Spectral Efficiency of Noncooperative Uplink Massive MIMO Systems with Joint Decoding

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***Abstract*—Massive multiple-input multiple-output (MIMO) systems have been drawing considerable interest. In the uplink, massive MIMO systems are commonly studied assuming that each base station (BS) decodes the signals of its user terminals separately and linearly while treating all interference as noise. Although this approach provides improved spectral efficiency (SE) in favorable channel conditions, it is generally sub-optimal from an information-theoretic perspective. In this work we characterize the SE of massive MIMO when the BSs are allowed to *jointly* decode the received signals. We consider two schemes for handling the interference, and derive their SEs for both finite and asymptotic number of antennas. Simulation tests of the proposed methods illustrate their gains in SE compared to standard separate linear decoding, and show that the standard approach fails to capture the actual achievable rates of massive MIMO systems, particularly when the interference is dominant.**

***Index terms*— Massive MIMO, spectral efficiency.**

I. Introduction

A major challenge of wireless systems is to meet the growing throughput demand. A promising method for increasing the spectral efficiency (SE) is to equip the base stations (BSs) with a large number of antennas. Such systems, referred to as *massive multiple-input multiple-output (MIMO) systems*, can provide improved throughput which scales with the number of antennas [1], and are the focus of considerable research attention.

Massive MIMO systems are noncooperative multi-user networks [2], where in each cell a set of single-antenna user terminals (UTs) are served by a multi-antenna BS. Each BS estimates the unknown channel to its UTs in a time-division duplex (TDD) manner prior to data transmission. The pioneering work of Marzetta [3] showed that, in certain favorable channel conditions and fixed number of UTs in each cell, and when the BSs perform separate linear decoding, the effects of channel estimation error and channel noise are made negligible as the number of BS antennas increases. Furthermore, performance is limited by pilot contamination, which is the interference caused by pilot reuse among cells. The impact of pilot contamination on SE was further studied in [4], [5]. The work [6] characterized the SE of linear decoders under more general channel conditions, when the number of UTs is proportional to the number of BS antennas. The tradeoff between SE and energy efficiency was studied in [7], while [8] treated the effect of UT allocation on SE. UT allocation schemes were considered in [9].

Focusing on the uplink, namely, on the communications from the UTs to the BSs, all the works above restricted the BSs to separately decode the signal of each UT based on some linear transformation of the channel output, such as matched filtering or minimum mean-squared error (MMSE) filtering, while interference is treated as noise. From an information-theoretic perspective, this approach is sub-optimal, as the massive MIMO network is a set of interfering multiple access channels (MACs). The capacity region of interfering MACs is unknown (actually, even the capacity region of simple two interfering point-to-point (PtP) channels is generally unknown [10, Ch. 6]). Thus, while separate decoding and treating interference as noise is generally a sub-optimal approach for such channels [10, Ch. 6], it is not clear how far it is from optimality. In fact, previous studies on the gap of massive MIMO schemes from optimality assumed no intercell interference, see, e.g., [1, Fig. 11] and [11, Fig. 4a]. Works studying similar channels without restricting the BSs to decode separately and treat interference as noise include [12], which studied the achievable ergodic sum-rate of MIMO MACs with interference and a-priori known channel in the asymptotic number of antennas regime; the works [13]–[15], which studied block-fading MIMO PtP channels; and [16], which focused on MIMO MACs with channel estimation and without interference.

In this work we study uplink massive MIMO systems, without restricting the BSs to decode separately. In addition, we do not collectively treat interference as noise, allowing the BSs to decode the interfering signals. We characterize the SE, measured as the achievable average ergodic rate over the entire network, of two approaches for handling the intercell interference, commonly studied in the network information theoretic context of intereference channels (ICs) [10, Ch. 6]: In the first scheme, each BS jointly decodes the signals of its corresponding UTs, and treats the intercell interference as noise. In the second approach, each BS decodes the signals of all the UTs in the network. Note that these methods do not consider how the UTs encode the transmitted signals, but only how the signals are decoded. For each approach we first characterize the SE for a finite number of BS antennas, and then analyze the SE in the massive MIMO regime, when the number of BS antennas approaches infinity, using results from random matrix theory.

