf{p}_0 & \cdots & \mathbf{p}_{L-1} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_\mathbf{p} & \bar{\mathbf{U}}_\mathbf{p} \end{bmatrix}.$$

The matrices $\mathbf{U}_\mathbf{p}$ and $\bar{\mathbf{U}}_\mathbf{p}$ are defined below:

$$\begin{aligned}
\mathbf{U}_\mathbf{p} &= \begin{bmatrix} \mathbf{p}_0 & \cdots & \mathbf{p}_{m-1} \end{bmatrix} \\
\bar{\mathbf{U}}_\mathbf{p} &= \begin{bmatrix} \mathbf{p}_m & \cdots & \mathbf{p}_{L-1} \end{bmatrix}.
\end{aligned}$$

In the above equation m is a known parameter representing the maximum delay of the channel and it is also the symbol duration of the cyclic prefix. In vector notation the received signal can be written as follows:

$$\mathbf{y} = \alpha \mathbf{p}_\tau + \mathbf{w}.$$

Multiplying both sides of the above equation by $\mathbf{U}$, or equivalently, performing circular correlation with the pilot sequence yields:

$$\mathbf{h} = \begin{bmatrix} h[0] & \cdots & h[L-1] \end{bmatrix}^T = \mathbf{U}^H \mathbf{y} = \alpha \mathbf{e}_\tau + \mathbf{U}^H \mathbf{w}. \tag{1}$$

The statistical properties of the additive noise, $\mathbf{U}^H\mathbf{w}$, in the above equation remain the same and $\mathbf{e}_\tau$ represents the τ^{th} column of the L-dimensional identity matrix.

From this transformed received vector the ML estimate of the channel can be shown to be:

$$\begin{aligned}
\hat{\tau}_{ML} &= \operatorname{argmax}_{n \in \{0,\cdots,m-1\}} \left| h[n] \right|, \\
\hat{\alpha}_{ML} &= h[\hat{\tau}_{ML}].
\end{aligned} \tag{2}$$

Our goal is to come up with an algorithm that is comparable to ML estimates and is yet capable of processing subsamples of the pilot signal in parallel. In our problem formulation we design a pilot signal that has an additional cyclic prefix whose length is greater than the delay spread of the channel. This allows us to focus our attention on circular auto-correlation and cross-correlation properties of the subsequences instead of linear correlation properties. That is, each decimated subsequence of the pilot must have good auto correlation property over the delay spread of the channel and the cross-correlation among the decimated sequences with different phase shifts must be low.

In the next section, we formulate a pilot assisted channel estimation algorithm using the afore-mentioned channel model.

3. SUB-SAMPLE BASED CHANNEL ESTIMATION

For our channel estimation algorithm we have chosen Zadoff-Chu (ZC) pilot sequence for its subsequence properties shown in [19]. ZC sequence is a constant amplitude zero auto-correlation (CAZAC) sequence. In addition to having perfect circular auto-correlation properties it also has a constant amplitude (polyphase sequence). This property allows a power saving power amplifier design, making it an attractive choice for wireless communication systems. In 4G LTE, ZC sequences are used extensively for synchronization and channel estimation purposes [20], [21]. More details on the ZC sequences can be found in [22], [23].

The ZC sequence $s[n]$ of length L and root u is defined as:

$$s[n] = \begin{cases} e^{-\frac{i\pi u n^2}{L}}, & \text{for } L \text{ even} \\ e^{-\frac{i\pi u n(n+1)}{L}}, & \text{for } L \text{ odd} \end{cases}$$

In this paper we represent this sequence by ZC(u,L). The root of the sequence u is relative prime to L. These sequences are periodic with a periodicity of L. It is easily seen that when these sequences are conjugated or scaled by a complex constant they preserve the constant amplitude, zero circular correlation (CAZAC) properties. Further, it is shown in [22] that adding a frequency offset by multiplying the sequences with the sequence $e^{i\frac{2\pi q}{L}n}, n = 0,\cdots,L-1$, where q is any integer, maintains the CAZAC sequence properties.

In [19], we prove that ZC sequences of length, $L = ND^2$, where N and D are any positive integer pair, when decimated by D, demonstrate the following desirable properties:

1) All D subsequences have zero circular cross-correlations among themselves.
2) The circular auto-correlation of each of the sub-sequences is non-zero at every N lag. Further, the magnitude of the non-zero entries of the circular auto-correlation have a constant amplitude and a linear phase drift.

