NOISY 1-BIT COMPRESSED SENSING WITH HETEROGENEOUS SIDE-INFORMATION

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ABSTRACT

We consider the problem of sparse signal reconstruction from noisy 1-bit compressed measurements using a statistically dependent signal, as an aid. We assume that this signal does not share joint sparse representation with the sparse signal and call it a heterogeneous side-information. We assume that compressed measurements are corrupted by additive white Gaussian noise before quantization and sign-flip errors after quantization. We propose a generalized approximate message passing-based algorithm for signal reconstruction from noisy 1-bit compressed measurements which leverages the dependence between the signal and the heterogeneous side-information. We model the dependence between signal and heterogeneous side-information using copula functions and show, through numerical experiments, that the proposed algorithm yields a better reconstruction performance than 1-bit CS-based recovery algorithms that do not exploit the side-information.

***Index Terms*—** sparse signal reconstruction, 1-bit compressed measurements, approximate message passing, heterogeneous side-information

1. INTRODUCTION

Compressed sensing (CS) [1, 2] deals with the reconstruction of high-dimensional sparse signals using fewer measurements than dictated by the Nyquist sampling theorem. The standard CS based reconstruction algorithms assume infinite precision for compressed measurements. However, compressed measurements must be quantized to finite number of bits before processing, transmission, and/or storage. 1-bit quantization is particularly attractive because the construction of the quantizer is simple and cost-effective. It also provides savings in scarce resources like communication bandwidth, transmit/processing power and storage. 1-bit compressed sensing was introduced in [3] where the signal was reconstructed from the sign-information of the compressed measurements. Several reconstruction algorithms have been proposed that allow sparse signal reconstruction from its 1-bit compressed measurements [4–9].

The performance of reconstruction of a sparse signal from its 1-bit compressed measurements is susceptible to noise. The works in [7–11] have improved the reconstruction performance of 1-bit CS either by taking into account noise during signal reconstruction or using multiple measurement vectors. Further improvement in the reconstruction performance is possible if the receiver has extra information regarding the signal and exploits it during reconstruction. In many applications, previously reconstructed signals are either similar to or are statistically dependent with the signal that the receiver needs to estimate. For example, in the case of dynamic MRI, a reference scan is used to shorten the acquisition time or improve the signal-to-noise ratio of the reconstructed images [12]. Similarly, a previously reconstructed video frame can be used in reconstruction of the current video frame. We refer to the signal that aids in signal reconstruction as side-information (SI). Several algorithms have been proposed that incorporate the SI during reconstruction. Most of the works [13–15] assume that the SI and the sparse signal are similar. However, there are cases when the SI and the signal are significantly different but dependent, e.g., multimodal signals, and hence can aid the sparse signal reconstruction. We refer to this signal as heterogeneous SI (HSI). Some recent works [16–18] have addressed the problem of HSI-aided sparse signal reconstruction from compressed measurements. All of these works consider only the real-valued compressed measurements. Authors in [10] have looked into the problem of sparse signal reconstruction from 1-bit compressed measurements when the SI is similar to the sparse signal. To the best of our knowledge, the problem of HSI-aided sparse signal reconstruction based on 1-bit compressed measurements has not been explored in the literature.

We consider the problem of sparse signal reconstruction from its noisy 1-bit compressed measurements in a Bayesian framework when HSI is available at the receiver. We impose a sparsity-inducing Gaussian mixture prior on the signal. We model the statistical relationship between HSI and the compressed signal using a copula density function. We then pose the problem of sparse signal reconstruction from noisy 1-bit compressed measurements in the presence of HSI as a problem of MMSE estimation of the posterior density. The MMSE estimator of the posterior density is approximated using a message-passing algorithm called Generalized Approximate Message Passing (GAMP) [19]. It is an efficient iterative algorithm that approximates the mean of the posterior density.

We emphasize that the reconstruction of sparse signals from its 1-bit compressed measurements is very sensitive to

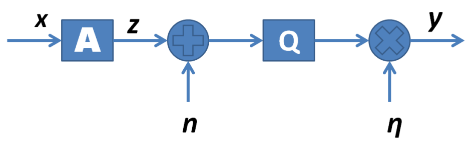

Fig. 1: 1-bit CS with pre-quantization and post-quantization noise.

noise. Through numerical simulations, we show that, incorporating HSI at the decoder leads to improved reconstruction performance.

