JOINT CODEBOOK DESIGN FOR MULTI-CELL NOMA

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ABSTRACT

For spreading-based multiple access, whether orthogonal (OMA) or non-orthogonal (NOMA), the spreading sequences (signatures) are selected from a predefined codebook. When operating in a cellular system, intercell interference will be inherently present between close base stations that share the same resources. If the codebook is reused across the different cells, then intercell interference can cause a full collision of the interfering users in the code-domain, thus deteriorating their performance, especially those at the cell-edge. In this paper, we propose a method for reducing intercell interference by means of jointly designed codebooks. The criterion for the codebook design is to minimize the maximum cross-correlation between the signatures of the interfering cells. In order to obtain such codebooks, we employ the algorithm of alternating projection. We finally apply the method to two interfering cells for both NOMA and OMA systems, with the users being uniformly distributed in the cells, and show that it can provide a considerable gain to the cell-edge users.

Index Terms— NOMA, codebook design, intercell interference, multi-cell, alternating projection.

1. INTRODUCTION

Non-orthogonal multiple access (NOMA) has gained considerable attention recently, due to its capability of providing massive connectivity and low latency operation [1, 2]. These requirements are the main drivers for the future services in 5th generation (5G) wireless systems and beyond, such as massive machine-type communications [3], and cellular-assisted vehicular communications [4, 5]. Among the various NOMA schemes [6], those that rely on spreading sequences with low cross-correlation, can offer a very high overloading capability [7]. When applied to cellular systems, intercell interference will be present between the base stations that share the same resources of time, frequency and space. In the case of a spreading-based system, whether orthogonal multiple access (OMA), or NOMA, a codebook with low cross-correlation properties is usually designed and defined in the standard. If this codebook would then be reused as it is by the interfering cells, then a full collision of the interfering users in the code-domain would occur. This can lead to a deterioration of the performance of the users, especially those at the cell-edge. An alternative to reusing the codebooks in a multi-cell NOMA system is to design the codebooks in a joint manner, such that the impact of the intercell interference is reduced. In [8], the author considers the joint design of codebooks for a multi-cell overloaded code-division multiple-access (CDMA) system; however, that method assumes that the codebooks of the different cells are orthogonal with respect to each other, and therefore the joint design would impact the internal correlation properties of the codebooks.

In this work, we investigate reducing the intercell interference in multi-cell NOMA systems, by assigning the interfering cells codebooks that are jointly designed with low codebook cross-correlation, while still preserving the correlation properties of a single-cell codebook. The criterion for the codebook design is to minimize the maximum cross-correlation between the signatures of the different codebooks. In order to obtain such codebooks, we use the iterative algorithm of alternating projection [9, 10], and apply it in a fashion similar to [11], in which the authors used it to perform packing of subspaces in the Grassmannian manifold. We show how the algorithm can be modified to support NOMA codebooks, that is, codebooks with non-orthogonal vectors, and how to perform the packing with respect to the maximum cross-correlation. Note that including power constraints into the design procedure can lead to a better design of the signatures [8, 12]; however, in this work, we assume a passive scheme, in which the base station is unaware of the transmit powers of the interfering users from the interfering cells, and that the users are uniformly distributed within the cells. In Sec. 4, we apply the designed codebooks to two interfering cells for both an OMA and a NOMA system with various interference levels, in which the results show a considerable gain for the cell-edge user.

