COMPLEMENTARY SEQUENCE ENCODING FOR 1D AND 2D CONSTANT-MODULUS OFDM TRANSMISSION AT MILLIMETER WAVE FREQUENCIES

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ABSTRACT

This paper develops a method for constructing an OFDM signal from a pair of complementary sequences so that the resulting signal is constant-modulus. A recursive method of constructing complementary sequences is developed such that the length of the signal grows linearly with the number of information symbols encoded. The constant-modulus property is exploited at each stage of the backwards symbol-decoding iteration through simple means to progressively reduce noise. For diversity comparing purposes, the new scheme is compared in symbol and frame error rates to m-PSK transmission with time-diversity in AWGN. The constant-modulus signal construction is then extended to two dimensions for the motivating application where we expect this work to have the greatest impact: massive MIMO at mmWave frequencies.

***Index Terms*—** Complementary sequences, OFDM, constant modulus, massive MIMO, mmWave

1. INTRODUCTION

There is significant prior work regarding the development of information-carrying, complementary sequences for use in conjunction with OFDM to reduce the Peak-to-Average Power Ratio (PAPR) [1],[2]. In this paper, we present a novel scheme for how to map the sequence values of the complementary pair to even and odd carrier frequencies to form a truly constant-modulus OFDM signal. This facilitates the use of a nonlinear amplifier while offering the benefits of OFDM (e.g., cyclic prefix) and constant-modulus exploitation.

With respect to the construction of information-carrying, complementary sequence pairs, prior research focused on determining the number of complementary sequence pairs for a given length L when the values of the sequence are restricted to a specific symbol set or finite alphabet [3],[4]. The number of length L sequences that form complementary pairs is on the order of $\log L$, meaning the length of the sequence grows exponentially with the number of information symbols to be transmitted. In contrast, we develop a recursive technique for constructing complementary sequence pairs such that the sequence length grows linearly with the number of information symbols encoded. Finally, we extend the constant-modulus OFDM signal generation to two dimensions, for use in massive MIMO at mmWave frequencies.

2. SYMBOL ENCODING OF A COMPLEMENTARY SEQUENCE PAIR FOR OFDM

Consider an ODFM signal equal to a sum of orthogonal sinewaves.

$$s(t) = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{T}t} \quad -\frac{T}{2} < t < \frac{T}{2} \tag{1}$$

The sinewaves are orthogonal according to:

$$\frac{1}{T}\int_{-T/2}^{T/2} e^{jk\frac{2\pi}{T}t} e^{-j\ell\frac{2\pi}{T}t} dt = \delta[k-\ell] \tag{2}$$

Consider dividing the sinewaves into even and odd carriers as

$$s_e(t) = \sum_{k=0}^{N/2-1} c_1[k] e^{j(2k)\frac{2\pi}{T}t} \quad -\frac{T}{2} < t < \frac{T}{2} \tag{3}$$

$$s_o(t) = \sum_{k=0}^{N/2-1} c_2[k] e^{j(2k+1)\frac{2\pi}{T}t} \quad -\frac{T}{2} < t < \frac{T}{2} \tag{4}$$

Each even carrier is periodic, repeating at least every $T/2$ such that the segment in $-\frac{T}{2} < t < 0$ is equal to the segment $0 < t < \frac{T}{2}$. For the odd-carriers, the segment in $-\frac{T}{2} < t < 0$ is the negative of the segment in $0 < t < \frac{T}{2}$.