Notation

Vectors and matrices are represented by boldface and upper boldface characters such as $\mathbf{x}$ and $\mathbf{A}$, respectively. Hadamard product, i.e., element wise product is denoted by $\odot$. We represent the Gaussian pdf with mean m and variance v by $\mathcal{N}(.|m,v)$. We define $I_n(a,b;m,v) = \int_a^b x^n \mathcal{N}(x|m,v)dx$ and $PI_n(\tau,m,v) = \int x^n \Phi(x/\sqrt{\tau})\mathcal{N}(x|m,v)dx$. Further, $\Phi(x) = I_0(-\infty,x;0,1)$, and $\phi(x) = \mathcal{N}(x|0,1)$.

2. MEASUREMENT MODEL AND PROBLEM FORMULATION

First, we discuss the signal model, and the measurement model used in this work.

2.1. Signal Model

We consider the elements of the input signal $\mathbf{x} \in \mathbb{R}^n$ to be random and i.i.d. with joint pdf

$$p_{\mathbf{x}}(\mathbf{x}) = \prod_{i=1}^{N} p_{x_i}(x_i), \tag{1}$$

where each component x_i is a Gaussian Mixture

$$p_{x_i}(x_i) = (1-\lambda)\mathcal{N}(x_i|0,v_1) + \lambda\mathcal{N}(x_i|0,v_2), \tag{2}$$

where λ is the probability of having non-zero values, and $v_1 << v_2$ with v_1 close to zero to impose sparse structure on the signal.

2.2. Measurement Model

Figure 1 shows the measurement model of noisy 1-bit CS with pre- and post-quantization noise. The sparse signal $\mathbf{x} \in \mathbb{R}^N$ is compressed to a lower-dimensional vector $\mathbf{z} \in \mathbb{R}^M$ using the random measurement matrix $\mathbf{A} \in \mathbb{R}^{M\times N}$. The vector, $\mathbf{z}$, is assumed to be corrupted by additive i.i.d. Gaussian noise vector, $\mathbf{n}$, with mean vector zero and covariance matrix $v_n\mathbb{I}_M$, where $\mathbb{I}_M$ is the $M\times M$ identity matrix. This corrupted compressed vector is quantized element-wise to +1 or -1 based on the sign of the signal.

We assume that the channel between the 1-bit quantizer and the receiver is noisy. The quantized measurements are corrupted by sign-flip noise. Formally, the noisy 1-bit quantized measurement model can be written as

$$y_j = \eta_j Q(\mathbf{a}_j^T\mathbf{x} + n_j), \quad j = 1,\cdots,M \tag{3}$$

where $\mathbf{a}_j^T$ is the j-th row of $\mathbf{A}$, n_j is the j-th element of $\mathbf{n}$, the quantizer $Q : \mathbb{R} \to \{-1,+1\}$ is the sign quantizer

$$Q(\zeta_j) = \begin{cases} +1, & \text{if } \zeta_j > 0, \\ -1, & \text{if } \zeta_j \leq 0, \end{cases} \tag{4}$$

and $\eta_j \in \{-1,+1\}$ is the post-quantization noise. η_j is assumed to follow i.i.d. Bernoulli distribution with $Pr(\eta_j = 1) = \gamma$. We define the inverse of the quantization function, $Q^{-1}(.)$, as

$$Q^{-1}(y_j) = \begin{cases} (-\infty,0\,], & \text{if } y_j \leq 0, \\ (0,\infty), & \text{if } y_j > 0, \end{cases} \tag{5}$$

where y_j is the j-th element of $\mathbf{y}$.