2. SYSTEM MODEL

Our focus is on the uplink of a NOMA system, where the number of active users is larger than the spreading length. Let the spreading length be N, the number of active users that are simultaneously transmitting be M, then the received signal at the base station for some instant is given by

$$\mathbf{y} = \sum_{m=1}^{M} h_m \sqrt{P_m} \mathbf{s}_m x_m + \mathbf{n}\,, \tag{1}$$

where $x_m \in \mathbb{C}$, $\mathbf{s}_m \in \mathbb{C}^{N\times 1}$, $\sqrt{P_m} \in \mathbb{R}^+$, $h_m \in \mathbb{C}$ are the transmit symbol, unit-norm signature, transmit power, fading coefficient for the m^{th} user, respectively, and $\mathbf{n} \in \mathbb{C}^{N\times 1}$ is the additive white Gaussian noise. The fading coefficient being a scalar across the entire spreading length (i.e., flat fading) is a valid assumption when the spreading length is short enough compared to the channel coherence length. At the base station, a codeword-level (CWL) minimum mean squared error (MMSE) with successive interference cancellation (SIC) receiver (or its enhancements [13]) is used to detect the users, removing them in a successive manner based on their post-equalization signal-to-interference-plus-noise ratio (SINR).

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From the code-domain perspective, the performance of the system depends on the codebook of signatures defined as

$$\mathbf{S} \in \mathbb{C}^{N\times M} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \ldots & \mathbf{s}_M \end{bmatrix} . \tag{2}$$

It can be clearly seen that a codebook having low cross-correlation between the signatures $\mathbf{s}_m$ is desired, as it allows for a better user separability at the base station. In the case of an OMA system, it is possible to have signatures that are orthogonal, that is, $\mathbf{s}_n^H\mathbf{s}_m = 0$, $n \neq m$. This can be achieved by employing any $N\times N$ unitary matrix as the codebook. For a NOMA system, however, the signatures can no longer be orthogonal, and therefore the goal then is to find signatures with low cross-correlation. One criterion for the signatures design is to minimize the maximum cross-correlation [14], stated as

$$\underset{\mathbf{S}\in\mathbb{C}^{N\times M}}{\arg\min} \max_{\mathbf{s}_n,\mathbf{s}_m\in\mathbf{S}} |\mathbf{s}_n^H\mathbf{s}_m| , \quad n\neq m . \tag{3}$$

Optimal solutions to this problem are known as Grassmannian frames, in connection to the Grassmannian line packing problem [15]. Finding a closed-form solution to this optimization problem is only possible for a handful combinations of N and M [16]. Alternatively, numerical methods such as [17,18] can be used. Once the codebook with desired properties is designed (e.g., according to (3)), it can be applied to the cellular system. Extra terms have to be added to (1), in order to account for the contribution of the users from the interfering cells.

3. JOINT CODEBOOK DESIGN

3.1. Codebook Design Problem

As we mentioned in the introduction, we do not include any power constraints into the design problem, but rather assume a passive system where the base station is unaware of the power of the interfering users from the interfering cells, and that the received power of the users from the primary and interfering cells is uniformly distributed, as clarified later in the results section. Instead of reusing the codebook $\mathbf{S}$ across the interfering cells, we use different codebooks that have the same correlation properties as of $\mathbf{S}$, however, they are designed jointly, in an attempt to reduce the impact of one codebook on another. The reason for requiring them to have the same correlation properties is because we do not want to reduce the intercell interference at the cost of an increased intracell interference, and this is achieved by preserving the internal structure of the codebooks. The correlation properties of the codebook are fully captured by the Gramian $\mathbf{S}^H\mathbf{S}$. Therefore, we first have to determine what freedom do we have in designing those codebooks, given that their Gramian is the same. The answer to that is, codebooks with the same Gramian are equivalent up to an isometry. In our case, since all the signatures have unit-norm, then the isometry is just a rotation (including reflections). To understand why this is true, consider two codebooks $\mathbf{S}_1$ and $\mathbf{S}_2$ with identical Gramian,

$$\mathbf{S}_1^H\mathbf{S}_1 = \mathbf{S}_2^H\mathbf{S}_2 . \tag{4}$$

We can introduce two unitary matrices $\mathbf{U}_1, \mathbf{U}_2 \in \mathbb{C}^{N\times N}$ ($\mathbf{U}_1^H\mathbf{U}_1 = \mathbf{U}_2^H\mathbf{U}_2 = \mathbf{I}_N$) without altering the equality