While these techniques are computationally more complex than traditional separate decoding, characterizing their SE quantifies how much can be gained by removing the restrictions of the traditional approach and by properly treating massive MIMO systems as a set of interfering MACs. Furthermore, while the complexity of optimal joint decoding grows exponentially with the number of UTs, its performance can be approached using interference cancellation [17, Pg. 540], whose complexity only grows linearly with the number of UTs, i.e., the same complexity order as separate linear decoding [18], at the cost of increased decoding latency. Alternatively, recent developments in machine learning suggest that deep neural networks can perform accurate joint decoding at reduced complexity and latency, based on a sufficiently large training data, see, [19].

Our numerical study demonstrates that substantial gains in SE can be obtained by allowing the BSs perform joint decoding and by properly applying methods for handling the interference. This indicates that the approach of separately decoding a linear transformation of the channel output fails to capture the fundamental limits of massive MIMO networks. For example, we illustrate that when the intercell interference is dominant, a relevant scenario for future cellular networks [20], the traditional approach results in a SE which approaches zero, while, when the BSs are allowed to jointly decode the interference, non-negligible average ergodic rates are achieved.

The rest of this paper is organized as follows: Section II presents the system model. Section III derives the SE of the considered schemes. Section IV provides simulation examples.

Throughout the paper, we use boldface lower-case letters for vectors, e.g., $\mathbf{x}$; Matrices are denoted with boldface upper-case letters, e.g., $\mathbf{M}$, and $\mathbf{I}_n$ is the $n \times n$ identity matrix. Hermitian

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transpose, transpose, and stochastic expectation are written as $(\cdot)^H$, $(\cdot)^T$, and $\mathcal{E}\{\cdot\}$, respectively. Let $\stackrel{d}{=}$ denote equality in distribution. All logarithms are taken to base-2.

II. System Model

We consider a noncooperative multi-cell multi-user MIMO system with n_c cells, focusing on the uplink. In each cell, a BS equipped with n_t antennas serves n_u single-antenna UTs. We assume that n_t and n_u are sufficiently large to carry out large scale (asymptotic) analysis, and fix the ratio of the number of UTs to the number of antennas $\kappa \triangleq \frac{n_u}{n_t}$.

Let $\mathbf{D}_{k,l}$ be an $n_u \times n_u$ random diagonal matrix with positive diagonal entries $\{D_{k,l,m}\}_{m=1}^{n_u}$ representing the attenuation between the m-th UT of the l-th cell and the k-th BS, $k, l \in \{1, \ldots, n_c\} \triangleq \mathcal{N}_c$. We assume that $\{D_{k,l,m}\}_{m=1}^{n_u}$ are mutually independent, and identically distributed (for a fixed k, l). Let $\mathbf{H}_{k,l} \in \mathcal{C}^{n_t \times n_u}$ be a random proper-complex zero-mean Gaussian matrix with i.i.d. entires of unit variance, representing the instantaneous channel response between the UTs of the l-th cell and the k-th BS, $k, l \in \mathcal{N}_c$. The random matrices $\{\mathbf{H}_{k,l}\}_{k,l \in \mathcal{N}_c}$ are mutually independent, and independent of $\{\mathbf{D}_{k,l}\}_{k,l \in \mathcal{N}_c}$. Let $\mathbf{G}_{k,l} = \mathbf{H}_{k,l}\mathbf{D}_{k,l}$ be the random channel matrix from the UTs in the k-th cell to the l-th BS. We assume a block-fading model for $\{\mathbf{H}_{k,l}\}_{k,l \in \mathcal{N}_c}$, i.e., they remain constant only for a coherence duration of τ_c symbols. As in, e.g., [8], each BS knows its corresponding attenuation coefficients i.e., the k-th BS knows $\{\mathbf{D}_{k,l}\}_{l \in \mathcal{N}_c}$. Let $\mathbf{w}_k[i] \in \mathcal{C}^{n_t}$ be an i.i.d. zero-mean proper-complex Gaussian signal with covariance matrix $\sigma_W^2 \mathbf{I}_{n_t}$, representing the additive noise at the k-th BS.