If complex-amplitude sequences $c_1[n]$ and $c_2[n]$ form a complementary pair such that their respective autocorrelations sum to a scalar multiple of a Kronecker Delta function

$$r_{11}[\ell] + r_{22}[\ell] = (E_1 + E_2)\delta[\ell] \tag{5}$$

where $r_{kk}[\ell] = c_k[\ell] * c_k^*[-\ell]$ is the autocorrelation sequence and E_k is the energy for $c_k[n], k = 1, 2$, it easy to show that

$$|s_e(t)|^2 + |s_o(t)|^2 = \text{constant independent of } t \tag{6}$$

As a result, it follows that the signal below is a constant-modulus, i.e., constant-amplitude, phase-only signal.

$$x(t) = s_e(t) + je^{j\phi_c(t)} s_o(t) = \text{constant modulus signal} \tag{7}$$

where the phase-correction (spreading) signal is given by

$$e^{j\phi_c(t)} = e^{j\{\angle s_e(t) - \angle s_o(t)\}} \quad (8)$$

To maintain orthogonality between the even carriers and the phase-corrected odd carriers, we force the phase-correction in the segment $-\frac{T}{2} < t < 0$ to be the same as the phase-correction in the segment $0 < t < \frac{T}{2}$. Relative to eq. (7), this is equivalent to implementing $-j$ in $-\frac{T}{2} < t < 0$, while implementing a $+j$ in $0 < t < \frac{T}{2}$. The net result is to maintain mutual orthogonality between the even carriers and the phase-corrected odd carriers for computational simplicity.

Since the phase of the composite odd carrier signal is lost in the signal formed above, a single transmission with all necessary information to recover the original sequences is

$$\tilde{c}_1[n] = c_1[n] + c_2[n - \frac{N}{4}] \quad \tilde{c}_2[n] = c_1[n] - c_2[n - \frac{N}{4}] \quad (9)$$

where $c_1[n]$ and $c_2[n]$ are complementary sequences of length $L = N/4$ such that $\tilde{c}_1[n]$ and $\tilde{c}_2[n]$ are complementary sequences of length $N/2$. $\tilde{c}_1[n]$ are the complex-amplitudes for the $N/2$ even carriers, while $\tilde{c}_2[n]$ are the complex-amplitudes for the $N/2$ odd carriers. This has the added benefit of achieving additional frequency diversity.

3. COMPLEMENTARY SEQUENCE CODING OF INFORMATION SYMBOLS

The basic building blocks for forming a complementary sequence pair at each iteration from the symbols is described below, where I_k is the $k-th$ information symbol equal to a member of a finite alphabet and r is a small, real-valued, positive constant on the order of 0.1 for a typical FFT length such as $N = 1024$ ($L = 256$).

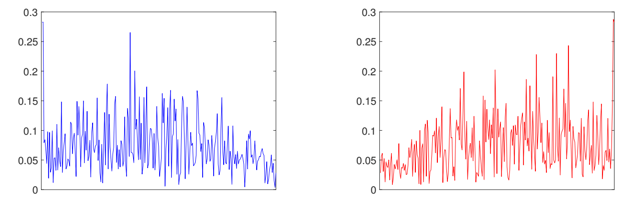
$$c_1^{(k+1)}[n] = I_k c_1^{(k)}[n] - r c_2^{(k)}[n-1] \quad (10)$$
$$c_2^{(k+1)}[n] = r c_1^{(k)}[n] + I_k^* c_2^{(k)}[n-1] \quad (11)$$

The small value of r limits the magnitudes of the terms summed in the middle of the sequence, maintaining a reasonable range of sequence values. We typically commence this iteration with the simple length-2 complementary sequences

$$c_1^{(0)}[n] = \{1, 1\} \qquad c_2^{(0)}[n] = \{1, -1\} \quad (12)$$

The reasons that the above iteration yields a new pair of complementary sequences at each iteration, with the length growing by 1 at each iteration, are two-fold: (1) a time-shift does not affect autocorrelation and (2) a 2×2 unitary transformation applied to a pair of complementary sequences yields another pair of complementary sequences. The unitary transformation used here is

$$\mathbf{S}_k = \frac{1}{r^2 + |I_k|^2} \begin{bmatrix} I_k & -r \\ r & I_k^* \end{bmatrix} \quad (13)$$