$$\begin{aligned} \mathbf{S}_1^H\mathbf{U}_1^H\mathbf{U}_1\mathbf{S}_1 &= \mathbf{S}_2^H\mathbf{U}_2^H\mathbf{U}_2\mathbf{S}_2 , \\ (\mathbf{U}_1\mathbf{S}_1)^H(\mathbf{U}_1\mathbf{S}_1) &= (\mathbf{U}_2\mathbf{S}_2)^H(\mathbf{U}_2\mathbf{S}_2) , \\ \Rightarrow \mathbf{U}_1\mathbf{S}_1 &= \mathbf{U}_2\mathbf{S}_2 , \end{aligned} \tag{5}$$

from which follows that

$$\mathbf{S}_2 = \mathbf{U}_2^{-1}\mathbf{U}_1\mathbf{S}_1 . \tag{6}$$

The quantity $\mathbf{U}_2^{-1}\mathbf{U}_1$ is just another unitary matrix, and therefore the two codebooks are related by a unitary transformation. In other words, $\mathbf{S}_2$ is a rotation (including reflections) of $\mathbf{S}_1$.

Next, we need to answer two questions; what determines a good rotation, is there a metric for it? and then, given the target metric, how to perform such rotation in the first place? Ultimately, we would like to have $\mathbf{S}_1^H\mathbf{S}_2 = \mathbf{0}$, that is, the codebooks are orthogonal and do not interfere with each other. However, in our case, this is not possible at all, because we usually have $M \geq N$, and according to (3), we design the signatures such that they are as far apart as possible in the ambient space $\mathbb{C}^N$. Therefore, we end up having $\text{span}\{\mathbf{S}_1\} = \text{span}\{\mathbf{S}_2\} = \mathbb{C}^N$. Since the codebooks cannot be orthogonal with respect to each other, we turn our attention to getting close to orthogonality by means of some metric $\|\mathbf{S}_1^H\mathbf{S}_2\|$. Metrics such as the Frobenius or spectral norm cannot be used here. To see why, consider the trace definition of the Frobenius norm

$$\|\mathbf{S}_1^H\mathbf{S}_2\|_F^2 = \text{trace}\left(\mathbf{S}_1^H\mathbf{S}_2\mathbf{S}_2^H\mathbf{S}_1\right) . \tag{7}$$

A class of optimal solutions to (3) are equiangular tight frames (ETFs) satisfying $\mathbf{S}\mathbf{S}^H = \frac{M}{N}\mathbf{I}_N$ [17]. Therefore, if our codebooks are ETFs, then $\mathbf{S}_1\mathbf{S}_1^H = \mathbf{S}_2\mathbf{S}_2^H = \frac{M}{N}\mathbf{I}_N$, and thus

$$\begin{aligned} \|\mathbf{S}_1^H\mathbf{S}_2\|_F^2 &= \frac{M}{N}\text{trace}\left(\mathbf{S}_1^H\mathbf{S}_1\right) \\ &= \frac{M}{N}\|\mathbf{S}_1\|_F^2 . \end{aligned} \tag{8}$$

This is a constant that does not depend on how $\mathbf{S}_2$ is rotated with respect to $\mathbf{S}_1$. In a similar fashion, using the eigenvalue definition of the spectral norm, we can show that it also takes a constant value for such codebooks. Based on that, we recognize that the chosen metric should be valid, irrespective of the internal structure of the codebooks.