Channel estimation is carried out in a TDD fashion, where the coherence duration τ_c is divided into a channel estimation phase, consisting of τ_p pilot symbols, and a data transmission phase, consisting of $\tau_d = \tau_c - \tau_p$ data symbols. During channel estimation, each UT sends a deterministic orthogonal pilot sequence (PS), where the PSs are the same in all cells. Letting $s_k[i]$ denote the i-th pilot symbol of the k-th UT, $k \in \{1, \ldots, n_u\} \triangleq \mathcal{N}_u$, and defining $\mathbf{s}[i] \triangleq [s_1[i], \ldots, s_{n_u}[i]]^T$, the channel output at the k-th BS, $k \in \mathcal{N}_c$, is given by

$$\mathbf{y}_k[i] = \sum_{l=1}^{n_c} \mathbf{G}_{k,l}\mathbf{s}[i] + \mathbf{w}_k[i], \qquad i = 1, \ldots, \tau_p. \tag{1}$$

The PS length, τ_p, satisfies $\tau_p \geq n_u$ [3, Sec. III-A].

During data transmission, we assume equal unit power Gaussian codebooks among all UTs, i.e., the transmitted signal of the UTs in the k-th cell, $k \in \mathcal{N}_c$, denoted $\mathbf{x}_k[i] \in \mathcal{C}^{n_u}$, is a zero-mean Gaussian vector with identity covariance. The channel output at the k-th BS is given by

$$\mathbf{y}_k[i] = \sum_{l=1}^{n_c} \mathbf{G}_{k,l}\mathbf{x}_l[i] + \mathbf{w}_k[i], \qquad i = \tau_p + 1, \ldots, \tau_c, \tag{2}$$

where $\{\mathbf{x}_l[i]\}_{l \neq k}$ represents the intercell interference.

Our goal is to characterize the achievable average ergodic rate of noncooperative multi-cell multi-user MIMO systems. Letting $r_{k,m}$ be the achievable ergodic rate of the m-th UT in the k-th cell, the achievable average ergodic rate is defined as

$$r \triangleq \frac{\tau_d}{\tau_c} \cdot \frac{1}{n_c \cdot n_u} \sum_{k=1}^{n_c} \sum_{m=1}^{n_u} r_{k,m}, \tag{3}$$

where the factor $\frac{\tau_d}{\tau_c} = 1 - \frac{\tau_p}{\tau_c}$ follows since only τ_d symbols of each coherence interval are used for data transmission. Specifically, we study the achievable average ergodic rate in the *massive MIMO regime*, namely, when the number of BS antennas, n_t, grows infinitely large while the ratio $\kappa = \frac{n_u}{n_t}$ is kept fixed and finite. As explained in [6, Sec. 3], this analysis provides tight approximations of the SE of practical systems, where both n_t and n_u are large yet finite. This setup is different from that in [3], where only n_t grows arbitrarily large.

Separate Linear Decoding: The standard approach in the massive MIMO literature, e.g., [3]–[8], is to restrict the BSs to separately decode the signal of each UT from some linear transformation of the channel output. We henceforth refer to this approach as *separate linear decoding*. Here, in order to recover the symbol of the m-th UT in the k-th cell, the BS computes the inner product between the received vector $\mathbf{y}_k[i]$ and some linear filter $\mathbf{q}_{k,m} \in \mathcal{C}^{n_t}$, and uses the result to decode only the symbol of the m-th UT. The SE of this approach is given by

$$r^{\mathrm{SEP}} = \frac{\tau_d}{\tau_c} \cdot \frac{1}{n_c \cdot n_u} \sum_{k=1}^{n_c} \sum_{m=1}^{n_u} \mathcal{E}\left\{\log\left(1 + \gamma_{k,m}\right)\right\}, \tag{4}$$

where $\gamma_{k,m}$ is an random variable (RV) representing the signal-to-interference-and-noise ratio (SINR) of the channel relating the m-th UT of the k-th cell and its corresponding BS, $k \in \mathcal{N}_c, m \in \mathcal{N}_u$. The SINR is determined by the filter $\mathbf{q}_{k,m}$, the attenuation coefficients $\{\mathbf{D}_{k,l}\}_{l \in \mathcal{N}_c}$, and the noise power σ_W^2, see, e.g, [6, Sec. III]. The novel aspect of our analysis is that *we allow the BSs to use joint multi-user detection*. While multi-user detection is inherently more complex than separate linear decoding, especially for a large number of UTs, the resulting analysis captures the fundamental properties of massive MIMO systems, and quantifies how much is lost, in terms of SE, due to the restriction to use separate linear decoding. Furthermore, the additional complexity is required only at the BSs, and no additional processing is required at the UTs. Finally, the performance of optimal multi-user detection can be approached at a significantly reduced complexity using deep learning algorithms, as indicated in [19], or using iterative algorithms, whose complexity only grows linearly with the number of UTs, at the cost of increased decoding delay, see [18].