Sequence 1 Sequence 2

Fig. 1. Magnitudes of typical pair of complementary sequences of length $L = 256$, $r = 0.1$

where, again, r is real-valued. Fig. 1 shows the magnitudes of a typical pair of complementary sequences of length $L = 256$, using $r = 0.1$. We use the two respective values of each sequence at both the beginning, $n = 0$, and at the end, $n = N_k$, to extract the information symbols at each iteration as we progressively work backwards. Focusing on the endpoints, we have

$$\begin{bmatrix} c_1^{(k+1)}[0] & c_2^{(k+1)}[0] \\ c_1^{(k+1)}[N_k] & c_2^{(k+1)}[N_k] \end{bmatrix} = \begin{bmatrix} c_1^{(k)}[0] & 0 \\ 0 & c_2^{(k)}[N_k] \end{bmatrix} \mathbf{S}_k$$

This leads to the following primary mechanism for estimating the k-th symbol at the k-th iteration

$$\hat{I}_k^* c_1^{(k+1)}[0] + r c_2^{(k+1)}[0] = 0 \quad (14)$$
$$-r c_1^{(k+1)}[N_k] + \hat{I}_k c_2^{(k+1)}[N_k] = 0 \quad (15)$$

Since r is a known value, the value of $\hat{I}_k$ can be calculated from both eq. (14) and eq. (15). By encoding information symbols using eq. (10) and eq. (11), the length of each sequence at each iteration, denoted $N_k + 1$, grows by 1 with the incorporation of each new information symbol; the two sequences form a complementary pair at each and every iteration.

From the two received complementary sequences $\hat{c}_1^{(L)}[n]$ and $\hat{c}_2^{(L)}[n]$, the symbol estimate $\hat{I}_{i,L}$ for $i = 1, 2$ can be estimated from eq. (14) and eq. (15), respectively. These estimates are independent in additive white gaussian noise, so the arithmetic mean provides the best estimate of the received symbol. Choosing the nearest symbol $\hat{I}_L$ to the mean $I_{L,est}$, we can form the estimated inverse of eq. (13) (conjugate transpose, since S_k is Hermitian), allowing us to estimate the two sequences, $\hat{c}_1^{(L-1)}[n]$ and $\hat{c}_2^{(L-1)}[n]$ using

$$\hat{c}_1^{(L-1)}[n] = \hat{I}_L^* \hat{c}_1^{(L)}[n] + r\hat{c}_2^{(L)}[n] \quad (16)$$
$$\hat{c}_2^{(L-1)}[n+1] = -r\hat{c}_1^{(L)}[n] + \hat{I}_L \hat{c}_2^{(L)}[n] \quad (17)$$

This transformation will always create a complementary sequence pair of length L, regardless of the value of $\hat{I}_L$, since we are using the conjugate transpose of eq. (13). However, if the symbol estimate is correct, eq. (14) and eq. (15) are satisfied, meaning that $\hat{c}_1^{(L-1)}[L] = \hat{c}_2^{(L-1)}[-1] = 0$, and the two estimated sequences are also a complementary sequence pair of length $L-1$. The process can then be repeated on the two shortened sequences to get $\hat{I}_k$ for $k = L-1, ..., 1$.

Although, with properly selected r, any symbol alphabet is possible in this scheme (PSK, QAM, etc.), an m-PSK symbol alphabet, where $|I_k| = 1$, provides a decoding advantage. A symbol error can then be viewed as a multiplication by eq. (14), with $e^{j\phi_e}$, the difference between estimated and actual symbol, in place of I_k. For sufficiently small r, with m-PSK symbols, this is approximately a rotation of the entire sequences $\hat{c}_1^{(k)}[n]$ and $\hat{c}_2^{(k)}[n]$ by $\pm\phi_e$ (the sequences will remain complementary, since this is a unitary transformation). This phase rotation will rotate symbol estimates $\hat{I}_{k-1}$ and beyond by the same phase error. Differential encoding can protect from this error (to first order) at the cost of one encoded symbol. Pilot symbols can also be inserted into the series of information symbols, in order to rotate the sequence back to the correct phase. This phase rotation also means that although, in principle, $I_{k,est}$ could have been used instead of $\hat{I}_k$ in eq. (16) and eq. (17), a soft decoding rule will propagate phase differences and increase error rates.