2.3. Bayesian Formulation

We assume that HSI, $\widetilde{\mathbf{x}}$, is available at the receiver. HSI is assumed to be dissimilar to but statistically dependent with the sparse signal. Using Bayesian rule, $p(\mathbf{x}|\mathbf{y},\widetilde{\mathbf{x}}) \propto p(\mathbf{x})p(\widetilde{\mathbf{x}},\mathbf{y}|\mathbf{x}) = p(\mathbf{x})p(\widetilde{\mathbf{x}}|\mathbf{x})p(\mathbf{y}|\mathbf{x})$, the posterior distribution of $\mathbf{x}$ given the noisy 1-bit quantized measurement, $\mathbf{y}$, and HSI, $\widetilde{\mathbf{x}}$, at the receiver is

$$p(\mathbf{x}|\mathbf{y},\widetilde{\mathbf{x}}) \propto p(\mathbf{y}|\mathbf{x})p(\mathbf{x},\widetilde{\mathbf{x}}) \propto \prod_{j=1}^{M} \mathbb{I}_{z_j \in \{Q^{-1}(\eta_j y_j)\}} \prod_{i=1}^{N} p(x_i,\widetilde{x}_i), \tag{6}$$

where $\mathbb{I}_{(.)}$ represents the indicator function, and $\propto$ represents equality up to a proportional constant, and $p(x_i,\widetilde{x}_i)$ is the joint density function that gives the joint statistical characterization of the sparse signal and the HSI. The minimum mean square error (MMSE) estimator of $\mathbf{x}$ is the mean of the posterior distribution (6), i.e., $\mathbb{E}[p(\mathbf{x}|\mathbf{y},\widetilde{\mathbf{x}})]$. As the evaluation of the MMSE estimator of (6) is intractable, we develop a message passing-based algorithm to approximate the MMSE estimator.

3. GAMP ALGORITHM UPDATE EQUATIONS

The GAMP algorithm was introduced in [19] which is a generalized version of the AMP algorithm [20]. It is a message passing based algorithm which efficiently approximates the computationally intractable high-dimensional integration required for evaluating the mean of the posterior density in an iterative fashion. In this section, we develop the GAMP-based algorithm which reconstructs the sparse signal from its noisy 1-bit compressed measurements when the receiver has access to HSI. We encourage readers to read [19] for a detailed exposition of the GAMP algorithm.

3.1. Heterogeneous Side-Information (HSI)

HSI is a signal which is assumed to be of different modality than the compressed signal and hence may not share joint-sparse representation as considered in the literature previously [12, 13, 15]. So, we cannot impose the Laplace distribution between HSI and the signal during reconstruction as in [10]. Instead, we assume that the HSI is dependent with the sparse signal, and use the Copula function [21] to model the statistical dependence between the signal and the HSI.

Algorithm 1 GAMP Algorithm for noisy 1-bit CS with heterogeneous SI (HSI-GAMP)

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1. Initialization: Set t=0 and initialize $\hat{\mathbf{x}}^t$, τ_x^t, and $\hat{\mathbf{s}}^t$ as $\hat{\mathbf{x}}^t = \mathbb{E}[\mathbf{x}]$, $\tau_x^t = \text{var}[\mathbf{x}]$, $\hat{\mathbf{s}}^t = 0$, where the expectation and variance of $\mathbf{x}$ are with respect to p_x.
2. **Measurement Update**
   - Linear Step
     $$\tau_p^{t+1} = (\mathbf{A} \odot \mathbf{A})\tau_x^t, \ \ \hat{\mathbf{p}}^{t+1} = \mathbf{A}\hat{\mathbf{x}}^t - \tau_p^{t+1} \odot \hat{\mathbf{s}}^t,$$
   - Non-Linear Step
     $$\hat{\mathbf{s}}^{t+1} = F_1(\mathbf{y}, \hat{\mathbf{p}}^{t+1}, \tau_p^{t+1}), \tag{7}$$
     $$\tau_s^{t+1} = F_2(\mathbf{y}, \hat{\mathbf{p}}^{t+1}, \tau_p^{t+1}), \tag{8}$$
     where F_1 and F_2 are applied element-wise and are defined as
     $$\begin{aligned} F_1\big(y, \hat{p}, \tau_p\big) &= \frac{1}{\tau_p}\Big(\mathbb{E}[z|y] - \hat{p}\Big), \\ F_2\big(y, \hat{p}, \tau_p\big) &= \frac{1}{\tau_p}\Big(1 - \frac{\text{var}[z|y]}{\tau_p}\Big). \end{aligned} \tag{9}$$
     The expectation and variance are evaluated with respect to $z \sim \mathcal{N}(\hat{p}, \tau_p)$.
3. **Estimation Update**