Consider the element-wise maximum norm defined as

$$\|\mathbf{A}\|_{\max} = \max_{k,l} |[\mathbf{A}]_{kl}| , \tag{9}$$

where $[\mathbf{A}]_{kl}$ is the element at the k^{th} row and l^{th} column of the matrix $\mathbf{A}$. By applying it to our problem, we obtain

$$\begin{aligned} \|\mathbf{S}_1^H\mathbf{S}_2\|_{\max} &= \max_{k,l} |[\mathbf{S}_1^H\mathbf{S}_2]_{kl}| \\ &= \max_{\mathbf{a}\in\mathbf{S}_1,\, \mathbf{b}\in\mathbf{S}_2} |\mathbf{a}^H\mathbf{b}| . \end{aligned} \tag{10}$$

Minimizing this metric would then guarantee that the maximum cross-correlation between the signatures of the different codebooks is kept low. In other words, minimizing (10) guarantees that a maximum separation (or angle) between the codebooks is maintained. Therefore, we adopt this metric in our joint design method. Note that minimizing (10) is a min-max problem, which is difficult to solve. Instead, we seek to bring it below a certain level μ.

Let the number of codebooks be K; the goal is to find codebooks $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_K$ with the following conditions

$$\begin{aligned} \mathbf{S}_i^H\mathbf{S}_i &= \mathbf{S}^H\mathbf{S} , \quad \text{for every } i , \\ \|\mathbf{S}_i^H\mathbf{S}_j\|_{\max} &\leq \mu , \quad \text{for every } i\neq j . \end{aligned} \tag{11}$$

The first condition defines the internal structure of the codebooks (e.g., according to (3)), while the second condition enforces the cross-correlation between the signatures of the different codebooks to go below a specific level μ. Note that this level μ cannot be arbitrarily small, but it is rather limited by the maximum packing possible in the ambient space. We are unaware of lower bounds to the packings of codebooks with non-orthogonal vectors, but μ is certainly larger than the Grassmannian bounds of (3), such as the Welch bound [19], and it cannot take values larger than 1. Our choice of μ is rather experimental, as explained later in the results section.

The construction of such rotated codebooks can be performed using the iterative algorithm of alternating projection, in a fashion similar to the problem of subspace packing in the Grassmannian manifold [11]. In our case, we do not have subspaces, but rather codebooks that span the whole ambient space with generally non-orthogonal vectors (signatures), and we pack those codebooks with respect to the $\|.\|_{\max}$ norm.

3.2. Alternating Projection

Let $\mathbf{\Sigma} = \begin{bmatrix}\mathbf{S}_1 & \mathbf{S}_2 & \dots & \mathbf{S}_K\end{bmatrix}$ be the matrix containing the codebooks. The Gramian of the codebooks is given by

$$\mathbf{G} = \mathbf{\Sigma}^H\mathbf{\Sigma} = \begin{bmatrix}\mathbf{S}_1^H\mathbf{S}_1 & \mathbf{S}_1^H\mathbf{S}_2 & \cdots & \mathbf{S}_1^H\mathbf{S}_K \\ \mathbf{S}_2^H\mathbf{S}_1 & \mathbf{S}_2^H\mathbf{S}_2 & \cdots & \mathbf{S}_2^H\mathbf{S}_K \\ \vdots & \dots & \ddots & \vdots \\ \mathbf{S}_K^H\mathbf{S}_1 & \mathbf{S}_K^H\mathbf{S}_2 & \cdots & \mathbf{S}_K^H\mathbf{S}_K\end{bmatrix}. \quad (12)$$

The properties of the Gramian matrix $\mathbf{G}$ meeting the conditions in (11) are

- $\mathbf{G}$ is Hermitian.
- The diagonal blocks satisfy $\mathbf{G}_{ii} = \mathbf{S}^H\mathbf{S}$.
- Every off-diagonal block satisfy $\|\mathbf{G}_{ij}\|_{\max} \leq \mu$.
- $\mathbf{G}$ is positive semi-definite.
- $\mathbf{G}$ has a rank of N.
- $\mathbf{G}$ has a trace equal to MK.