These properties are preserved in the circularly shifted ZC sequences as well. Let us define the decimated subsequence of a ZC sequence delayed by an integer τ and with the j^{th} phase offset, where $j = 0,1,\ldots,D$-1, as:

$$s_{j,\tau}[k] = s[j-\tau+Dk], k = 0,\cdots,ND-1$$

This expression can be simplified to separate out the constant phase term, the linear phase term and the linear frequency term respectively, for k=0,…,ND-1, as follows:

$$s_{j,\tau}[k] = \begin{cases} e^{\frac{-i\pi u(j-\tau)^2}{ND^2}} e^{\frac{-i2\pi u(j-\tau)}{ND}k} e^{\frac{-i\pi u k^2}{N}}, & N \text{ even, } D \text{ even/odd} \\ e^{\frac{-i\pi u(j-\tau)(j-\tau+1)}{ND^2}} e^{\frac{-i2\pi u(j-\tau+\frac{D}{2})}{ND}k} e^{\frac{-i\pi u k(k+1)}{N}}, & N \text{ odd, } D \text{ even} \\ e^{\frac{-i\pi(j-\tau)(j-\tau+1)}{ND^2}} e^{\frac{-i2\pi u(j-\tau+\frac{(D+1)}{2})}{ND}k} e^{\frac{-i\pi u k(k+1)}{N}}, & N \text{ odd, } D \text{ odd.} \end{cases}$$

The third term is ZC(u,N) repeated D times and it neither depends on the phase of the subsequences nor the delay in the channel. This term can be removed from the received signal vector by multiplying each sample by the complex conjugate of the ZC(u,N) sequence. This operation does not change any statistical properties of the AWGN because of the constant amplitude property of the ZC sequence.

The remaining term of the j^{th} subsequence is a complex sinusoid. The discrete frequency $\omega_{j,\tau} = \frac{2\pi}{ND}k_{j,\tau}$ and the phase $\varphi_{j,\tau}$ of the sinusoidal term of the j^{th} subsequence is given below:

$$k_{j,\tau} = \begin{cases} \left(\left(-u(j-\tau)\right)\right)_{ND}, & N \text{ even, } D \text{ even/odd} \\ \left(\left(-u(j-\tau+\frac{D}{2})\right)\right)_{ND}, & N \text{ odd, } D \text{ even} \\ \left(\left(-u(j-\tau+\frac{(D+1)}{2})\right)\right)_{ND}, & N \text{ odd, } D \text{ odd} \end{cases}$$

$$\varphi_{j,\tau} = \begin{cases} \frac{-\pi u(j-\tau)^2}{L}, & N \text{ even} \\ \frac{-\pi u(j-\tau)(j-\tau+1)}{L}, & N \text{ odd.} \end{cases}$$

From the above expressions it is evident that, for each subsequence there is one-to-one mapping of the discrete frequency, $\omega_{j,\tau}$, $\tau = 0,\cdots,ND-1$ to the channel delay, $\tau \in \{0,\cdots,ND-1\}$. This is because the ZC root u is relative prime to ND.

Consequently, after removing the ZC(u,N) term the problem of estimating the channel model parameters becomes a single sinusoidal parameter estimation problem for each received subsequence. The ML estimate of the single sinusoid can be achieved by taking ND-point DFT of each of the subsequences. The one-to-one mapping of the DFT frequency index, $k_{j,\tau}$ to delay τ produces the impulse response of the channel with a phase rotation given by $\varphi_{j,\tau}$ for each subsequence $j = 0,\cdots,D-1$. After removing the phase rotations, the peak of the channel response corresponds to the gain in the channel and the corresponding delay is the integer delay in the channel. The variances of the additive noise in the impulse responses from the subsequences can be reduced by a factor of D by averaging all of the D subsequences. Note that the channel estimation can be independently performed from each subsequence, therefore, channel estimation from sub-Nyquist sampling is possible. Following is a block diagram of the channel estimation algorithm using all D subsequences of the received ZC pilot sequence.

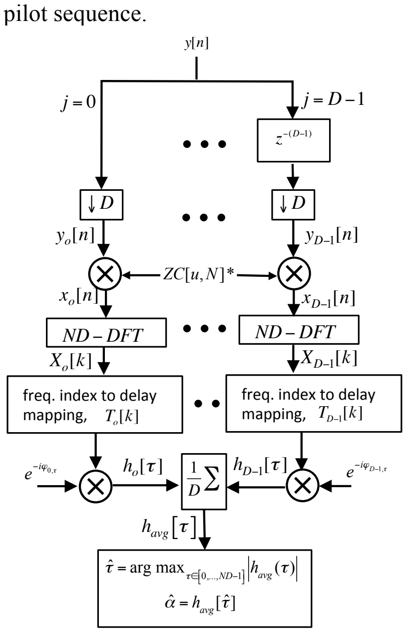

Figure 1. Block diagram of sub-sample based algorithm

In the diagram the received signal is the ZC(u,L) with AWGN as follows:

$$y[n] = \alpha \frac{1}{\sqrt{L}} s[n-\tau] + w[n], \quad n = 0, L-1. \tag{3}$$

3.1. A comparison with the ML estimation:

With the selection of the pilot sequence in (3), the ML estimate of the channel is given by (2) where,

$$\mathbf{p}_\tau = \frac{1}{\sqrt{L}}\left[\ s[\tau]\ \cdots\ s[L-1]\ \ s[0]\ \cdots\ s[\tau-1]\ \right]^T.$$