If all symbols are decoded correctly, the end result will be a scalar multiple of eq. (12). A difference from these sequences alerts the receiver to an error in decoding, and can prompt a request for retransmission.

4. RECOVERING COMPLEMENTARY SEQUENCES FROM THE TRANSMITTED OFDM SIGNAL

Since the OFDM signal from section 2 is constant modulus, there are powerful denoising and channel estimation techniques that can be applied, including the constant-modulus (CM) algorithm [5] or a variant [6],[7]. After processing, the received sequence $\tilde{c}_{1,r}[n]$ can be recovered directly from the even carriers via standard OFDM demodulation. The even carrier estimates of the original sequences, $\hat{c}_{1,e}[n]$ and $\hat{c}_{2,e}[n]$ are the first and second halves of $\tilde{c}_{1,r}[n]$, respectively.

Given the estimates from the even carriers, the phase-correction for the odd carriers can be estimated as $\phi_c(t) = \angle s_{e,r}(t) - \angle \hat{s}_o(t)$, where $s_{e,r}(t)$ is the received even carrier signal, and $\hat{s}_o(t)$ is the estimate of the (phase uncorrected) odd carrier signal using $\hat{c}_{1,e}[n]$ and $\hat{c}_{2,e}[n]$ in eq. (9) and eq. (4). Multiplying the odd carriers by $e^{-j\hat{\phi}_c(t)}$ and demodulating the OFDM signal gives $\tilde{c}_{2,r}[n]$. The odd carrier estimates $c_{1,o}[n]$ and $c_{2,o}[n]$ are then the first half and the negative of the second half of $\tilde{c}_{2,r}[n]$, respectively. Since this second estimate is affected by the noise on both the even and odd carriers, the odd carrier sequence estimate is less accurate than the even carriers. However, the odd carriers do contain information that can be used to improve the estimates of $\hat{c}_1^{(L)}[n]$ and $\hat{c}_2^{(L)}[n]$, especially after processing.

Using the information encoding scheme of section 3, the sequences are complementary at all iterations $k = 1, ..., L$,

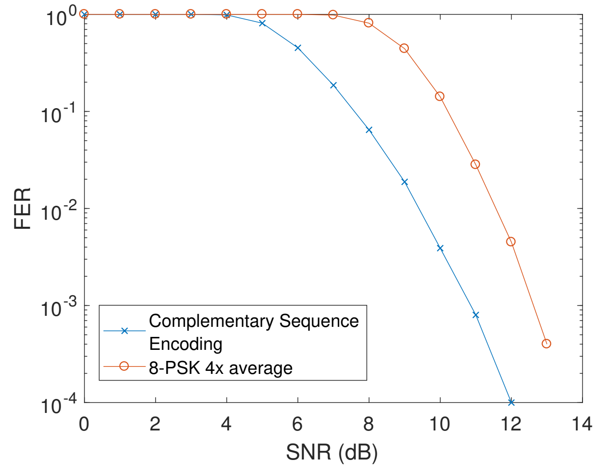


Fig. 2. Frame Error Rate

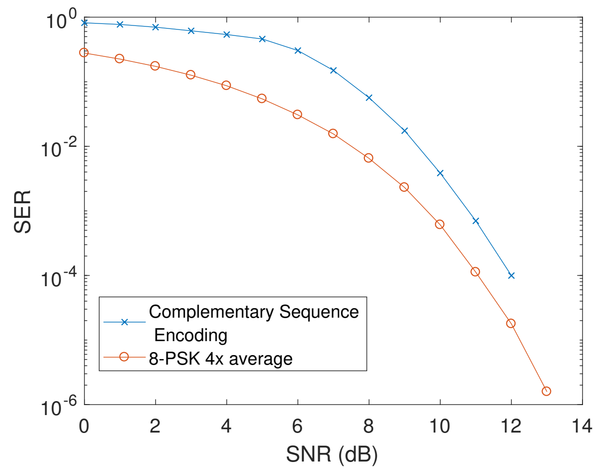


Fig. 3. Symbol Error Rate

so at any iteration of the decoding in section 3, the constant-modulus signal of section 2 can be re-formed, and any CM exploiting algorithm can be re-run to further reduce noise.