The first three properties are structural properties, while the last three are spectral. Define the structural constraints set as

$$\mathcal{H} = \{\mathbf{H} \in \mathbb{C}^{MK\times MK} : \mathbf{H} = \mathbf{H}^H, \mathbf{H}_{ii} = \mathbf{S}^H\mathbf{S}, \|\mathbf{H}_{ij}\|_{\max} \leq \mu, \forall\, i \neq j\}, \quad (13)$$

and the spectral constraints set as

$$\mathcal{G} = \{\mathbf{G} \in \mathbb{C}^{MK\times MK} : \mathbf{G} \succcurlyeq 0, \text{rank}(\mathbf{G}) = N, \text{trace}(\mathbf{G}) = MK\}. \quad (14)$$

We use the alternating projection algorithm to find a matrix $\mathbf{G}$ satisfying both constraint sets. Let the maximum number of iterations be T, the algorithm is summarized as follows [11]

1. Start with a random Hermitian $\mathbf{G}^{(0)} \in \mathbb{C}^{MK\times MK}$.
2. Set the iteration number $t = 0$.
3. Solve a nearest matrix problem to the set $\mathcal{H}$
$$\mathbf{H}^{(t)} = \underset{\mathbf{H}\in\mathcal{H}}{\arg\min}\|\mathbf{H} - \mathbf{G}^{(t)}\|_F\,. \quad (15)$$
4. Solve a nearest matrix problem to the set $\mathcal{G}$
$$\mathbf{G}^{(t+1)} = \underset{\mathbf{G}\in\mathcal{G}}{\arg\min}\|\mathbf{G} - \mathbf{H}^{(t)}\|_F\,. \quad (16)$$
5. Break if $t = T$. Otherwise, increase t and go to 3).

Once $\mathbf{G}^{(T+1)}$ is obtained, we can retrieve $\mathbf{\Sigma}$ by means of an eigendecomposition.

Two nearest matrix problems need to be solved. Let us start with (15). From the constraints set $\mathcal{H}$, the diagonal blocks are forced to be $\mathbf{H}_{ii} = \mathbf{S}^H\mathbf{S}$. Therefore, the nearest matrix problem is then concerned with the off-diagonal blocks only ($i \neq j$)

$$\mathbf{H}_{ij} = \underset{\mathbf{A}\in\mathbb{C}^{M\times M}}{\arg\min}\,\|\mathbf{A} - \mathbf{G}_{ij}\|_F^2\,, \quad \|\mathbf{A}\|_{\max} \leq \mu\,. \quad (17)$$

Both the objective and constraint functions are convex. Therefore, a unique solution exists.

Proposition 1. For the optimization problem in (17), the solution is given by

$$[\mathbf{H}_{ij}]_{kl} = \begin{cases}[\mathbf{G}_{ij}]_{kl}\,, & [\mathbf{G}_{ij}]_{kl} \leq \mu\,, \\ \mu[\mathbf{G}_{ij}]_{kl}/|[\mathbf{G}_{ij}]_{kl}|\,, & [\mathbf{G}_{ij}]_{kl} > \mu\,.\end{cases} \quad (18)$$

That is, every element of the off-diagonal block $\mathbf{G}_{ij}^{(t)}$ which has its magnitude larger than μ, is scaled to have a magnitude of exactly μ.

Proof. The key observation here is that the constraint $\|\mathbf{A}\|_{\max} \leq \mu$ applies element-wise. It does not enforce a relationship across the entire structure of matrix $\mathbf{A}$ from the feasible set. Expand the Frobenius norm in (17)

$$\|\mathbf{A} - \mathbf{G}_{ij}\|_F^2 = \sum_k\sum_l |[\mathbf{A}]_{kl} - [\mathbf{G}_{ij}]_{kl}|^2\,. \quad (19)$$

