LOW-COMPLEXITY DETECTION AND PERFORMANCE ANALYSIS FOR DECODE-AND-FORWARD RELAY NETWORKS

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ABSTRACT

We re-examine the problem of designing low-complexity detectors and their performance analysis for the 3-node one-way decode-and-forward (DF) relay network, where the destination has the statistical channel state information (CSI) of the source-relay link, and the instantaneous CSI of both the source-destination and relay-destination links. Recently proposed detection schemes, such as the piece-wise linear detector (PLD), achieve near-optimal error performance with linear complexity (with respect to the modulation size). In this paper, we propose two new detectors with near-optimal error performance. One has linear complexity for general modulations and the other has constant complexity for PAM signals. Additionally, an algorithm for computing the upper bound on the symbol error rate (SER) of the new detectors for the static source-destination and relay-destination links is proposed. Simulation results are presented to verify the efficiency of the proposed detectors and the tightness of the proposed SER bound.

Index Terms— Relay networks, decode-and-forward, low complexity detection, performance analysis

1. INTRODUCTION

Relay-assisted cooperative communication, introduced by van der Meulen [1], has been recognized as an effective technique for increasing the transmission rate [2], [3], [4]. There exist various relaying protocols, among which the amplify-and-forward (AF) and decode-and-forward (DF) relaying are two main protocols [5], [6]. Unlike the AF protocol that preserves a linear input-output relationship, in the DF protocol, the hard demodulation operation performed at the relay complicates the detector design and the associated performance analysis.

In this work, we are interested in the scenario that the destination has the instantaneous channel state information (CSI) of both the source-destination and relay-destination links, and only the statistical CSI of the source-relay link in the 3-node one-way DF relay network. One remark is that the results are not restricted to this protocol only [7]. In this scenario, [8] proposes an almost maximum likelihood detector (AMLD)[1] and a piece-wise linear detector (PLD), with quadratic and linear complexity[2], respectively, to achieve near-optimal error performance. These two detectors are the state-of-the-art for the underlying system model, to the best of our knowledge. The derivations of such detectors involve several approximations such as the nearest neighbor approximation to simplify the SER expressions [8], [9], [10].

The main contributions of this paper are as follows: 1) Two new detectors are proposed. One is the maximum AMLD (MAMLD), which has linear complexity for the general case of any M-ary modulation. The other is the near maximum likelihood detector (NMLD) with constant complexity for the case of M-PAM. 2) An algorithm for computing the SER upper bound for both MAMLD and NMLD for the case of static source-destination and relay-destination links is proposed. Additionally, our simulation results show that the bound is tight in the high SNR regime.

2. SYSTEM MODEL AND EXISTING WORKS

2.1. System Model

Consider the 3-node DF relay network with one source (S), one relay (R) and one destination (D). R works in the half-duplex mode. We also assume perfect channel knowledge at R. However, D is assumed to have the instantaneous CSI of the S-D and R-D links and only the statistical CSI of the S-R link. Time division multiplexing (TDM) is performed and one transmission session consists of two time slots. In the first time slot, S broadcasts its signals to R and D. In the second slot, S remains silent while R performs detection and transmits the detected symbols to D.

Assume that S selects and transmits symbols with equal probability from the alphabet $\mathcal{X}$ of an M-ary constellation. The received signals at all the nodes are modeled as $y_{\mathrm{X,Y}} = h_{\mathrm{X,Y}} x_{\mathrm{X}} + n_{\mathrm{X,Y}}$, where $\mathrm{X, Y} \in \{\mathrm{s, d, r}\}$ denote the transmitting and receiving nodes, respectively, x_{X} is the transmitted symbol of node X, and $h_{\mathrm{X,Y}}$ is the complex channel coefficient between link X and Y. It is also assumed

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[1]The proposed MLD in [8] is referred to here as AMLD. Because its derivation involves some approximations, which will be detailed in Sec 2.2.

[2]We describe the complexity order with respect to the modulation size.

that the noise $n_{\mathrm{X,Y}}$ is the circularly symmetric complex additive white Gaussian noise (AWGN) with zero mean, i.e., $n_{\mathrm{X,Y}} \sim \mathcal{CN}(0, N_{\mathrm{X,Y}})$. It is further assumed that the optimal maximum likelihood detector (MLD) is adopted at R.