   Linear Step
   $$\tau_r^{t+1} = ((\mathbf{A} \odot \mathbf{A})^T \tau_s^t)^{-1}, \ \ \hat{\mathbf{r}}^{t+1} = \hat{\mathbf{x}}^t + \tau_r^{t+1} \odot (\mathbf{A}^T \hat{\mathbf{s}}^{t+1}),$$
   where the inversion is performed element-wise

   Non-linear Step
   $$\hat{\mathbf{x}}^{t+1} = G_1(\hat{\mathbf{r}}^{t+1}, \tau_r^{t+1}), \tag{10}$$
   $$\tau_x^{t+1} = G_2(\hat{\mathbf{r}}^{t+1}, \tau_r^{t+1}), \tag{11}$$
   where G_1 and G_2 are applied element-wise and are defined as
   $$G_1(\hat{r}, \tau_r) = \mathbb{E}[x|\hat{r}, \tilde{x}], \ \ G_2(\hat{r}, \tau_r) = \text{var}[x|\hat{r}, \tilde{x}]. \tag{12}$$

   The expectation and variance are evaluated with respect to $p_{x|\hat{r}} \propto \mathcal{N}(\cdot; \hat{r}, \tau_r) p_x(\cdot) p_{\tilde{x}|x}(\cdot)$.

   Set t = t+1 and return to step 2.

3.2. Copula Functions

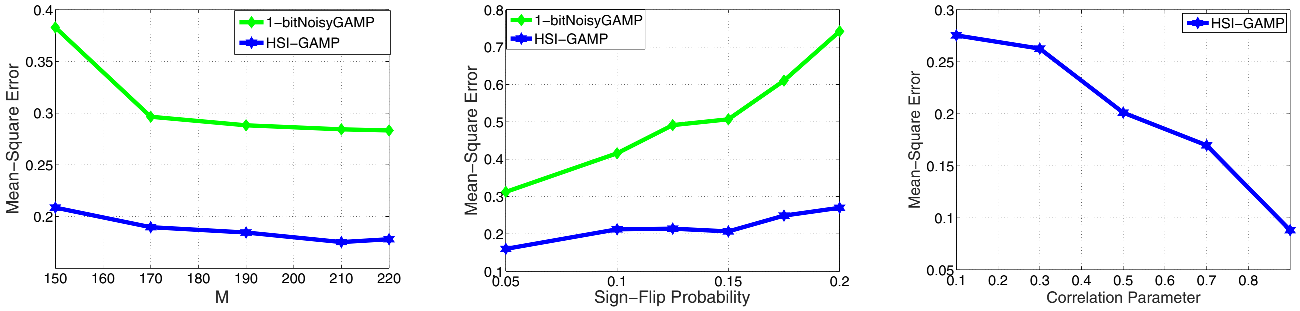
Copula functions allow the signals produced from different modalities to have arbitrary marginal distributions, while merging them into a joint multivariate probability distribution function. Let F_{X_i} and $F_{\tilde{X}_i}$ represent the marginal cumulative distribution of the i-th signal and HSI coefficients, respectively. According to Sklar's Theorem [21], if $F_{X_i,\tilde{X}_i}(x_i, \tilde{x}_i)$ is the 2-dimensional joint distribution of X_i and $\tilde{X}_i$, there exists a unique 2-dimensional copula function $C : [0,1]^2 \to [0,1]$ such that
$$F_{X_i,\tilde{X}_i}(x_i, \tilde{x}_i) = C[F_{X_i}(x_i), F_{\tilde{X}_i}(\tilde{x}_i)]. \tag{13}$$
Differentiating the above joint distribution, we obtain the joint probability density function of signal and its HSI
$$p(x_i, \tilde{x}_i) = c[F_{X_i}, F_{\tilde{X}_i}] \ p(x_i) \ p(\tilde{x}_i), \tag{14}$$
where $c[F_{X_i}, F_{\tilde{X}_i}]$ represents the bivariate copula density function. There are several copula densities such as Gaussian copula, Clayton, and Frank copula which represent different correlation structure between random variables. Among several copula functions, the one that captures the dependencies between the signal and the HSI should be selected [21]. For simplicity of exposition, we assume that the Gaussian copula models the correlation between the signal and the HSI. The distribution of Gaussian copula is defined as
$$c_g(\mathbf{w}) = |\mathbf{R}_g|^{-\frac{1}{2}} \exp\Big[-\frac{1}{2}\mathbf{w}_i^T \ (\mathbf{R}_g^{-1} - \mathbf{I}) \ \mathbf{w}_i\Big], \tag{15}$$
where $\mathbf{w}_i = [\Phi^{-1}(F_{X_i}), \Phi^{-1}(F_{\tilde{X}_i})]^T$, $\mathbf{I}$ is the identity matrix, and $\mathbf{R}_g = [1 \ \ \rho; \ \ \rho \ \ 1]$ is the Gaussian copula parameter.