Since there is no relationship between the elements, and since each of the sum terms is non-negative, then the minimum of $\|\mathbf{A} - \mathbf{G}_{ij}\|_F^2$ is obtained, when every sum term $|[\mathbf{A}]_{kl} - [\mathbf{G}_{ij}]_{kl}|^2$ is minimized individually. We can then reformulate (17) equivalently in terms of the elements

$$[\mathbf{H}_{ij}]_{kl} = \underset{[\mathbf{A}]_{kl}\in\mathbb{C}}{\arg\min}\,|[\mathbf{A}]_{kl} - [\mathbf{G}_{ij}]_{kl}|^2\,, \quad |[\mathbf{A}]_{kl}| \leq \mu\,, \quad (20)$$

from which directly follows that if $|[\mathbf{G}_{ij}]_{kl}| \leq \mu$, the solution is $[\mathbf{H}_{ij}]_{kl} = [\mathbf{G}_{ij}]_{kl}$. Otherwise, we look for a solution that satisfies the constraint and at the same time is the closest to $[\mathbf{G}_{ij}]_{kl}$, and this is equal to $[\mathbf{H}_{ij}]_{kl} = \mu[\mathbf{G}_{ij}]_{kl}/|[\mathbf{G}_{ij}]_{kl}|$. ☐

As for the second nearest matrix problem in (16), it is no different than the one in [11], for which the authors showed it can be solved efficiently by applying the Karush-Kuhn-Tucker (KKT) conditions [20].

The flexibility of the alternating projection algorithm allows it to produce signatures with various properties, such us signatures with low peak-to-average-power ratio (PAPR) [17]. This, of course, comes at the cost of more complicated constraints set, or by alternating to additional sets. However, as the codebooks are usually designed in an offline fashion, the extra computational complexity should not be a problem.

4. RESULTS

We consider the scenario of a spreading-based system consisting of two interfering cells, with spreading length $N = 8$. Our focus is on the uplink performance of the cell-edge user, as it is the one that suffers the most from the intercell interference. First, we construct a NOMA codebook $\mathbf{S}$ of dimensions 8×24 according to (3). The

Welch bound tells us that at such N and M, there is a possibility that a Grassmannian frame (codebook) with a maximum cross-correlation of 0.2949 exists. Using the method in [17], we were able to construct a codebook with a maximum cross-correlation of 0.2981. Next, we apply our method in Section 3 to find the rotated codebooks for the two cells. For the choice of μ, we start with a large value, say 0.9. If the algorithm succeeds in finding such packing, then we reduce μ to 0.8. If it again succeeds, then we further reduce it, and so on. The best packing we found was for $\mu = 0.52$. Going below that level caused the algorithm to fail in maintaining the first condition of (11). The reason for that is due to the limit on the maximum possible packing in the ambient space, which then would prevent finding a valid Gramian with such properties in the step of (16). Furthermore, we also performed the packing for an OMA system, by taking $\mathbf{S}$ to be some unitary matrix of dimensions 8×8. We then applied our method and obtained a best packing of the two codebooks for $\mu = 1/\sqrt{8}$.

To mimic an uplink transmission, the average signal-to-noise ratio (SNR) of the primary users at the base station is uniformly distributed in the range $[4, 20]$ dB. This corresponds to the variation between the received power of the users due to their position within the cell. The cell-edge user under consideration has its average SNR fixed to 4 dB. The average SNR of the interfering users from the interfering cell at the primary base station is distributed in the range $[-12, 4] - P_{\text{cell-edge}}/I_{\text{strongest}}$ dB, where $P_{\text{cell-edge}}/I_{\text{strongest}}$ is the ratio between the received power of our cell-edge user to the received power of the strongest interferer. When this ratio is zero, then the strongest interferer from interfering cell can be as strong as our cell-edge user.

We assume the users to transmit 256 symbols long messages, using 4-QAM with turbo coding with a code rate of $1/3$. During the transmission of a single message, the user employs the same spreading signature. On the transmission of the next message, it gets assigned a different signature, with the assumption that no collision occurs between the users of the same cell. For the channel, flat fading is assumed here, which holds well as an assumption for such a short spreading length. The channel estimation is ideal, that is, the receiver is assumed to have perfect knowledge of the channel. As for the multiuser detector, the CWL-MMSE-SIC receiver is used.