2.2. Almost Maximum Likelihood Detector

The maximum likelihood detection scheme should maximize the conditional joint probability density of the received data as: $\max_{x_s \in \mathcal{X}} \Pr(y_{s,d}|x_s) \sum_{x_r \in \mathcal{X}} \Pr(x_r|x_s) \Pr(y_{r,d}|x_r)$. The AMLD is obtained by approximating the transition probability term $\Pr(x_r|x_s)$ using the average SER of the detector at R, denoted as ε. Further, [8] assumes R erroneously decodes one symbol as any other symbol with equal probability to get their AMLD. Note that no performance analysis of the AMLD is given in [8], [10].

2.3. Piece-wise Linear Detector

AMLD has quadratic complexity since we need to enumerate over $(x_s, x_r) \in \mathcal{X}^2$, then the authors in [8] make several approximations to simplify it and obtain a linear complexity PLD. More specifically, to simplify the detection metric, the summations of some exponential terms (Gaussian mixture random variables [11]) are neglected, and the piece-wise linear approximation is made. The SER performance of PLD is shown to approach the AMLD in their simulation section.

The performance analysis specialized for PLD is provided in [8] and [10]. However, firstly they assume the transmitted symbol is wrongly decoded to only its nearest neighbors, which is the so-called nearest neighbor approach, to obtain the analytical SER expression. This is justified in traditional point-to-point channels [12] but not the DF relay network due to its non-linearity. Secondly the simulation results of the analytical SER expression are only shown for the Rayleigh fading links but not the static S-D and R-D links.

2.4. AMLD with Max Approximation

Here, we apply the widely-used max-log approximation [4], [13], [14], [15] to approximate the probability terms of the S-R-D link, which is the summation of M exponentials. Correspondingly, the following sub-optimal detection rule is obtained, in (1), with little loss of performance compared to AMLD, where $\eta \triangleq \log \frac{(1-\varepsilon)(M-1)}{\varepsilon}$.

$$\hat{x}_s = \arg\min_{x_s \in \mathcal{X}} \left\{ \frac{|y_{s,d} - h_{s,d}x_s|^2}{N_{s,d}} + \min \left\{ \frac{|y_{r,d} - h_{r,d}x_s|^2}{N_{r,d}}, \min_{x_r \in \mathcal{X}, x_r \neq x_s} \frac{|y_{r,d} - h_{r,d}x_r|^2}{N_{r,d}} + \eta \right\} \right\} \tag{1}$$

3. PROPOSED DETECTORS

In this section, the proposed MAMLD and NMLD are introduced. MAMLD can be applied to general M-ary modulations with linear complexity, and NMLD can be applied to M-PAM with constant complexity.

3.1. Detector at the Relay

The MLD at R is solving $x_r = \arg\min_{x_s \in \mathcal{X}} |y_{s,r} - h_{s,r}x_s|^2$, which is well established and studied for arbitrary constellations in point-to-point channels [12]. It is simply a least square problem with discrete inputs. The specific lattice structure of the PAM signal can be utilized and it is straightforward to obtain a closed-form solution by slicing and translation operations as

$$x_r = 2 \left\lfloor \frac{\Re\{y_{s,r}^* h_{s,r}\}}{2|h_{s,r}|^2} + \frac{M+1}{2} \right\rceil_M - (M+1), \tag{2}$$

where we explicitly define the alphabet $\mathcal{X} \triangleq \{-(M-1), \cdots, -1, 1, \cdots, M-1\}$ and the slicing operation as $\lfloor n \rceil_M \triangleq \arg\min_{m \in \mathbb{Z}, m \in [1,M]} |n-m|$, and $\mathbb{Z}$ is the integer set. In this way, the detection at R will be very efficient,