Results from Large Random Matrix Theory: In our study we rely on some existing results from the theory of large random matrices. We first recall the definition of the empirical eigenvalue cumulative distribution function (CDF): For an $n_t \times n_t$ random Hermitian matrix $\mathbf{A}$ with eigenvalues $\{\lambda_i(\mathbf{A})\}_{i=1}^{n_t}$, the (random) empirical CDF of its eigenvalues is given by $F_{\mathbf{A}}^{n_t}(x) = \frac{1}{n_t}\sum_{i=1}^{n_t} 1\{\lambda_i(\mathbf{A}) \leq x\}$, where $1\{\cdot\}$ is the indicator function. Note that $F_{\mathbf{A}}^{n_t}(x)$ is a random function of the scalar x. The following result, obtained from the eigenvalue distribution of large random matrices [22], is used in our analysis:

Theorem 1. [21, Thm. 2.39]: *Let $\mathbf{H} \in \mathcal{C}^{n_t \times n_u}$ be a proper-complex random matrix with i.i.d. entries with zero-mean and unit variance, and let $\mathbf{A} \in \mathcal{C}^{n_u \times n_u}$ be a Hermitian non-negative random matrix, independent of $\mathbf{H}$, whose empirical eigenvalue CDF converges almost surely to the nonrandom CDF of the non-negative scalar RV A. Then, for fixed $\frac{n_u}{n_t} = \kappa$,*

$$\frac{1}{n_t}\log\left|\mathbf{I}_{n_t} + \frac{1}{n_t}\mathbf{H}\mathbf{A}\mathbf{H}^H\right| \xrightarrow{\text{a.s.}} \kappa \cdot \mathcal{E}\left\{\log\left(1 + \eta \cdot A\right)\right\} - \log\eta + (\eta - 1)\log e \triangleq \nu(A, \kappa), \tag{5}$$

where $\xrightarrow{\text{a.s.}}$ denotes almost sure convergence in the limit $n_t \to \infty$, and $\eta \in (0, 1]$ is the solution to $\kappa = \frac{1-\eta}{1-\mathcal{E}\left\{\frac{1}{1+\eta \cdot A}\right\}}$.

When $\mathbf{A}$ is the deterministic matrix $\mathbf{I}_{n_u}$, (5) specializes to the limit in [21, Eq. (1.14)], characterizing the asymptotic capacity of Rayleigh fading PtP MIMO channels. Furthermore, as the left-hand side of (5) is a non-negative real-valued RV, the deterministic function $\nu(A, \kappa)$ is also non-negative real-valued.

III. Achievable Average Ergodic Rates

In order to compute the SEs, we recall that the uplink massive MIMO system is inherently a set of interfering MACs. In particular, in (2), $\mathbf{y}_k[i]$ is the MAC output, the entries of $\mathbf{x}_k[i]$ are the MAC inputs, and $\{\mathbf{x}_l[i]\}_{l\neq k}$ is the interference. Consequently, we consider two common approaches for handling the intercell interference: treating intercell interference as noise, and simultaneous decoding. These schemes determine only how each BS treats the intercell interference when decoding its input, and do not treat how the transmitted data is encoded.

To study these approaches, we first elaborate on the channel estimation phase in Subsection III-A. Then, in Subsections III-B–III-C, we discuss each method and its SE for a finite number of BS antennas and in the massive MIMO regime. The proofs of our results follow the same two steps for each approach: 1) To characterize the SE for a finite number of BS antennas we first divide the received signal into a signal which the BS decodes and an uncorrelated signal which is considered as noise. Then, we obtain the SE using worst-case uncorrelated noise arguments, see, e.g., [13], to obtain an expression for the SE. 2) To characterize the SE in the massive MIMO regime, we prove that the expression for the SE for a finite number of BS antennas satisfies the conditions of Theorem 1. Then, we apply Theorem 1 to explicitly obtain the SE in the massive MIMO regime. Due to page limitations, our results are given without proofs. The detailed proofs are provided in [23].