In our algorithm, after preprocessing the j^{th} subsequence of the received pilot signal, the vector, $\mathbf{h}_j = \left[\ h_j[0]\ \cdots\ h_j[ND-1]\ \right]^T$ has the same parametric model of the channel as in (1), except that the magnitude at the delay is scaled by a factor of $1/\sqrt{D}$. This is because the signal content of $\mathbf{x}_j = \left[\ x_j[0]\ \cdots\ x_j[ND-1]\ \right]^T$ is a complex sinusoid with gain $\alpha/\sqrt{L}$ and after ND point DFT the gain at the corresponding frequency becomes $\alpha\sqrt{ND}/\sqrt{L} = \alpha/\sqrt{D}$. The IID (independent and identical) Gaussian noise statistics are unaltered by the multiplication of constant amplitude ZC(u,N) sequence, the phase rotation operation, the DFT and the frequency index to delay mapping operation. The averaging process of all the D vectors scales the additive noise's standard deviation in $\mathbf{h}_{avg} = \left[\ h_{avg}[0]\ \cdots\ h_{avg}[ND-1]\ \right]^T$ by a factor of $1/\sqrt{D}$. Therefore, the parametric model of the channel of our method becomes identical to (1). This concludes the proof that applying the ML estimation on the transformed vector $\mathbf{h}_{avg}$ yields the ML estimates for the original channel estimation problem.

The computational complexity in ML estimation in (2) applying frequency domain circular correlation includes:

- L point DFT of the received L dimensional vector.
- Element wise complex multiplications of L-point-DFT with the DFT of the ZC(u,L) sequence.
- L-point IDFT.
- Processing clock at symbol rate.

The computational complexity in our approach is as follows:

- D ND-point DFTs.
- $2D$ element wise complex multiplications of ND-dimensional vectors. These computations reduce by a factor of 2 when $N = 1$, because ZC(u,1) = 1.
- Processing clock at $1/D^{\text{th}}$ of the symbol rate.

In the next section we display the simulation results using this algorithm that demonstrate that the algorithm is comparable to the ML estimates of the given channel model.

4. SIMULATION RESULTS

The pilot signal is a Zadoff-Chu sequence with a length $L = 1024 = ND^2$, where decimation factor is $D = 16$, and the root $u = 11$. The propagation channel is known to have a maximum delay spread of, $\tau_{\max} = 64$. The estimation is considered erroneous when the estimated delay differs from the actual delay in the channel. Error rates are calculated using 2000 Monte-Carlo simulation runs for each signal to noise ratio (SNR) level and channel delay. A theoretical error rate versus SNR is computed using (2) and numerically computing the distribution of $|h(n)|$, n=1, ..., m-1, for each SNR value.

The following figure plots the error rate in delay estimation for our proposed method (red-line in bottom plot), error rate in delay estimation for ML estimator (red-line in top plot, as in equation (2)) and the theoretical error rate for an ML estimator (black-line in both plots). As seen in Figure 2, the error rate for both ML and our sub-sample based methods estimates closely matches the theoretical results. Hence, the simulation result confirms our theoretical assessments.

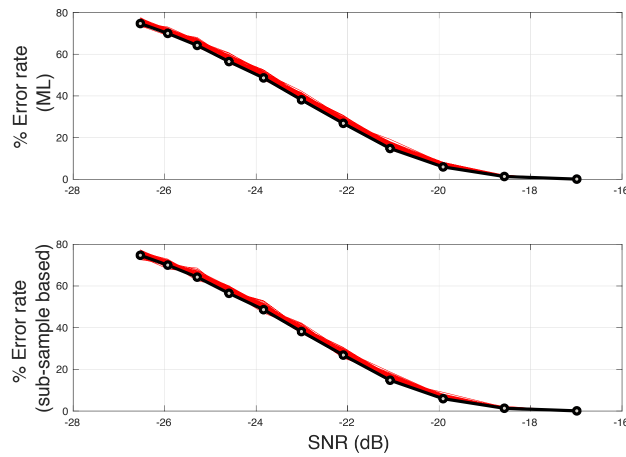

Figure 2. Error rate vs SNR

5. CONCLUSION

In this paper we propose a low complexity channel estimation algorithm that utilizes pilot sequence structure to reduce power consumption in ultra-wideband mmWave communication systems. The mmWave channel is assumed to have an integer symbol delay and the range of possible delays in the channel is a known parameter. The pilot signal is a Zadoff-Chu sequence, whose length, L is any multiple of D^2. Subsequences of such pilot sequences that are decimated by a factor D have very special circular correlation properties. The analysis and simulation results show that our channel estimation method is equivalent to an ML estimation for the given channel model. The main contributions of the algorithm are: 1) the capability to process the D decimated received samples independently, thus reducing implementation complexity, processing time, and energy consumption in the channel estimation process; 2) the ability to reduce the ADC speed to a fraction of the symbol rate and perform channel estimation with a reduced performance.

In summary, we have developed a method that can achieve an ML performance for a single path channel model that saves power using sub-sampling of sequences. This work lays the foundation for estimating more complex channel models such as those with multipath components with fractional delays.

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