3.2. Proposed Detectors at the Destination

We focus on (1) and develop the proposed detectors. Based on the observation that $\frac{|y_{r,d} - h_{r,d}x_s|^2}{N_{r,d}} < \frac{|y_{r,d} - h_{r,d}x_s|^2}{N_{r,d}} + \eta$ holds when $\eta > 0$, i.e., when $1 - \varepsilon > 1/M$, which is generally true, we can remove the constraint $x_r \neq x_s$ and get the proposed MAMLD in (3):

$$\hat{x}_s = \arg\min_{x_s \in \mathcal{X}} \left\{ \frac{|y_{s,d} - h_{s,d}x_s|^2}{N_{s,d}} + \min \left\{ \frac{|y_{r,d} - h_{r,d}x_s|^2}{N_{r,d}}, \frac{|y_{r,d} - h_{r,d}s_r|^2}{N_{r,d}} + \eta \right\} \right\}, \tag{3}$$

where $s_r = 2 \left\lfloor \frac{\Re\{y_{r,d}^* h_{r,d}\}}{2|h_{r,d}|^2} + \frac{M+1}{2} \right\rceil_M - (M+1)$. This has linear complexity for the general case of any M-ary modulation since we only need to enumerate over $x_s \in \mathcal{X}$.

Then we exploit the advantage that the $\min$ operations are exchangeable and get the proposed NMLD for PAM signals in (4):

$$\hat{x}_s = \arg\min \left\{ \min_{\mathbf{x} \in \mathcal{X}^2, x_s = x_r} f(\mathbf{x}), \min_{\mathbf{x} \in \mathcal{X}^2} f(\mathbf{x}) + \eta \right\}, \tag{4}$$

where $f(\mathbf{x}) \triangleq \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$, $\mathbf{y} = \left[\frac{\mathrm{Re}\{y_{s,d}^* h_{s,d}\}}{|h_{s,d}|}, \frac{\mathrm{Re}\{y_{r,d}^* h_{r,d}\}}{|h_{r,d}|} \right]^T$, $\mathbf{H} = \begin{bmatrix} |h_{s,d}| & 0 \\ 0 & |h_{r,d}| \end{bmatrix}$, $\mathbf{x} = [x_s, x_r]^T$, and $\mathbf{y} \sim \mathcal{N}(\mathbf{H}\mathbf{x}, \mathbf{\Sigma})$ with $\mathbf{\Sigma} = \frac{1}{2} \begin{bmatrix} N_{s,d} & 0 \\ 0 & N_{r,d} \end{bmatrix} \triangleq \frac{1}{2}\mathbf{I}$ for ease of presentation and without loss of generality.

The decision rules for our NMLD are defined as: $\hat{x}_s = s_0$ if $f([s_0, s_0]^T) < f([s_1, s_r]^T) + \eta$, and $\hat{x}_s = s_1$ otherwise, where $s_0 \triangleq \arg\min_{\mathbf{x} \in \mathcal{X}^2, x_s = x_r} f(\mathbf{x})$, $(s_1, s_r) \triangleq \arg\min_{\mathbf{x} \in \mathcal{X}^2} f(\mathbf{x})$. Essentially to get s_0, s_1, s_r, we only need to solve three point-to-point channel detection problems similar to (2) with different equivalent channel coefficients and observations. Therefore, the complexity of the proposed detector is invariant of the modulation size.

4. SER UPPER BOUND FOR PROPOSED DETECTORS

In this section, the SER upper bound is analyzed for M-PAM in the case when the channel coefficients of the S-D and R-D links are static. Note that since the SER performance of the NMLD is essentially the same as the MAMLD, our proposed bound is for both detectors, while the performance analysis in [8] and [10] is only for the PLD, not their proposed AMLD.