A. Channel Estimation

As stated in the system model, the first τ_p symbols of each coherence interval are orthogonal PSs used by the BSs to produce the MMSE estimate of their corresponding channel responses. Define the $n_t \times \tau_p$ random matrices $\mathbf{Y}_k \triangleq \big[\mathbf{y}_k[1],\ldots,\mathbf{y}_k[\tau_p]\big]$, $\mathbf{W}_k \triangleq \big[\mathbf{w}_k[1],\ldots,\mathbf{w}_k[\tau_p]\big]$, and the $n_u \times \tau_p$ deterministic matrix $\mathbf{S} \triangleq \big[\mathbf{s}[1],\ldots,\mathbf{s}[\tau_p]\big]$. From (1) it holds that:

$$\mathbf{Y}_k = \sum_{l=1}^{n_c} \mathbf{G}_{k,l}\mathbf{S} + \mathbf{W}_k, \qquad \forall k \in \mathcal{N}_c. \tag{6}$$

Note that $\mathbf{S}\mathbf{S}^H = \tau_p \cdot \mathbf{I}_{n_u}$ as the PSs are orthogonal. Let $\mathbf{M}$ be an $n_t \times n_u$ zero-mean proper-complex Gaussian random matrix with i.i.d. unit variance entries, and define the RVs

$$B_{k,l,m} \triangleq \frac{\tau_p D^2_{k,l,m}}{\sigma_W^2 + \tau_p \sum\limits_{l'=1}^{n_c} D^2_{k,l',m}}, \qquad k,l \in \mathcal{N}_c, m \in \mathcal{N}_u, \tag{7}$$

and the $n_u \times n_u$ diagonal matrices $\{\mathbf{B}_{k,l}\}_{k,l\in\mathcal{N}_c}$ with diagonal entries $\{B_{k,l,m}\}_{m=1}^{n_u}$. The MMSE channel estimate and its statistical characterization are stated in the following lemma:

Lemma 1. *The MMSE estimate of $\mathbf{G}_{k,l}$ from $\mathbf{Y}_k$ and $\{\mathbf{D}_{k,l}\}_{l\in\mathcal{N}_c}$ is given by*

$$\hat{\mathbf{G}}_{k,l} = \tau_p^{-1}\mathbf{Y}_k\mathbf{S}^H\mathbf{B}_{k,l}. \tag{8}$$

Furthermore, the MMSE estimate $\hat{\mathbf{G}}_{k,l}$ is distributed as $\hat{\mathbf{G}}_{k,l} \stackrel{d}{=} \mathbf{M}\mathbf{B}_{k,l}^{1/2}\mathbf{D}_{k,l}$ and its estimation error $\tilde{\mathbf{G}}_{k,l} \triangleq \mathbf{G}_{k,l} - \hat{\mathbf{G}}_{k,l}$ is distributed as $\tilde{\mathbf{G}}_{k,l} \stackrel{d}{=} \mathbf{M}\left(\mathbf{I}_{n_u} - \mathbf{B}_{k,l}\right)^{1/2}\mathbf{D}_{k,l}$.

B. Decoding Scheme 1 - Interference as Noise

We first study the SE when each BS treats the intercell interference as noise. Here, the k-th BS jointly decodes the signals of the UTs associated with the k-th cell, $\mathbf{x}_k[i]$, and treats the signals transmitted by all other UTs, $\{\mathbf{x}_l[i]\}_{l\neq k}$, as noise. The main difference between this decoding scheme and previous massive MIMO works, e.g., [3]–[8], which also assumed that the BSs treat intercell interference as noise, is that these works restricted each BS to decode the signals transmitted from each of its UTs *separately*, thus the channel from the UTs to the BS is treated as a set of PtP channels. By allowing the BSs to *jointly* decode the signals of their UTs, we exploit the fact that the channel from the UTs to their associated BS is a MAC.

Using the MMSE channel estimate $\hat{\mathbf{G}}_{k,k}$ and its estimation error $\tilde{\mathbf{G}}_{k,k}$, the received signal (2) can be written as

$$\mathbf{y}_k[i] = \hat{\mathbf{G}}_{k,k}\mathbf{x}_k[i] + \tilde{\mathbf{G}}_{k,k}\mathbf{x}_k[i] + \sum_{l=1,l\neq k}^{n_c} \mathbf{G}_{k,l}\mathbf{x}_l[i] + \mathbf{w}_k[i]. \tag{9}$$

By treating interference as noise, the equivalent noise signal is defined as $\mathbf{v}_k^{\text{IAN}}[i] \triangleq \tilde{\mathbf{G}}_{k,k}\mathbf{x}_k[i] + \sum\limits_{l=1,l\neq k}^{n_c} \mathbf{G}_{k,l}\mathbf{x}_l[i] + \mathbf{w}_k[i]$, and the received signal can be written as

$$\mathbf{y}_k[i] = \hat{\mathbf{G}}_{k,k}\mathbf{x}_k[i] + \mathbf{v}_k^{\text{IAN}}[i], \quad i = \tau_p+1,\ldots,\tau_c. \tag{10}$$