5. ERROR RATES IN AWGN

Figures 2 and 3 show frame and symbol error rates, respectively, vs SNR, generated from Monte Carlo simulations using AWGN with 10,000 runs at each SNR, using a block length of 256 8-PSK information symbols. Frame errors were counted as any symbol in the block being decoded incorrectly. Since the proposed method sends $N/4$ information symbols on N carrier frequencies, error rates are compared to transmitting the same 8-PSK symbols (with no pulse shaping) four times and averaging the received values. We used 8-PSK vice a multi-level alphabet to allow for differential encoding.

Other than differentially encoding the symbols, our method was used in a bare bones mode with no pilot symbols (see below and section 3), minimal (simple) exploitation of the constant-modulus property, and the suboptimal (but simple) symbol-by-symbol decoding scheme represented by eq. (14) and eq. (15). The symbols are clearly coupled in the complementary sequences, so some form of joint decoding is required for optimality. As a simple means of exploiting the CM property of the signal, the received signal was clipped, essentially retaining only phase information. This was repeated at each decoding step by synthesizing a sum of sinewaves from the *current* complementary pair similar to that prescribed previously at the full-length, and clipping to progressively reduce the noise. The odd carrier estimates were averaged with the even estimates via a weighted average starting at 0 and geometrically trending to even weighting.

The proposed method outperforms the 8-PSK with 4x time-diversity when viewed from a block perspective, but the overall symbol error rate is higher. The latter is due to the coupling of symbols, as discussed above, such that even a single error can cause substantial error propagation with the suboptimal symbol-by-symbol decoding scheme and no pilot symbols. In the 8-PSK case, an error in decoding one symbol has no effect on the remaining symbols. Again, this was a bare bones demodulation scheme for our method. Despite that, a frame error was significantly less likely to occur with our scheme. To combat error propagation, pilot symbols should be included, as discussed in section 3, and joint decoding should be employed. A Viterbi-like decoder at the symbol level is currently under development, since eq. (10) and eq. (11) can be viewed as convolutions:

$$c_1^{(k+1)}[n] = c_1^{(k)}[n] * \{I_k, 0\} + c_2^{(k)}[n] * \{0, -r\} \tag{18}$$

$$c_2^{(k+1)}[n] = c_1^{(k)}[n] * \{r, 0\} + c_2^{(k)}[n] * \{0, I_k^*\} \tag{19}$$

where $\{a, b\}$ denotes a length-2 sequence. Finally, these results include minimal exploitation of the constant-modulus property of the transmitted signal when decoding. Additional advantages could be obtained with more sophisticated methods, especially for channels with intersymbol interference.

6. EXTENSION OF CONSTANT MODULUS TRANSMISSION TO 2 DIMENSIONS

As discussed briefly, the primary motivating application for this work is for use at mmWave frequencies with massive MIMO. Assume a uniform linear array of M (equi-spaced by d) identical antennas at mmWave frequencies. DFT vectors of length M are used to form co-phasal beams toward spatial angles equi-spaced in μ-space, where $\mu = \frac{2\pi}{\lambda} d \cos\theta$ with λ denoting the wavelength and θ equal to the conical (physical) angle relative to the line of the array. What we propose is that each encoded sequence value be carried by a 2D sinewave equal to the outer-product of a length-N DFT vector (corresponding to a DT (discrete-time) temporal sinewave) and a length-M DFT vector corresponding to the DS (discrete-space) sinewave used as a beamforming vector. Note that the 1D case in section 2 was developed in Continuous-Time to emphasize that the transmitted OFDM signal is constant modulus for all time. In contrast, since the array is discrete, the 2D case is developed using the standard DT model for OFDM.