4.1. Pairwise Error Distance

We re-include the constraint $x_s \neq x_r$, then (4) is essentially a two-dimensional lattice decoding problem with $\mathbf{Hx}$ as the lattice points, and $\mathbf{y}$ as the observations. Geometrically, assume $\mathbf{a}$ and $\mathbf{b}$ are two distinct points with metric offset l_{X}, then we have $l_{\mathrm{X}} = \eta$ if $x_s \neq x_r$, and 0 otherwise, for $\mathrm{X} \in \{\mathbf{a}, \mathbf{b}\}$. The distance from the interest point $\mathbf{a}$ to the decision boundary of $\mathbf{a}$ and $\mathbf{b}$ can be calculated as:

$$d(\mathbf{a}, \mathbf{b}) = \frac{\|\mathbf{b} - \mathbf{a}\|}{2} + \frac{l_{\mathbf{b}} - l_{\mathbf{a}}}{2\|\mathbf{b} - \mathbf{a}\|}. \quad (5)$$

Using $d(\mathbf{a}, \mathbf{b})$, the pairwise error probability (PEP) for decoding $\mathbf{a}$ as $\mathbf{b}$ is given by $Q(d(\mathbf{a}, \mathbf{b})/\sigma)$, where $\sigma^2 = 1/2$ is the one-dimensional noise variance.

As mentioned earlier, existing works make several approximations to calculate the analytical SER [8] [10]. Here we give a more efficient way to verify the performance of our proposed schemes without making additional approximations by properly selecting a set which covers the Voronoi-relevant neighbor set (the definition is illustrated in [16]) of the interest point and using these sets to calculate the SER upper bound instead of the exact SER.

4.2. SER Upper Bound

For symmetry, only the lower triangular points are considered. We divide those points into M layers, the diagonal as layer-0, the second-diagonal as layer-1, etc. Points on layer-k ($k \in [0, M-1]$) are labeled as $A_{k,j}$, where $j \in [1, M-k]$ is indexed from right to left. Points with $j \in \{1, M-k\}$ are referred to as outer points while the rest are inner points. The labeling rules are illustrated in Fig. 1. For the diagonal points, the layer index k is omitted.

Here, we define $\mathcal{V}_{k,j} = \mathcal{V}_{k,j}^{d} \cup \mathcal{V}_{k,j}^{nd}$ as the Voronoi-relevant cover set of $A_{k,j}$, where $\mathcal{V}_{k,j}^{d}$ and $\mathcal{V}_{k,j}^{nd}$ contain the diagonal and non-diagonal neighbors, respectively.

It is obvious that for the diagonal point A_i, $\mathcal{V}_i^{d}$ can be chosen containing its geometrically nearest diagonal neighbors.

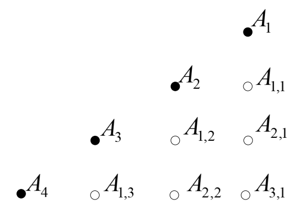


Fig. 1: Example of the labeling of the lower triangular points in case of 4-PAM. The black and white ones denote the diagonal and non-diagonal lattice points, respectively.

Algorithm 1: A high-level description of the algorithm

Input: $h_{s,d}, h_{r,d}, \eta, M, r_d, r_{nd}, \mathcal{W}_i, i \in [1, M], \mathbf{v}_{k,j}$, $k \in [1, M-1], j \in [1, M-k]$

Output: $\mathcal{V}_i^{nd}, i \in [1, M]$, $\mathcal{V}_{k,j}^{d}, k \in [1, M-1], j \in [1, M-k]$

/* Procedure 1: update $\mathcal{V}_i^{nd}$ based on *selection condition 1* */

for $i = 1 : M$ **do**
  for $(m, n) \in \mathcal{W}_i$ **do**
    if $d_d(m, n) < r_d$ **then**
      $\mathcal{V}_i^{nd} \leftarrow \mathcal{V}_i^{nd} \cup (m, n)$;

/* Procedure 2: update $\mathcal{S}_k$ and $\mathcal{I}_k$ */

for $k = 1 : M-1$ **do**
  for $m = (1-M) : (M-k-1)$ **do**
    update $\mathcal{S}_k$ and $\mathcal{I}_k$;

/* Procedure 3: update $\mathcal{V}_{k,j}^{d}$ based on *selection condition 2* */

for $k = 1 : M-1$ **do**
  for $j = 1 : M-k$ **do**
    for $m \in \mathbf{v}_{k,j}$ **do**
      update $\mathcal{V}_{k,j}^{d}$;

Similarly, for the non-diagonal point $A_{k,j}$, $\mathcal{V}_{k,j}^{nd}$ can be chosen containing its geometrically nearest non-diagonal neighbors. Therefore, the algorithm we propose only aims to find $\mathcal{V}_i^{nd}$ and $\mathcal{V}_{k,j}^{d}$, which is sufficient.