We assume that the receiver knows the copula function that captures the correlation between the signal and the HSI.

3.3. Update Equations

In this subsection, we develop a GAMP based algorithm that reconstructs the sparse signal from noisy 1-bit compressed measurements when the dependence between the signal and the HSI is modeled by copula functions. In Algorithm 1, we summarize the steps required for the estimation of the sparse signal and refer to it as HSI-GAMP. The algorithm requires the evaluation of F_1, F_2, G_1, and G_2 as defined in (9), and (12). For F_1 and F_2, we require the evaluation of $\mathbb{E}[z|y]$ and $\text{var}[z|y]$. Following the steps as in [10], we obtain the following expressions for $\mathbb{E}[z|y]$ and $\mathbb{E}[z^2|y]$
$$\begin{aligned} \mathbb{E}[z|y] &= \frac{1}{C_1}\Big[\gamma\Big(PI_1(v_n, \hat{p}, \tau_p)\delta(y-1) + (\hat{p} - PI_1(v_n, \hat{p}, \tau_p))\delta(y+1)\Big) \\ &+ (1-\gamma)\Big((\hat{p} - PI_1(v_n, \hat{p}, \tau_p))\delta(y-1) + PI_1(v_n, \hat{p}, \tau_p)\delta(y+1)\Big)\Big], \\ \mathbb{E}[z^2|y] &= \frac{1}{C_1}\Big[\gamma\Big(PI_2(v_n, \hat{p}, \tau_p)\delta(y-1) + (\hat{p}^2 + \tau_p - PI_2(v_n, \hat{p}, \tau_p))\delta(y+1)\Big) \\ &+ (1-\gamma)\Big((\hat{p}^2 + \tau_p - PI_2)\delta(y-1) + PI_2(v_n, \hat{p}, \tau_p)\delta(y+1)\Big)\Big], \\ C_1 &= \gamma\Big(PI_0(v_n, \hat{p}, \tau_p))\delta(y-1) + \Big(1 - PI_0(v_n, \hat{p}, \tau_p)\Big)\delta(y+1)\Big) \\ &+ (1-\gamma)\Big(\Big(\hat{p} - PI_0(v_n, \hat{p}, \tau_p)\Big)\delta(y-1) + PI_0(v_n, \hat{p}, \tau_p)\delta(y+1)\Big). \end{aligned}$$

As $\text{var}[z|y] = \mathbb{E}[z^2|y] - (\mathbb{E}[z|y])^2$, F_1 and F_2 can now be evaluated. Next, we evaluate non-linear function G_1 and G_2 required in the estimation update in Algorithm 1.