Assume the 2D complex sequences $c_1[m, n]$ and $c_2[m, n]$ form a 2D complementary pair such that their respective 2D autocorrelations sum to a scalar multiple of a 2D Kronecker Delta function, i.e.,

$$r_{11}[\ell, k] + r_{22}[\ell, k] = (E_1 + E_2)\delta[\ell, k] \tag{20}$$

where $r_{jj}[k, \ell] = c_j[k, \ell] * * c_j^*[-k, -\ell]$ is the 2D autocorrelation sequence and E_j is the energy for $c_j[m, n]$, $j = 1, 2$.

Let $\mathbf{C}_j$ denote the $N \times M$ matrix representation of the 2D sequence $c_j[n, m]$, $j = 1, 2$. $\mathbf{W}_N$ denotes the $N \times N$ Inverse DFT matrix. $\mathbf{W}_N^{(E)}$ is formed from every other column of $\mathbf{W}_N$ starting with the first column, corresponding to the even carriers; $\mathbf{W}_N^{(O)}$ is formed from every other column of $\mathbf{W}_N$ starting with the second column, corresponding to the odd carriers. With $\mathbf{W}_M$, $\mathbf{W}_M^{(E)}$, and $\mathbf{W}_M^{(O)}$ defined similarly, the (n, m) element of the $N \times M$ matrix $\mathbf{X}$ below dictates the value transmitted by the m-th antenna at DT time n:

$$\mathbf{X}_1 = \mathbf{W}_N^{(E)} \mathbf{C}_1 \mathbf{W}_M^{(E)H} + \mathbf{W}_N^{(E)} \mathbf{C}_2 \mathbf{W}_M^{(O)H} \tag{21}$$

$$\mathbf{X}_2 = \mathbf{W}_N^{(O)} \mathbf{C}_2 \mathbf{W}_M^{(O)H} - \mathbf{W}_N^{(O)} \mathbf{C}_1 \mathbf{W}_M^{(E)H} \tag{22}$$

$$\mathbf{X} = \mathbf{X}_1 + j e^{j\angle \mathbf{\Phi}_c} \odot \mathbf{X}_2 \quad \mathbf{\Phi}_c = \mathbf{X}_1 \odot \mathbf{X}_2^* \tag{23}$$

where $\odot$ denotes the point-wise product and the exponentiation is point-wise. Each product $\mathbf{W}_N \mathbf{C} \mathbf{W}_M^H$ is equivalent to a double-sum (over the elements of $\mathbf{C}$) of 2D sinewaves equal to the outer products of the columns of $\mathbf{W}_N$ with the rows of $\mathbf{W}_M^H$. Similar to the 1D case, the top and bottom halves of the phase-correction matrix are equal. Constructed with eq. (23), $\mathbf{X}$ has constant magnitude in both time and space despite transmitting on multiple beams simultaneously!

The encoded 2D complementary sequences can be estimated from $\mathbf{X}$ according to

$$\mathbf{C}_1 = \mathbf{W}_N^{(E)H} \mathbf{X} \mathbf{W}_M^{(E)} \qquad \mathbf{C}_2 = \mathbf{W}_N^{(E)H} \mathbf{X} \mathbf{W}_M^{(O)} \tag{24}$$

This estimate effectively uses the $N/2$ even temporal carriers, as in the 1D case. The odd carrier estimates can be obtained after estimating $\mathbf{\Phi}_c$ like in section 4.

Future work includes encoding information onto 2D complementary sequence pairs and the development of a joint (Viterbi-like) decoding scheme as mentioned in section 5. A simple method to create a 2D complementary sequence pair would be to construct sequences of the appropriate lengths M and N using section 3, and then take the outer products according to the methods outlined in [8]. With further research, the methods outlined in this paper provide advantages both on transmit (nonlinear amplifiers) and receive (CM signal processing) in both the 1D (generic OFDM) case and especially the 2D (mmWave, massive MIMO) case.

7. REFERENCES

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