The idea is that we first define the original decision region of the interest point using at most four its geometrically nearest neighbors. Then a neighbor will be selected into the cover set if it satisfies the *selection condition* which will be defined later. The complete procedure is presented in Algorithm 1 to determine the non-diagonal neighbor sets for the diagonal points, and the diagonal neighbor sets for the non-diagonal points. We give some definitions in the following to support it.

Procedure 1 is to determine the non-diagonal neighbor sets for the diagonal points. Assume $\mathbf{b} = 2\mathbf{H}[m, n]^T$ with $(m, n) \in \mathbb{Z}^2$ is the vector representation of the non-diagonal point with the interest diagonal A_i as the origin. Adopting (5) and set $\mathbf{a}=\mathbf{0}$, the distance from A_i to the boundary produced by $\mathbf{b}$ is $d(\mathbf{0},\mathbf{b}) = \|\mathbf{b}\|/2 + \eta/(2\|\mathbf{b}\|) \triangleq d_d(m,n)$. With A_1 as the origin, $\mathcal{W}_1 \triangleq \{(m,n) : (m,n) \in \mathbb{Z}^2, (2-M) \le m \le 0, (1-M) \le n \le (m-1)\}$ denotes the offsets of all lower triangular non-diagonal points. $\mathcal{W}_i \triangleq \{(m,n) : (m-(i-1), n-(i-1)) \in \mathcal{W}_1\}$ is defined analogically with A_i as origin.

The original decision region for the inner diagonal points is generated using its four geometrically nearest non-diagonal points. The region is a rectangle whose minimal circumcircle has radius $r_d = \sqrt{d_d(1,0)^2 + d_d(0,-1)^2}$, and similarly for the outer points. The *selection condition 1* is defined as $d_d(m,n) < r_d$ for diagonal points. Here we omit the proof. $\mathcal{V}_i^{nd}$ contains the pairs of $(m,n) \in \mathcal{W}_i$ that satisfy *selection condition 1*.

Procedures 2 and 3 are to determine the diagonal neighbor sets for the non-diagonal points. Assume $\mathbf{b} = 2\mathbf{H}[m, m+k]^T$ is the vector representation of $A_i, i \in [1, M]$ with the interest non-diagonal $A_{k,j}$ as the origin. Adopting (5) and set $\mathbf{a} = \mathbf{0}$, the distance from $A_{k,j}$ to the boundary produced by $\mathbf{b}$ is $d(\mathbf{0},\mathbf{b}) = \|\mathbf{b}\|/2 - \eta/(2\|\mathbf{b}\|) \triangleq d_{nd}(m, m+k)$.

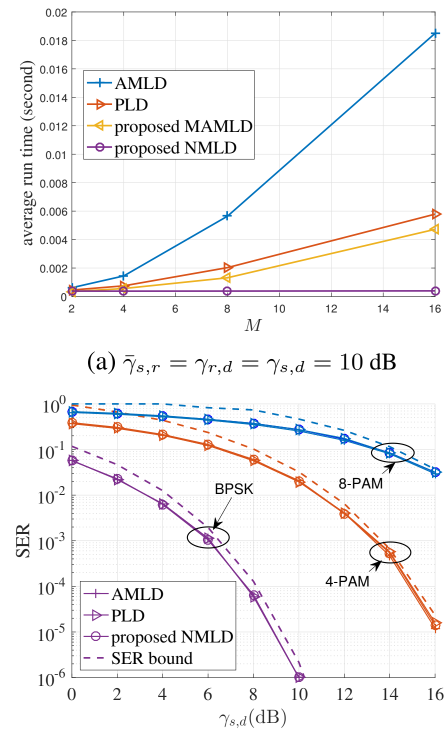
The original decision region is obtained by assuming equal prior information. Similar to the diagonal case, it is a rectangle whose minimal circumcircle is with radius $r_{nd} = \sqrt{|h_{s,d}|^2 + |h_{r,d}|^2}$.