To formulate the SE of (10), define the RV

$$T_k \triangleq \sum_{l=1}^{n_c} \text{trace}\big((\mathbf{I}_{n_u} - \mathbf{B}_{k,l})\mathbf{D}^2_{k,l}\big) + \sigma_W^2, \tag{11}$$

and the $n_u \times n_u$ random diagonal matrices

$$\mathbf{Q}_k^{\text{Net}} \triangleq T_k^{-1}\mathbf{B}_{k,k}\mathbf{D}_{k,k}^{-2}\sum_{l=1}^{n_c}\mathbf{D}^4_{k,l}; \tag{12a}$$

and

$$\mathbf{Q}_k^{\text{Int}} \triangleq T_k^{-1}\mathbf{B}_{k,k}\mathbf{D}_{k,k}^{-2}\sum_{l=1,l\neq k}^{n_c}\mathbf{D}^4_{k,l}. \tag{12b}$$

The SE for finite n_t is stated in the following proposition:

Proposition 1. *When the BSs treat intercell interference as noise, the following average ergodic rate is achievable:*

$$r_{n_t}^{\text{IAN}} = \frac{\tau_d}{\tau_c}\cdot\frac{1}{n_c \cdot n_u}\sum_{k=1}^{n_c}\Big(\mathcal{E}\left\{\log\left|\mathbf{I}_{n_t} + \mathbf{M}\mathbf{Q}_k^{\text{Net}}\mathbf{M}^H\right|\right\} - \mathcal{E}\left\{\log\left|\mathbf{I}_{n_t} + \mathbf{M}\mathbf{Q}_k^{\text{Int}}\mathbf{M}^H\right|\right\}\Big), \tag{13}$$

where the expectations are carried out with respect to the random matrices $\mathbf{M}$ and $\{\mathbf{Q}_k^{\text{Net}}, \mathbf{Q}_k^{\text{Int}}\}_{k\in\mathcal{N}_c}$.

Next, we use Proposition 1 to characterize the SE in the massive MIMO regime. To that aim, define the following RVs

$$A_k^{\text{Net}} \triangleq \frac{B_{k,k,1}D_{k,k,1}^{-2}\sum\limits_{l=1}^{n_c}D^4_{k,l,1}}{\kappa\sum\limits_{l=1}^{n_c}\mathcal{E}\{(1 - B_{k,l,1})\,D^2_{k,l,1}\}}; \tag{14a}$$

and

$$A_k^{\text{Int}} \triangleq \frac{B_{k,k,1}D_{k,k,1}^{-2}\sum\limits_{l=1,l\neq k}^{n_c}D^4_{k,l,1}}{\kappa\sum\limits_{l=1}^{n_c}\mathcal{E}\{(1 - B_{k,l,1})\,D^2_{k,l,1}\}}, \tag{14b}$$

for $k \in \mathcal{N}_c$. Letting $n_t \to \infty$ in (13) while fixing $\frac{n_u}{n_t} = \kappa$, we obtain the asymptotic SE, stated in the following theorem:

Theorem 2. *In the massive MIMO regime, the following average ergodic rate is achievable when treating intercell interference as noise:*

$$r^{\text{IAN}} \triangleq \lim_{\substack{n_t\to\infty\\ \frac{n_u}{n_t}=\kappa}} r_{n_t}^{\text{IAN}} = \frac{\tau_d}{\tau_c}\cdot\frac{1}{n_c\cdot\kappa}\sum_{k=1}^{n_c}\nu\left(A_k^{\text{Net}},\kappa\right) - \nu\left(A_k^{\text{Int}},\kappa\right). \tag{15}$$

Proposition 1 is proved by assuming that $\mathbf{v}_k^{\text{IAN}}$ is Gaussian. In standard separate linear decoding, this equivalent noise is also assumed to be Gaussian, and the SE in (4) is computed assuming that the decoder filters the received signal in (9) and decodes each entry separately. Thus, the SE of the standard approach is upper bounded by the SE in (13) and (15). In Section IV we demonstrate that treating intercell interference as noise is most beneficial when this interference is weak, in agreement with the theory of two-user Gaussian ICs [10, Ch. 6.4.3].