In Figure 1, the block error ratio (BLER) of the cell-edge user at different interference levels is shown for a full load OMA system; that is, $A_P = 8$ and $A_I = 8$, where A_P, A_I is the number of active users in the primary and interfering cells, respectively. The 'reuse' curve corresponds to the strategy where a single codebook is reused in each cell, while the 'joint' curve corresponds to the jointly designed codebooks obtained using our method. We observe that the jointly designed codebooks are able to sustain 2.5 dB higher interference power, while providing the same BLER as the reused codebooks. Next, we consider an overloaded NOMA system of $A_P = 24$, and different number of interfering users $A_I = 8, 16, 24$. For a spreading length of 8, the 8, 16, 24 activity corresponds to an overloading of 100%, 200%, 300%, respectively. We see a similar trend in Figure 2 compared to the OMA system; the jointly designed NOMA codebooks outperform the reuse strategy. We observe that the gain becomes smaller, as the number of interfering users increases. For $A_I = 8$, we see the biggest difference with a gain of 2 dB, and then for $A_I = 16$, the gap reduces to 1 dB, and finally to 0.6 dB for the case of $A_I = 24$. This is to be expected due to the high amount of interference experienced at such high overloading levels in both the primary and interfering cells. We plan next to apply the method to more than two interfering cells, and consider a more realistic simulation environment using our new Vienna 5G

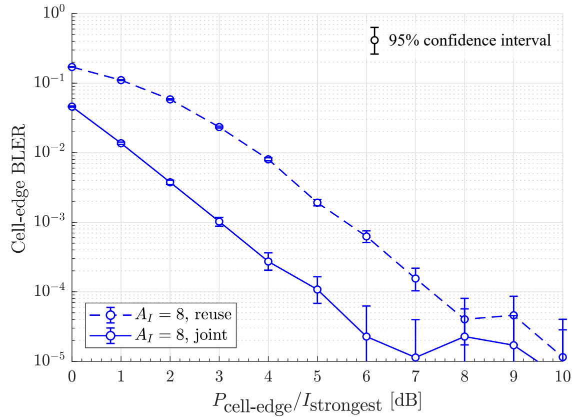

Fig. 1. Performance of the OMA cell-edge user for $A_P = 8$ (100%), and $A_I = 8$ (100%). Code rate of $1/3$ is considered here.

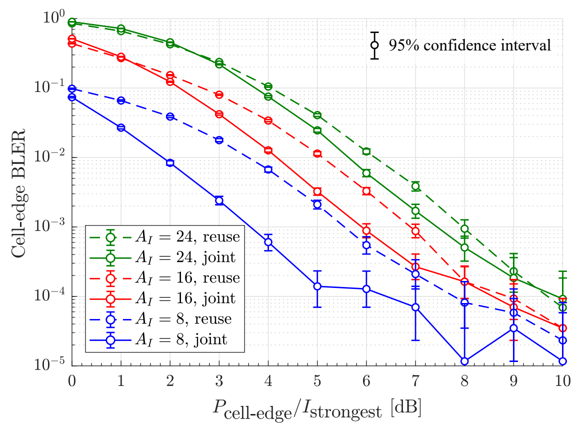

Fig. 2. Performance of the NOMA cell-edge user for $A_P = 24$ (300%), and $A_I = 8, 16, 24$ (100, 200, 300%). Code rate of $1/3$ is considered here.

Link- and System-Level Simulators[1] [21,22].

5. CONCLUSION

In this paper, we propose a method for reducing intercell interference in NOMA and OMA systems, by jointly designing the signature codebooks of the interfering cells. The considered criterion for the codebook design is to minimize the maximum cross-correlation across the signatures of the different codebooks. We use the algorithm of alternating projection to obtain those codebooks. We show the necessary modifications to the algorithm, and derive a close-form solution to the required optimization. We finally put the designed codebooks to test, by applying them to the uplink of two interfering cells, where the users are uniformly distributed in the cells, in which the results show a considerable gain for the cell-edge users.

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