With $A_{k,j}$ as the origin, $\mathbf{v}_{k,j} \triangleq [j-M, j-M+1, \cdots, j-1]$ contains the values of m for A_1-A_M. $\mathcal{U}_k \triangleq \cup_{j\in[1,M-k]} \mathbf{v}_{k,j} = \{m : (1-M) \le m \le (M-k-1)\}$ is defined accordingly, all layer-k points considered. Additionally, two sets are defined: $\mathcal{S}_k = \{m \in \mathcal{U}_k : d_{nd}(m, m+k) \in (-r_{nd}, r_{nd})\}, \mathcal{I}_k = \{m \in \mathcal{U}_k : d_{nd}(m, m+k) < -r_{nd}\}$. Note that $\mathcal{S}_k \cap \mathcal{I}_k = \emptyset$. The defined *selection condition 2* for $(m, m+k)$ for the outer points is $m \in (\mathcal{S}_k \cup \mathcal{I}_k)$, and for the inner points is $m \in \mathcal{S}_k$. $\mathcal{V}_{k,j}^d$ contains pairs of $(m, m+k), m \in \mathbf{v}_{k,j}$ that satisfy *selection condition 2*.

After obtaining all the cover sets for all the lower-triangular points, the SER upper bound can be calculated accordingly: $P_{SE}^u = \frac{1-\varepsilon}{M} P_{SE,d}^u + \frac{\varepsilon}{M(M-1)} P_{SE,nd}^u$, where $P_{SE,d}^u = 2\sum_{i=1}^{M} \sum_{\substack{(m,n)\in\mathcal{V}_i^{nd},\\ m\neq 0}} Q(d_d(m,n)/\sigma) + 2(M-1)Q(d_0(1,1)/\sigma)$, $P_{SE,nd}^u = 2\sum_{k=1}^{M-1}\sum_{j=1}^{M-k}[\sum_{\substack{(m,n)\in\mathcal{V}_{k,j}^d,\\ m\neq 0}} Q(d_{nd}(m,n)/\sigma) + \sum_{\substack{(m,n)\in\mathcal{V}_{k,j}^{nd},\\ m\neq 0}} Q(d_0(m,n)/\sigma)]$, $d(\mathbf{0},\mathbf{b}) = \frac{1}{2}\|\mathbf{b}\| \triangleq d_0(m,n)$ in the case that the interest point and $\mathbf{b}$ are both diagonal or non-diagonal.

5. SIMULATION RESULTS

In this section, the simulation results are shown to verify the efficiency and near-optimal SER performance of the proposed detectors and also the tightness of the proposed bound with a Rayleigh fading S-R link and static R-D and S-D links.



(a) $\bar{\gamma}_{s,r} = \gamma_{r,d} = \gamma_{s,d} = 10$ dB

(b) $\bar{\gamma}_{s,r} = \gamma_{r,d} = \gamma_{s,d}$

Fig. 2: Comparisons of various detectors and the proposed SER bound for the 3-node DF relay network with M-PAM based on (a) average run-time per symbol; (b) SER.

As shown in Fig. 2, for not too small M, the runtimes of the proposed MAMLD and NMLD are linear and invariant with M, respectively, while those of AMLD and PLD are quadratic and linear with M, respectively. The simulation results agree with our complexity order analysis. Additionally, the SER performance of NMLD approaches that of the AMLD, and the proposed bound is quite tight in the high SNR regime.[3]

6. CONCLUSION

We have proposed two near-optimal detectors, MAMLD and NMLD, with linear complexity for the general case of any M-ary modulation and constant complexity for the case of M-PAM, respectively, in the 3-node DF relay network. Their SER performance and complexity are compared favorably with the state-of-the-art detectors AMLD and PLD. Additionally, an SER bound is derived to analytically characterize the performance of the proposed detectors, and it is shown to be rather tight in the high SNR regime.

[3]The proposed MAMLD has the same SER performance as the proposed NMLD, and hence only the simulation results of the latter is shown in Fig. 2(b).

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