C. Decoding Scheme 2 - Simultaneous Decoding

The opposite approach to treating interference as noise is to decode the intercell interference. Here, each BS now jointly decodes the signals transmitted by all UTs in the network. This approach is optimal in the two-user Gaussian IC with strong interference [10, Ch. 6.4.2], and thus we expect it to achieve the best performance in networks where many UTs are not allocated to the BSs with best connectivity (a scenario which is not uncommon in wireless networks [9]). Thus, while this approach is more computationally complex than treating interference as noise, deriving its SE gives an indication of the fundamental limits of networks with strong intercell interference, which cannot be obtained by treating interference as noise.

From (7) and (8), it follows that $\hat{\mathbf{G}}_{k,l} = \hat{\mathbf{G}}_{k,k}\mathbf{D}_{k,k}^{-2}\mathbf{D}_{k,l}^{2}$. Thus, given $\{\mathbf{D}_{k,l}\}_{l\in\mathcal{N}_c}$, obtaining the MMSE estimate of all cross-cell channels, $\{\hat{\mathbf{G}}_{k,l}\}_{l\in\mathcal{N}_c}$, is equivalent to obtaining only $\hat{\mathbf{G}}_{k,k}$, and no additional pilots are required. The received signal at the k-th BS (2) can be written as

$$\mathbf{y}_k[i] = \sum_{l=1}^{n_c}\hat{\mathbf{G}}_{k,l}\mathbf{x}_l[i] + \sum_{l=1}^{n_c}\tilde{\mathbf{G}}_{k,l}\mathbf{x}_l[i] + \mathbf{w}_k[i]. \quad (16)$$

When decoding the intercell interference along with the data, the equivalent noise is $\mathbf{v}_k^{\rm SD}[i] \triangleq \sum_{l=1}^{n_c}\tilde{\mathbf{G}}_{k,l}\mathbf{x}_l[i] + \mathbf{w}_k[i]$, and the received signal can be written as

$$\mathbf{y}_k[i]=\hat{\mathbf{G}}_{k,k}\mathbf{D}_{k,k}^{-2}\sum_{l=1}^{n_c}\mathbf{D}_{k,l}^{2}\mathbf{x}_l[i]+\mathbf{v}_k^{\rm SD}[i], \quad i=\tau_p+1,\ldots,\tau_c. \quad (17)$$

The SE for finite n_t is stated in the following proposition:

Proposition 2. *When each BS decodes the intercell interference, the following average ergodic rate is achievable:*

$$r_{n_t}^{\rm SD} = \frac{\tau_d}{\tau_c}\cdot\frac{1}{n_c\cdot n_u}\min_{k\in\mathcal{N}_c}\left(\mathcal{E}\left\{\log\left|\mathbf{I}_{n_t}+\mathbf{M}\mathbf{Q}_k^{\rm Net}\mathbf{M}^H\right|\right\}\right), \quad (18)$$

where the expectations are carried out with respect to the random matrices $\mathbf{M}$ and $\{\mathbf{Q}_k^{\rm Net}\}_{k\in\mathcal{N}_c}$.

Next, we use Proposition 2 to characterize the SE in the massive MIMO regime. Letting $n_t \to \infty$ in (18) while fixing $\frac{n_u}{n_t} = \kappa$, we obtain the achievable average ergodic rate in the massive MIMO regime, stated in the following theorem:

Theorem 3. *In the massive MIMO regime, the following average ergodic rate is achievable when the BSs decode the intercell interference:*

$$r^{\rm SD} \triangleq \lim_{\substack{n_t\to\infty\\ \frac{n_u}{n_t}=\kappa}} r_{n_t}^{\rm SD} = \frac{\tau_d}{\tau_c}\cdot\frac{1}{n_c\cdot\kappa}\min_{k\in\mathcal{N}_c}\nu\left(A_k^{\rm Net},\kappa\right). \quad (19)$$

The minimization over k in (18)-(19) follows since each BS decodes the signals of all the UTs in the network. In Section IV we demonstrate that simultaneous decoding is most beneficial when the intercell interference is dominant, in agreement with its optimality for two-user Gaussian ICs [10, Ch. 6.4.2].