(a) $\rho = 0.7, \gamma = 0.10, v_n = 0.1, \lambda = 0.1$ (b) $M/N = 2, v_n = 0.1, \lambda = .1, \rho = 0.7$ (c) $M/N = 2, v_n = 0.1, \lambda = .1, \gamma = 0.1$

Fig. 2: Reconstruction performance of the proposed method.

$$\mathbb{E}[x|\hat{r},\tilde{x}] = \frac{1}{C_2}\int x\mathcal{N}(x|\hat{r},\tau_r)c[F_{X_i},F_{\widetilde{X}_i}]\ p(x_i)\ dx, \quad (16)$$

where C_2 is a normalization constant and is given by

$$C_2 = \int \mathcal{N}(x|\hat{r},\tau_r)c[F_{X_i},F_{\widetilde{X}_i}]\ p(x_i)dx. \quad (17)$$

Similarly,

$$\mathbb{E}[x^2|\hat{r},\tilde{x}] = \frac{1}{C_2}\int x^2\mathcal{N}(x|\hat{r},\tau_r)c[F_{X_i},F_{\widetilde{X}_i}]\ p(x_i)dx. \quad (18)$$

From Equations (16) and (18), we can clearly see that the method considered is general and be used for any bivariate coupla function. The evaluation of the closed-form expressions for (16), and (18) is possible if the copula density is Gaussian and the signal $\mathbf{x}$ follows Gaussian distribution. In other cases, we may need to resort to numerical integrations. Note that, the evaluation of the mean and the variance in (16) and (18), respectively requires unidimensional integrations and hence are computationally feasible. Note that, in both of the Measurement Update and the Estimation Update sections of Algorithm 1, per iteration computation is dominated by matrix multiplication. Hence, the per-iteration computation complexity of the algorithm is $O(MN)$.

4. SIMULATION RESULTS

We consider the problem of reconstructing a sparse signal of dimension N from M noisy 1-bit compressed measurements. The measurement matrix, $\mathbf{A}$, is drawn from an i.i.d. Gaussian distribution with zero-mean and $\frac{1}{M}$ variance. We assume that the side-information at the receiver is heterogeneous, which may or may not be sparse. We use the Gaussian copula, with copula parameter $\mathbf{R_g}$, to generate the sparse signal and the HSI. The signal, $\mathbf{x}$, follows the Gaussian mixture distribution in (2) with $\lambda = 0.1$, $v_1 = 0.1, v_2 = 5$, and $N = 100$. We perform 100 Monte Carlo runs and compare the mean square errors (MSE) of HSI-GAMP with 1-bitnoisyGAMP [10]. In several experiments, we study the reconstruction performance of the proposed algorithm. In the first experiment, we study the reconstruction performance of the proposed algorithm as a function of M. In Figure 2(a), we plot the MSE values of the HSI-GAMP algorithm and the 1-bitNoisyGAMP against M. In the second experiment, we study the effect of the sign-flip noise on the reconstruction performance of the proposed algorithm. Figure 2(b) shows the results of the second experiment. In the third experiment, we study the effect of the correlation parameter of the Gaussian copula, ρ, on the reconstruction performance of HSI-GAMP. Figure 2(c) shows the results of the third experiment. From Figure 2(a), we can see that incorporating HSI at the receiver yields improved reconstruction performance when compared to 1-bitnoisyGAMP. From Figure 2(b), we can see that the reconstruction performance of the proposed algorithm is more robust to the sign-flip noise when compared to 1-bitNoisyGAMP. HSI-GAMP exploits the dependence between the signal and the HSI to get better MSE performance. From Figure 2(c), we can see significant improvement in the reconstruction performance when the dependence between the signal and the HSI is large. When the dependence is large, the receiver has more information regarding the sparse signal through its statistical characterization which leads to improved reconstruction performance. In the third experiment, we obtain the average MSE value of 0.292 for 1-bitNoisyGAMP and is always larger than HSI-GAMP.

5. CONCLUSION

In this work, we proposed an algorithm for sparse signal recovery from noisy 1-bit compressed sensing when the receiver has access to HSI. We showed that taking into account the heterogeneous side-information during reconstruction yields an improved performance. We also showed that the proposed algorithm is robust to sign-flip noise and can significantly reduce the reconstruction error when the signal and the HSI are highly dependent. In future work, we plan to extend the proposed algorithm for the joint reconstruction of heterogeneous sparse signals.

References

[1] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[2] E. J. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[3] P. T. Boufounos and R. G. Baraniuk, "1-bit compressive sensing," in *42nd Annual Conference on Information Sciences and Systems*, 2008, pp. 16–21.

[4] K. Knudson, R. Saab, and R. Ward, "One-bit compressive sensing with norm estimation," *IEEE Trans. Inf. Theory*, vol. 62, no. 5, pp. 2748–2758, 2016.

[5] U. S. Kamilov, A. Bourquard, A. Amini, and M. Unser, "One-bit measurements with adaptive thresholds," *IEEE Signal Process. Lett.*, vol. 19, no. 10, pp. 607–610, 2012.

[6] R. G. Baraniuk, S. Foucart, D. Needell, Y. Plan, and M. Wootters, "Exponential decay of reconstruction error from binary measurements of sparse signals," *IEEE Trans. Inf. Theory*, vol. 63, no. 6, pp. 3368–3385, 2017.

[7] O. Musa, G. Hannak, and N. Goertz, "Generalized approximate message passing for one-bit compressed sensing with AWGN," in *2016 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*. IEEE, 2016, pp. 1428–1432.

[8] Y. Plan and R. Vershynin, "Robust 1-bit compressed sensing and sparse logistic regression: A convex programming approach," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 482–494, 2013.

[9] L. Jacques, J. N. Laska, P. T. Boufounos, and R. G. Baraniuk, "Robust 1-bit compressive sensing via binary stable embeddings of sparse vectors," *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2082–2102, 2013.

[10] S. Kafle, T. Wimalajeewa, and P. K. Varshney, "Generalized approximate message passing for noisy 1-bit compressed sensing with side-information," in *2018 Asilomar Conference on Signals, Systems and Computers*, 2018.

[11] S. Kafle, V. Gupta, B. Kailkhura, T. Wimalajeewa, and P. K. Varshney, "Joint sparsity pattern recovery with 1-bit compressive sensing in distributed sensor networks," *IEEE Transactions on Signal and Information Processing over Networks*, 2018.

[12] L. Weizman, Y. C. Eldar, A. Eilam, S. Londner, M. Artzi, and D. B. Bashat, "Fast reference based MRI," in *37th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC)*, 2015, pp. 7486–7489.

[13] J. F. Mota, N. Deligiannis, and M. R. Rodrigues, "Compressed sensing with side information: Geometrical interpretation and performance bounds," in *IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, 2014, pp. 512–516.

[14] W. Lu and N. Vaswani, "Regularized modified BPDN for noisy sparse reconstruction with partial erroneous support and signal value knowledge," *IEEE Trans. Signal Process.*, vol. 60, no. 1, pp. 182–196, 2012.

[15] A. Ma, C. Rush, D. Baron, D. Needell *et al.*, "An approximate message passing framework for side information," *arXiv preprint arXiv:1807.04839*, 2018.

[16] N. Deligiannis, J. F. Mota, E. Zimos, and M. R. Rodrigues, "Heterogeneous networked data recovery from compressive measurements using a copula prior," *IEEE Trans. Commun.*, vol. 65, no. 12, pp. 5333–5347, 2017.

[17] J. F. Mota, E. Tsiligianni, and N. Deligiannis, "A framework for learning affine transformations for multimodal sparse reconstruction," in *Wavelets and Sparsity XVII*, vol. 10394. International Society for Optics and Photonics, 2017, p. 103941T.

[18] E. Zimos, J. F. Mota, M. R. Rodrigues, and N. Deligiannis, "Bayesian compressed sensing with heterogeneous side information," in *Data Compression Conference (DCC), 2016*. IEEE, 2016, pp. 191–200.

[19] S. Rangan, "Generalized approximate message passing for estimation with random linear mixing," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 2168–2172.

[20] D. L. Donoho, A. Maleki, and A. Montanari, "Message-passing algorithms for compressed sensing," *Proceedings of the National Academy of Sciences*, vol. 106, no. 45, pp. 18 914–18 919, 2009.

[21] R. B. Nelsen, *An Introduction to Copulas*. Springer Science & Business Media, 2007.