IV. Numerical Results and Discussion

We now evaluate the SEs of the schemes discussed in Section III in a simulations study. We consider a network consisting of $n_c = 5$ cells. The coherence duration is $\tau_c = 1000$ symbols. The attenuation coefficients are generated as $D_{k,l,m} = \frac{Z_{k,l,m}}{C_{k,l,m}^2}$, where $\{Z_{k,l,m}\}$ are the shadow fading coefficients, independently randomized from a log-normal distribution with standard deviation of 8 dB, and $\{C_{k,l,m}\}$ represent the range between the m-th UT of the l-th cell and the k-th BS [3, Sec. II-C]. Let $((\cdot))_{n_c}$ be the modulo n_c operator, and $\{U_{k,l,m}\}$ be i.i.d. RVs uniformly distributed over $[1,2]$. To capture various interference profiles, we use two setting of $C_{k,l,m}$: 1) $C_{k,l,m}^2 = e^{3((k-l))_{n_c}}\cdot U_{k,l,m}$, referred to as *weak interference*. 2) $C_{k,l,m}^2 = e^{-1((k-l))_{n_c}}\cdot U_{k,l,m}$, referred to as *strong interference*. Expectations are averaged over 2000 Monte Carlo simulations. The asymptotic SEs of our schemes are compared to the corresponding rates achievable using separate decoding with matched filtering and with MMSE filtering, computed via (4), where the SINR is computed using [6, Thm. 3]. Here, the number of BS antennas is $n_t = 800$, and the number of UTs in each cell is $n_u = 80$. We fix the number of symbols used for channel estimation to $\tau_p = 100$, and let the signal-to-noise ratio (SNR), $1/\sigma_W^2$, vary between $[-30, 30]$ dB.

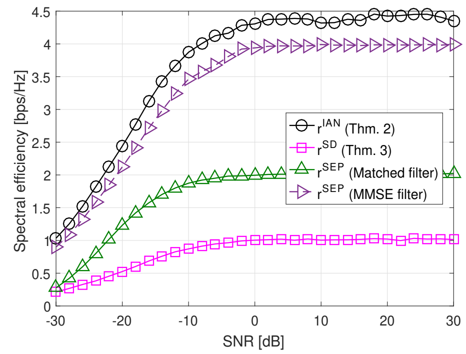

Fig. 1. SE vs. SNR, weak interference.

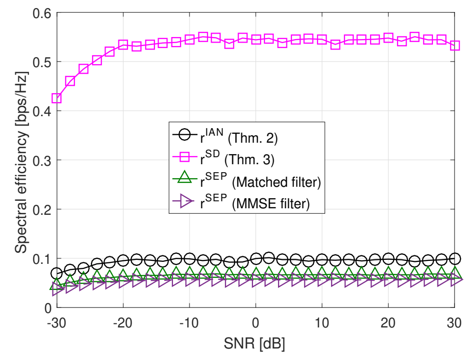

Fig. 2. SE vs. SNR, strong interference.

The results for the weak interference and strong interference settings the are depicted in Figs. 1-2, respectively. We observe in Fig. 1 that in the weak interference setting, although both the rates of Theorem 2 and [6, Thm. 3] are computed assuming that intercell interference is treated as noise, the SEs of Theorem 2 are higher, with gains of 2.2 bps/Hz and 0.45 bps/Hz compared to matched filtering and MMSE filtering, respectively, at high SNRs, indicating that the SE of massive MIMO networks can be improved by allowing the BSs to perform joint decoding. Note that an SE gain of 0.45 bps/Hz is translated into an overall SE gain of over 36 bps/Hz in a cell with $n_u = 80$ UTs. In the strong interference scenario, we observe in Fig. 2 that the optimized scheme as well as simultaneous decoding achieve an average ergodic rate of 0.55 bps/Hz, while separate decoding results in negligible achievable rates. Thus, the fundamental limits of such channels are much higher than those achieved using standard separate linear decoding and treating interference as noise. We note that for all schemes, the achievable rates hardly vary with SNR at high SNRs, settling with the observation in [3, Sec. IV].

These results demonstrate the SE benefits of properly acknowledging the nature of massive MIMO systems as interfering MACs. Furthermore, our results indicate the fundamental limits of massive MIMO systems, and how far the conventional approach is from capturing these characteristics.

V. Conclusions

In this paper we studied the SE of uplink massive MIMO systems allowing the BSs to jointly decode the received signals. We characterized the SEs of two schemes for handling the intercell interference. Simulation results demonstrate the gains obtained by our schemes, and indicate that when the interference is dominant, standard separate linear decoding fails to capture the fundamental limits of massive MIMO systems.

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